INTERMEDIATE
&
HIGHER SECONDARY
PHYSICS

BASU & CHATTERJEE

HIGHER SECONDARY & INTERMEDIATE PHYSICS

[Also for Central Modern, Pre-Engineering, Pre-Professional and Higher Secondary Courses of Studies]

BY

NAGENDRA NATH BASU, M. Sc.,
Late Professor of Physics
Ripon College, Calcutta.

AND

JYOTIRMOY CHATTERJEE, M. Sc.,
Emeritus Professor of Physics
Scottish Church College, Calcutta.
Also
Honorary Professor of Physics
Ramkrishna Vidyamandir, Belur.
and sometime Professor of Physics
Maharaja Manindra Chandra College

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PREFACE TO THE FIRST EDITION

The book is written according to the Intermediate Syllabi of Physics of different Indian Universities. The book has a special recommendation, inasmuch as it is neither too bulky for a student of average merit, nor too meagre in its information for an inquisitive student.

The subject matter herein has been treated in a systematic way and in a simple and clear style with ample illustrations and explanatory diagrams. A sufficient number of experiments, the end of each Chapter and Exercises containing Questions of different Universities with direct reference to the Articles form special features of the book. Numerous problems of different types put herein will make a student independent of any separate book on Examples. Articles marked with asterisks deal with more advanced portions, and may, therefore, be omitted at the first reading·

We take this opportunity to express our gratitude to Prof. Rajanikanta De, M.A., of the Scottish Church College, Calcutta, who has kindly given us permission to use some of the blocks prepared for his well-known books on Physics.

Calcutta
June, 25, 1936

N. BASU
J. CHATTERJEE

PREFACE TO THE SILVER JUBILEE EDITION

In this Edition special attention has been given to include in its context the courses of studies prescribed by the different provincial Higher Secondary Boards as also the syllabus of the Central Modern Schools guided by the Education Department of the Government of Indian Republic. I had an opportunity of going through some books recommended in various Engineering Schools and Military Academies and I have attempted to include in this Book almost all the topies in those syllabi.

Although teaching in provincial languages has deepened its root almost every where and books in different provincial languages with slightly different syllabi have been published catering the need of the students, it has one advantage that the students can more easily grasp the idea without having any difficulty of a foreign language. But the authors of books on regional languages have been content just to include the particular courses of studies recommended in their respective provinces. As a result they havc not been able to treat the whole subject systematically in such a small compass to the entire satisfaction of teachers as also to arouse the interest of the students. I should not be misunderstood to have blamed the authors. They have been compelled to write strictly according to syllabus and within page limit.

In conclusion, I may make a modest statement that in view of higher studies particularly in Sciences, we have still to depend upon new books and periodicals written in western languages and if we have to choose any one of such languages, it is definitely English. With such intent and purpose I have thoroughly revised the Book, included some new topics as Vectors, Kinetic theory of gases, Physical optics and a little of nuclear Physics. I have tried to correct printing mistakes to the best of my ability and included a few more solved examples.

J. CHATTERJEE

PREFACE TO THE REPRINTED SILVER JUBILEE EDITION

The number of Editions are the passing phases of a book which may speak of itself. I should feel obliged to get any suggestion and criticism of my venture. Hearty thanks to inmmerable Colleagues and students who have always partonised my efforts. I convery my best regards to B. Chand & Sons for their co-operation in publishing this book.

Sree Panchami
February 3, 1987

J. CHATTERJEE

PART I—General Physics, Heat and Sound.
PART II—Light, Magnetism and Electricity.
Recommended for Intermediate,
Central Modern, Pre-Engineering
and
Higher Secondary Courses
of Indian Universities and various
Boards of Studies

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GENERAL PHYSICS

CHAPTER I

UNITS AND MEASUREMENTS

1. **Knowledge got through Sense Organs**—Senses of touch, smell, sight, taste and sound are the chief sources through which we gain experiences of things around us. On looking at an object, we get an idea of its size, shape and colour ; and so the eye is the agent to give some idea of the object. We can feel with our fingers whether an object is hard or soft, hot or cold, and such experiences as these we get by our sense of touch. In a similar way, the use of various organs of our system gives us some knowledge regarding the nature and behaviour of the material objects.

In childhood our senses remain undeveloped. As we advance in years, senses become more and more developed giving us a deeper insight of things and events. Thus, with time passing on, the experiences gained by us are utilised in making further studies of material objects and their behaviour.

2. **Natural Sciences**—The word Science means a knowledge acquired after a careful observation and reasoning. In earlier days Physical Sciences comprised the study of the whole of natural phenomena and were therefore termed **Natural Philosophy.** It thus included subjects like *Physics*, *Astronomy*, *Zoology*, *Botany*, *Chemistry*, *Geology*, etc. But with time advancing as these subjects grew more extensive, Physical Sciences were subdivided and each one became a separate subject for study. The branch of Physics now deals with a detailed study of the properties of *Matter* and *Energy* and their interactions with each other.

Matter is that which occupies some space and which we can feel by our senses. It may exist in any of the three forms,—solid, liquid and gaseous. Thus a piece of iron, a quantity of water or air is each an example of matter. **Energy** is that, which an agent or a material system must possess, in order to be able to do some work. Under suitable conditions work can be obtained from energy,—energy being the cause and work being the effect. When a material body moves, we can utilise its motion to do some work ; for example, work is done in the transport of luggages by a motor car. Hence, a moving car possesses energy. More about work and energy would be discussed in Chapter V of General Physics of this volume.

Subdivisions of Physics—There are six principal forms in which energy becomes apparent to us; namely, *elasticity* and *gravitation, heat, light, sound, magnetism* and *electricity*. These are separately studied in the following six branches of Physics :—General Physics [including gravitation and properties of matter], Heat, Light, Sound, Magnetism and Electricity.

3. **Measurements and their utility**—Although by direct observation of sight, touch, etc., we may estimate some property of material bodies, such an estimate is rough or even sometimes erroneous. For example everybody can say that an average football is bigger than a cricket ball ; but it may be difficult to say all at once which of the two is bigger,—a hockey ball or a cricket ball. The sun and the moon appear to be nearly of same size, although we know that the former is many thousand times larger than the latter. A range of hills sometimes seems to an observer to be within half-an-hour's walk, but actually it may be far beyond that. In Fig. 1, two straight lines are perpendicular to each other. Which appears longer to you ? Most probably the vertical line appears longer. But on measurement you will find that they are equal in length. The horizontal lines in Fig. 2 may not appear parallel to you ; but they are actually parallel lines. The quadrilateral in Fig. 3 appears to have the left base corner angle slightly acute. Place a set square and see that it is a right angle. These illustrations suggest that without the help of the measuring instruments, we are liable to make mistakes in our estimations.

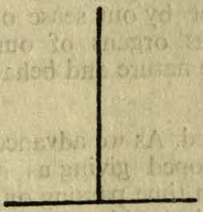


Fig. 1

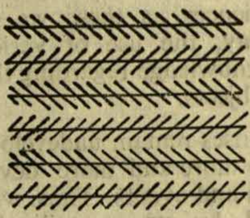


Fig. 2

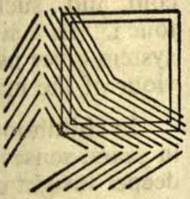


Fig. 3.

4. **Units of Measurements**—We acquire knowledge about natural laws through measurement of certain quantities. These quantities are mainly *length*, *mass* and *time*. In measuring any of these quantities, some definite and convenient quantity of the *same kind* is taken as the standard, in terms of which the quantity as a whole is estimated. This is called the **unit** for that particular quantity. The *number* of times the unit is contained in the quantity is called the *numerical measure* of that quantity. Thus when we say that a rod is 3 feet long, we mean that a certain length, called the foot, has been taken as the unit of measurement and the number of this unit contained in the length

of the rod is 3. Hence the numerical measure is 3 and the unit of length is a *foot*. The mass is a measure of the *quantity of matter* in a body.

Fundamental and Derived Units—Different units are to be chosen for measuring different kinds of physical quantities and so there would be as many kinds of units as there are types of physical quantites. It has been found that the units of *Length*, *Mass* and *Time* are found to be independent of each other and are, therefore, called *fundamental units*. The units of all other quantities can be expressed in terms of the fundamental units and hence they are called *derived units*. Thus a *unit of area* is the area of a square, each side of which is of unit length ; a *unit volume* is the volume of a cube, each side being of unit length. So the unit of area and that of volume are *derived* from the units of length. It is found that the derived units bear a simple relation to the fundamental units.

Systems of Units—In all measurements, two systems of units are generally used, namely (1) the British or the **foot-pound-second** system (F. P. S. system), and (2) the French or *metric* or the **centimetre-gramme-second** system (C. G. S. system). The units of length and mass on the *British system* are respectively the **foot** and the **pound**, and on the *metric system* they are respectively the **centimetre** and the **gramme**. The unit of time is the **second** in both the systems. There is a modified metric system, known as M. K. S. (metre-kilogram-second), which is now gaining popularity.

5. Units of Length—The standard unit of length in the F. P. S. system is the *British Imperial Yard*. The yard is defined by an Act of British Parliament as the straight line or the distance between the centres of two transverse lines on two gold plugs on a bronze bar kept at a temperature of 62°F. This standard is kept at the Office of the Board of Trade, England. The usual unit of length on this system is, however, a **foot**. The following are the multiples and submultiples of a foot.—

12″ (inches)=1′ (foot or ft.) ; 220 yd.=1 furlong ;
3 feet =1 yard (yd.) ; 1760 yd.=1 mile ;
6 feet =1 fathom ; 6080 ft.=1 knot.

The standard unit of length on the C. G. S. system, called the International Prototype Metre, is now conveniently defined as the distance between the ends of a certain rod of platinum-iridium preserved at the International Bureau of Weights and Measures at Sev′res near Paris, at a temperature of 0°C. The usual unit of length on this system is the **centimetre**.

The following are the submultiples and multiples of a *metre* :—

10 millimetres (mm.) = 1 centimetre (cm.) ;
10 centimetres (cm.) = 1 decimetre (dm.) ;
10 decimetres (dm.) = 1 metre (m.) ;
1000 metres (m.) = 1 kilometre (km.).

Relation between the units of length on the two systems :—

1 inch = 2·54 cm.
1 metre = 39·37 in.
1 kilometre = 0·6213 mile.
100 metres = 109·36 yards.

Units of Area and Volume are derived form the units of length. The unit of area on the F. P. S. system is one *square foot* and that on the C. G. S. system is one *square centimetre*. The **volume** of a body is the space occupied by it. The unit of volume on the F. P. S. system is a cubic foot (cu. ft.) and that on the C. G. S. system it is one cubic centimetre (c.c.). As measures of capacity for liquids the units of volume on the two systems are respectively a *gallon* and a *litre*. **A gallon** equal to the volume of 10 lb. (avoir.) of distilled water at 62°F, while one **litre** is equal in volume to 1000 cubic centimetres of pure water at 4°C. 1 gallon equals 4·54 litres. One cubic centimetre is sometimes called a milli-litre.

6. Units of Mass—The standard unit of mass on the F. P. S. system is the **Pound Avoirdupois**. It is defined to be the mass of a certain lump of platinum marked "P.S. 1844, 1 lb." and is preserved at the Office of the Board of Trade, England.

The standard unit of mass on the C. G. S. system is the **kilogramme.** It is now defined to be the mass of certain cylinder of platinum-iridium, preserved in the Archives at International Bureau of Weights and Measures at Sev'res near Paris. The unit of mass usually adopted is **gramme** (or **gram**) which is one-thousandth part of the standard kilogramme. For practical purposes, one gram may be taken to be equal to the mass of one cubic centimetre of pure water at 4°C.

Submulitples and multiples of a Pound :—

16 drams = 1 ounce (oz.) ; 16 ounces = 1 Pound (lb.) ;
28 pounds = 1 quarter (qr.) ; 4 quarters = 1 hundred weight (cwt.) ;
20 cwt = 1 Ton = 2240 lbs.

Submultiples and multiples of a gramme :—

1 gram (gm.) = 100 centigrams = 1000 milligrams (mg.) ;
1 kilogram (kg.) = 1000 grams. 100 kg. = 1 Quintal ;
10 Quintals = 1 Metric Tonne.

Relation between the units of mass on the two systems :—

1 grain = 0·0648 gram ; 1 gram = 15·432 grains ;
1 pound = 453·56 grams ; 1 kilogram = 2·2047 pounds ;
1 Metric Tonne=2204·7 lbs=0·98 Ton.
1 Indian Tola=12 grams=185·18 grains.

Examples :

1. The Nanda Devi, the highest mountain peak in Indian Republic, is 25,660 ft. in height. Express the altitude in miles and kilometres.

Ans. Since 1760 yds.=5280 ft.=1 mile. $\therefore$ $1 \text{ ft.} = \frac{1}{5280} \text{ mile.}$

$$\therefore\ 25660 \text{ ft.} = \frac{25660}{5280} \text{ miles} = 4{\cdot}8598 \text{ miles.}$$

Again, since 39·37 in.=3·2808 ft.=1 metre, $\therefore$ $1 \text{ ft.} = \frac{1}{3{\cdot}2808} \text{ metre.}$

$$\therefore\ 25660 \text{ ft.} = \frac{25660}{3{\cdot}2808} \text{ m.} = 7821{\cdot}3 \text{ m.} = 7{\cdot}8213 \text{ km.}$$

2. Madhya Pradesh is the largest State of India with an area of 131,686 sq, miles. Express the figure in sq. kilometres.

Ans. Since 1 inch=2·54 cm., 36 in=1 yd.=91·44 cm.=0·9144 metre and so 1760 yd.=1 mile=(1760 × ·9144) m.=1609·3 m=1·6093 km.

Thus 1 sq. mile=1 mile × 1 mile=(1·6093 × 1·6093) sq. km.=2·5898 sq. km.

$\therefore$ 131686 sq. miles=(131686 × 2·5898) sq. km.=341040 sq. km.

3. The capacity of a certain flask is found to be 10 litres. What would be its capacity in gallons ?

Ans. We have 4·54 litres=1 gallon, or $1 \text{ litre} = \frac{1}{4{\cdot}54} \text{ gallon.}$

$$\therefore\ 10 \text{ litres} = \frac{10}{4{\cdot}54} \text{ gallons} = 2{\cdot}2 \text{ gallons.}$$

7. Unit of Time—Time is an entity by which we differentiate between past, present and future. The time interval between the sun-rise of yesterday and that of today is called one day. The length of such solar days varies continuously throughout a year which consists of 365¼ days. If the periods of the solar days in a year be added together and the sum be divided by the total number of days in a year, we obtain an interval of time, which is called the **mean solar day**. The mean solar day is divided into 24 *hours*, each hour into 60 *minutes* and each minute into 60 *seconds.* The unit of time on both the systems is the *mean solar second*, which is thus 86,400th part of a mean solar day. Our clocks, watches and chronometers keep mean solar time. Accurate Astronomical observations have proved that the period of solar year is slightly changing year to year by a small fraction of a second. So the tropical year 1900 was taken as the standard year from which the mean solar second was derived.

The interval of time, that elapses between two successive transits of any fixed star across the meridian of a place, is found to be a constant and is called the **sidereal day**. The sidereal day is about four minutes shorter than the mean solar day.

8. Measurement of Length—For practical purposes, distances from several thousand miles to a very small fraction of a centimetres are to be measured accurately. A particular type of measuring instrument is suitable and convenient for any particular range of length. Only the general principle of the working of the instruments will be dealt with in this book. (For experimental details of the various apparatus, the readers are referred to the *Intermediate Practical Physics* by J. Chatterjee). The different appliances, generally used for the measurement of length, are the following :

(i) **Metre Scale**—It is usually made of box-wood or steel with one edge generally graduated in inches and the other in centimetres (Fig. 4). The graduations on the upper side are in inches and those on the lower side are in centimetres. The inch scale is as shown, further subdivided into 10 equal parts so that the distance between any two consecutive small marks on the upper side is 0·1 inch. Some inch scales are subdivided into $\frac{1}{8}$ or $\frac{1}{10}$ inch. The centimetre scale also is divided into 10 equal parts, giving each small division in millimetre.

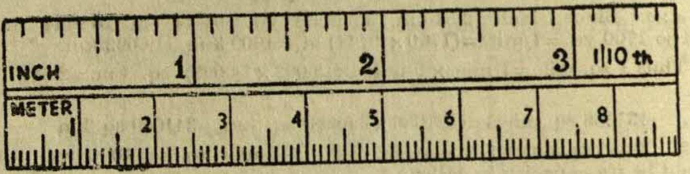
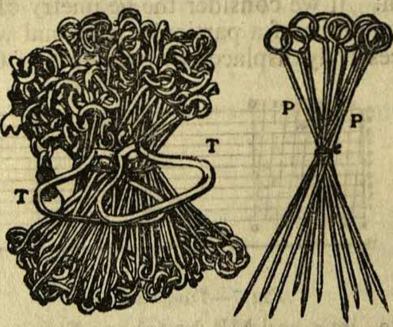


Fig. 4—Metre Scale

In measuring the length of an object, the scale is placed directly alongside the object, so that one of the graduations coincides exactly with one end of the object and the length is then read off from the graduation of the scale coinciding with the other end. If this end does not coincide with any one division, the fraction of a scale division is ascertained by eye estimation. In measuring a length, the ends of the metre scale should be avoided as far as possible, since they wear out and bocome rounded with continued handling.

(ii) **Measuring Chain**—It is used for longer distances or for measuring distance along curved lines. A chain consists of inter-connected steel rods, each one foot long, so as to make a total length of 60 to 100 ft. When not in use, a chain can be folded and bunched [Fig. 5(*a*)]. To measure a distance the starting end of the chain is fixed to a pin P [shown in a bundle in Fig. 5(*b*)] and the chain is made straight, so that it does not bend or sag anywhere along its length. The other end is fixed to another pin P, so that distance between these two points is equal to the length of the chain. In this way, the entire distance is measured chain after chain. It is widely used is surveying lands.



(*a*) (*b*)

Fig. 5 – Measuring Chain

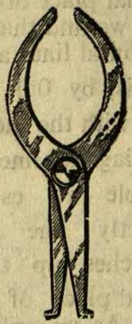


Fig. 6 – Simple Callipers

(iii) Simple Callipers—It consists of two curved pieces of metal, hinged like a pair of scissors (Fig. 6). It can be used for measuring the internal and the external diameters of pipes, hollow vessels, etc., the upper pair of legs being designed for external diameter and the lower pair for internal diameter.

(iv) Diagonal Scale—The accuracy of reading a fraction of a millimetre or an inch is very much hampered by eye-estimation, since different persons might guess different fractions for the same length. To reduce this uncertainty, modified centimetre or inch scales, known as diagonal scales, may be used.

Fig. 7 represents an inch diagonal scale engraved on a rectangular metal sheet. The usual inch graduation being from the zero of the bottom line to the right. The width FW of the scale is divided into 10 equal parts by horizontal lines. There is also an inch extension to the left of the zero line subdivided into 10 equal parts at the top and the bottom. As shown in the diagram, the subdivisions are connected by a series of parallel oblique straight lines. These straight lines are generally known as diagonal lines whence the name of the scale is derived.

Each subdivision of the extension inch evidently represents 0·1 inch. So lengths correct to first place of decimal can be directly read from the scale. Moreover, the device of the diagonal lines enables us to find out the length correct to the second decimal place of an inch. If we consider the geometry of these lines, we find that the junctions of a particular diagonal with the horizontal lines are successively displaced by one-tenth of 0·1 inch that is, by 0·01 inch. Thus with the help of this diagonal lines, it is possible to estimate correctly the length in inches up to the second place of decimals. As an an example, let a rod MR be taken. From a preliminary measurement, it is found to be a bit longer than 2″. To find the length, its one end is made to coincide with the 2-inch graduation of the scale and the other end is allowed to extend beyond the zero mark. It is actually found to extend even slightly beyond the third subdivision of the extension inch. So its actual length is a bit more than 2·3″. To get the second place of decimals, MR is bodily slided parallel to the side of the scale, always keeping one end on the 2-inch graduation line. Now, the position of coincidence of the other end with a particular junction of the fourth diagonal lines is noted. It is actually the 8th junction from the bottom. As each junction corresponds to a displacement of 0·01 inch, the fractional part in the second decimal place is 8×0·01″ = 0·08″. So the total length of MR is 2·38″ correct to second decimal place. Similarly, ST represents a length of 1·13″. Thus the uncertainties of eye-estimation are almost eliminated.

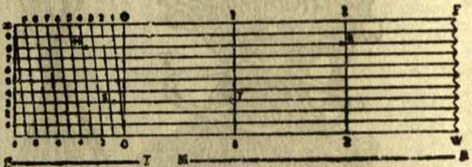


Fig 7—Diagonal Scale.

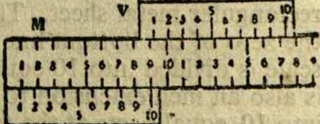


Fig. 8—Vernier Scale.

The Vernier—This is so named after its inventor Pierre Vernier, a seventeenth-century French mathematician. It consists of a short auxiliary scale V called the *vernier scale*, which can slide along the edge of the main scale M (Fig. 8) whereby lengths can be determined correct to some particular fraction of the unit on the main scale. The graduations on the main and the vernier scales are so designed that a certain number of divisions of the vernier scale say n are made coincident with $n-1$ or $n+1$ number of divisions of the main scale. The vernier is then capable of being read with an accuracy of $1/n$ of the main scale division. In Fig. 8, 10 divisions of the vernier scale are equal to 9 divisions of the main scale. The smallest measurable distance with such a scale is called the *vernier constant*. The ordinary vernier scales are designed to measure up to one-tenth part of a millimetre. To *measure small angles* circular verniers also are constructed on the same principle.

(v) **Vernier Callipers**—The principle of vernier scale is utilised in a *slide callipers*, also called a *vernier callipers.* It consists of a thin steel scale with a jaw fixed at one end at right angles to its length (Fig. 9), while the other jaw is movable along the scale and can be fixed to any position by a screw. The movable jaw carries two vernier scales, one on each side, sliding over the graduations on the steel bar. When the two jaws are brought in contact, the zero of the vernier should ordinarily coincide with the zero of the main scale, so that when the jaws are separated, the distance between them is directly read from the two scales. The object, whose length is to be measured, is placed between the jaws, with one end against the fixed jaw, while the movable jaw is brought just in contact with the other end. The reading is taken in the usual way from the main and the vernier scales. If the vernier zero stands after the mth division of the main scale and if the rth division of the vernier is found to be *in line* with a main scale graduation, the length of the object is $(m+rv)$ in proper units, where v represets the vernier constant. Vernier callipers may be used to measure a length correct to one-tenth or one-fiftieth part of a millimetre.

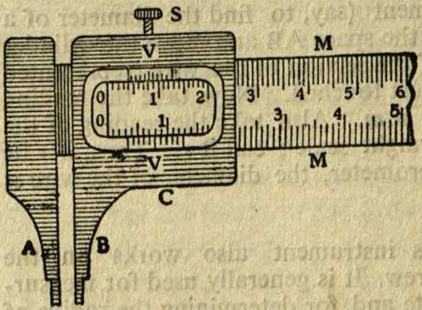


Fig. 9—Vernier Callipers

(vi) Micrometer Screw Gauge—It is used for measuring accurately very short lengths. It consists of a U-shaped body F of

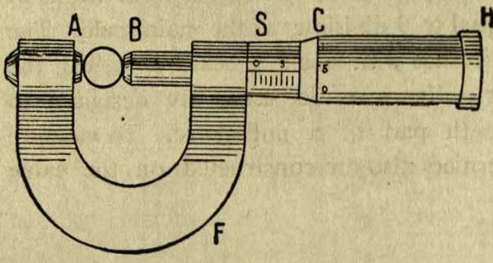


Fig. 10—Screw Gauge

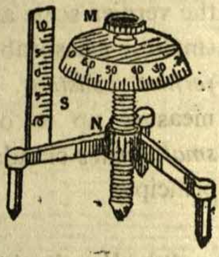


Fig. 11—Spherometer

solid metal, one arm of which carries a fixed abutment A (Fig. 10) with a carefully planed surface. To the other arm is attached a hollow cylinder having a straight scale S in millimetres etched on it, which is known as the *pitch scale*. Inside the hollow cylinder moves an accurate screw, the pitch of which is usually 0·5 mm. The head H of the screw carries a cap whereby the cylinder is rotated. The bevelled edge C of the cylinder is provided with a circular scale with 50 or 100 equal divisions engraved round it. The shortest distance measurable with a screw gauge, known as *least count* of the micrometer, is usually ·001 cm.

To make any measurement (say, to find the diameter of a wire), place the wire within the space AB and move the cylinder so that the opposite ends of the section of the wire (represented by the circular section) are just touched. Now take the reading of the straight scale ; call it m. Also take the reading of the circular head against the straight scale ; call it r. Thus, if v be the least count of the micrometer, the diameter of the wire is $m + rv$ in proper units.

(vii) Spherometer—This instrument also works on the principle of a micrometer screw. It is generally used for measuring the thickness of a plate and for determining the radius of curvature of a spherical surface. It consists of metal framework, supported on three fixed legs (Fig. 11) of equal length. Through the centre of the frame a fine screw works which terminates at the top in a milled head M with a large graduated circular disc C. The lower end T of this screw forms the central leg of the instrument. A small vertical scale S is fixed at one end of the frame with its graduations close to those on the disc. This scale is usually graduated in half-millimetres, while the edge of the circular disc is divided into a large number of equal divisions, the number being usually either 50 or 100.

To start an experiment with the spherometer, the pitch of the screw carrying the circular scale is first determined, whence the least count of the instrument is found. It is then placed upon the glass plate known as the *base plate* and the usual reading is then taken. The plate, whose thickness is to be measured, is placed on the base plate and the central screw is made to touch the upper surface of the plate. Another reading is then taken. The difference of the two readings gives the thickness of the plate. Spherometers of high precision may be used to measure a thickness accurately up to a thousandth part of millimetre.

Measurement of Radius of Curvature—The instrument may also be used to measure the radius of curvature of a spherical surface. The following is the procedure : The spherometer is placed on a plane sheet of glass and the milled head is turned, so that the central screw *just* touches the surface. Now the reading of the spherometer is taken. Let the reading be e. The central screw is then raised up, if necessary, and the spherometer is placed on the spherical surface such that the fixed legs rest on this surface. The screw is then turned down slowly so as to touch the spherical surface. Let the reading of the spherometer at this position be b. Then the difference of readings of the spherometer at these two positions is $b \sim e = h$, say.

Next the central screw is raised up and the spherometer is placed on a piece of white paper and is slightly pressed. Three dots corresponding to fixed legs are imprinted on the paper. The distance between the dots are measured with a pair of dividers and the metre scale or more accurately with vernier microscope. The mean value of such distances is found. Let it be c. Then the radius of curvature R of the spherical surface is given by the expression :

$$R = \frac{c^2}{6h} + \frac{h}{2}.$$

Proof—Imagine a plane to pass through the tips A, B and C of the spherometer legs when placed on the spherical surface. This plane cuts the spherical surface in a circular section ABPC having O as its centre (Fig. 12). The point O may be obtained by producing the direction of the central screw so as to cut the circular section. Since the fixed legs are equidistant, $\triangle ABC$ is equilateral and $\angle BAC = 60^\circ$. Again arc ABP=arc ACP, being each a semicircle and arc AB=arc AC, standing on equal chords. Therefore arc PC=arc PB and ∠PAC=∠PAB=30°.

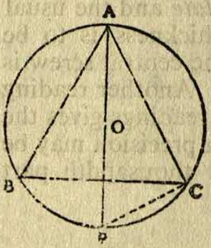


Fig. 12

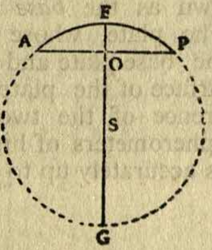


Fig. 13

Now from the rt.-∠ed. △ACP, AC=AP cos 30°=2AO cos 30°. If AO=a,

$$a=\frac{\text{AC}}{2\cos 30^\circ}=\frac{\text{AC}}{\sqrt{3}}=\frac{c}{\sqrt{3}}.$$

Imagine another vertical circle to pass through the points A which is one leg-tip of the spherometer, and E which is the point of the central screw on the spherical surface. The part of the circle, will pass through the point P and the part of this circle lying beyond the actual spherical surface, is shown by the dotted line (Fig. 13). The radius of this circle is evidently the radius of curvature of the spherical surface. The screw of the spherometer moves along EO. The length EO represents the depth of the plane ABC below the point E and is thus the difference of readings of the spherometer on a plate glass and the spherical surface, which is h. The line EO being produced passes through the centre S of the circle PEAG. Now since the diameter EG and the chord AP intersect at right angles, elementary geometry leads to,

$$\text{EO}\times\text{OG}=\text{AO}\times\text{OP, or, EO}\times(\text{EG}-\text{OE})=\text{AO}\times\text{OP}$$

$$\therefore\quad h(2\text{R}-h)=a^2=\frac{c^2}{3},\ \text{whence R}=\frac{a^2}{2h}+\frac{h}{2}=\frac{c^2}{6h}+\frac{h}{2}.$$

Thus by measuring h and c or a, the radius of curvature R may be experimentally determined.

Example :

The fixed legs of a spherometer are at the corners of an equilateral triangle of 4 cm. side. When adjusted on the surface of a spherical mirror, the instrument reads 1·500 mm. Find the radius of curvature of the mirror taking zero error of the instrument to be zero. Prove the formula you use. [U. P. B.–1951]

Ans. The distance c between the fixed legs=4 cm. The height or depth h of the spherical surface above or below the level of the fixed legs as obtained with the central screw head=1·5 mm.=0·15 cm. Hence by the expression for the radius of curvature

$$\text{R}=\frac{c^2}{6h}+\frac{h}{2}=\left(\frac{4^2}{6\times\cdot15}+\frac{\cdot15}{2}\right)\text{cm.}=17{\cdot}85\text{ cm.}$$

9. Measurement of Area—The measurement of an area involves two linear measurements.

Area of Regular Geometric Surfaces—Areas of regular surfaces can be obtained from the following relations :

Area of a Rectangle $=$ length $\times$ breadth ;
,, ,, Square $= (\text{length})^2$
,, ,, Triangle $= \frac{1}{2} \times$ base $\times$ altitude ;
,, ,, Trapezium $= \frac{1}{2} \times$ (sum of parallel sides) $\times$ altitude
,, ,, Parallelogram $=$ one side as base $\times$ altitude, *i.e.*, perpendicular distance between that side and the opposite one ;
,, ,, Circle $= \pi \times (\text{radius})^2$;
,, ,, an Ellipse $= \pi \times$ semi-major axis $\times$ semi-minor axis;
,, ,, a Sphere $= 4\pi \times (\text{radius})^2$;
,, ,, Cylinder $= \pi \times$ diameter $\times$ length.

Suppose that you are required to measure the surface area of a rectangular metal sheet. First measure the length of the sheet a number of times by any suitable apparatus and find the mean length. Let it be x cm. Then measure the breadth of the sheet in a similar way. Let the mean breadth be y cm. Then the area of the sheet is $x \times y$ sq. cm. If, however, you are supplied with a circular sheet, measure the diameter of the circle at several regions and find the mean diameter. Half of the mean diameter is the mean radius (say, r cm.). Then the area of the circular sheet is $3{\cdot}14 \times r^2$ sq. cm. (as $\pi = 3{\cdot}14$).

Area of Irregular Surfaces—The area of an irregular surface may be obtained by dividing the surface into suitable areas in the form of triangles, rectangles or circles and then by adding up the calculated areas of the parts.

By Squared Paper—The area of a small irregular surface may be determined by placing it on a piece of squared paper, tracing the outline and counting the number of small squares included within its boundary. If half or more than half of a square lies inside the surface, it is taken as one whole and if more than half lies outside, that one is neglected. The area of each square being known that of the required surface is readily obtained.

An irregular figure has been traced out on a graph paper

(Fig. 14). To find its area, it is divided into as many rectangular figures as possible, *e.g.*, PQRS and ABDC. The remaining irregular regions, such as APR, SBQ, BXD, DYC and CZA together with the two rectangular parts make up the area of irregular figure. Now the area PQSR=55 squares. Similarly, area ABDC=224 small squares ; part APR=12, QSB=6, BXD=25, CYD=31, CZA =17. Hence, the total area is the algebraic sum of the number of squares, *i.e.*, 370. If the area of each small square be one sq. mm., the total area of the given figure is very nearly 370 sq. mm.

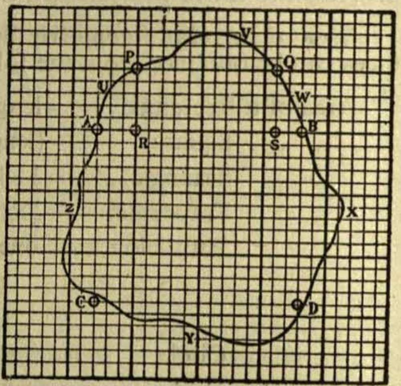
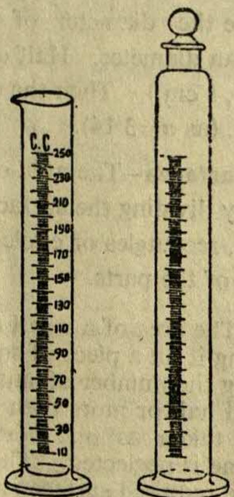


Fig. 14—Measurement of Area

The area of any extended surface, *e.g.*, that of a country, can be determined on this principle by drawing to a scale a map of its boundary on a piece of squared paper. The usual method of measuring the area of large tracts of land is by Trigonometrical survey (*vide* J. Chatterjee's *Intermediate Practical Physics*).

10. Measurement of Volume—The volume of a *solid of any form* can be determined by the following apparatus :



(*a*) (*b*)

Fig. 15

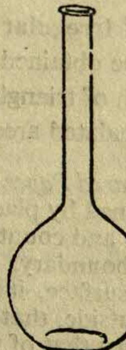


Fig. 16

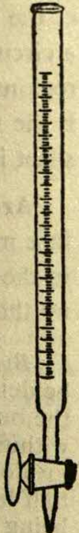


Fig.17

The volume of a liquid can be measured with a graduated cylinder or flask. Fig. 15 (*a*) and (*b*) represent two types of cylinders graduated in cubic centimetres. Fig. 16 is a flask, the capacity of which is marked on its body. The liquid is poured into the cylinder and the upper surface of the liquid is read. This gives the volume of the liquid in units marked on the vessel. Fig. 17

shows a burrette and is used when small volume of liquid is transferred from it.

Volume of Irregular Solid—The volume of a small piece of solid, irregular or regular, can be determined as noted below,—

By Displacement of Liquid—A liquid, which does not corrode or dissolve the solid, is taken in a graduated cylinder (Fig. 18) and the position on the liquid level is carefully noted. Suppose that the original level is at A. The solid is then completely immersed in the liquid when the level of the liquid is found to rise. The position of meniscus is noted again. Suppose that the new level is at B. The difference of the two readings gives the volume of the solid.

By Weighing—The volume of a solid can be determined by a method based on the principle of Archimedes. (*Vide* Chap. IX) :

Volnme of a Parallelopiped $=$length$\times$breadth$\times$height ;
" " Cube $=(\text{length})^3$
" " Cylinder $=$(area of one end)$\times$length ;
" " Sphere $=\frac{4}{3}\pi\times(\text{radius})^3$;
" " Pyramid or Cone$=\frac{1}{3}\times$(area of base)$\times$(vertical length).

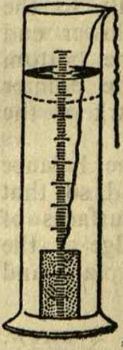


Fig. 18

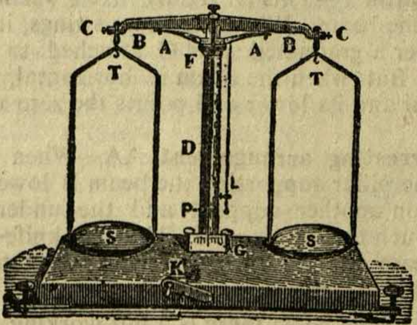


Fig. 19—Physical Balance

11. Measurement of Mass—The *mass* of a body can be determined by weighing in a balance. Everybody has seen a grocer's balance and knows how he uses it in weighing a substance. It consists of a straight uniform rod called the *beam*, from the ends of which two identical *pans* are suspended. The beam is pivoted at its middle point, so that with empty pans, the beam remains horizontal when suspended. The substance to be weighed is placed on one pan and standard weights are placed on trial on the other pan till the beam becomes horizontal. Then the total mass of the weights is equal to the mass of the substance. Such a balance is not, however, sensitive to very light objects.

Common Balance—The *common balance* is a contrivance for comparing the masses of different bodies. The masses, which are used as standards of reference, are called '*weights*'. The essential *parts* of a common balance are (Fig. 19):

(1) The **beam** B—It is a horizontal metal rod, which is capable of turning freely about a sharp steel or agate **knife-edge** F at the middle and is called the *fulcrum*. To minimise friction the sharp end of the knife-edge rests on a small plate of steel or agate. At the ends of the beam two similar agate knife-edges are attached with their sharp edges pointing upwards.

(2) The **stirrups** or pan supporters TT—These rest on the terminal knife-edges. In the better type of balances, these are provided with agate pieces attached to the lower side of the stirrups. Each stirrup carries a hook from which the pan is suspended. The distance from the fulcrum to the centre of gravity of the stirrup is called the *arm* of the balance. The arms are equal in length.

(3) The **scale** pans SS, on which standard 'weights' and bodies to be weighed, are placed.

(4) The **pillar** D supports the beam when at rest—

The pillar is a vertical rod which can be raised or lowered by a key K at the base, whenever required. The pillar has an agate plate at the top upon which the central knife-edge of the beam rests.

(5) The **pointer** P—This is rigidly fixed vertically to the middle part of the beam. When the beam swings, its lower end moves freely over a graduated scale G attached to the bottom of the pillar. But when the beam is horizontal, the pointer becomes vertical and its lower end points the zero mark of the scale.

(6) The **arresting arrangement** AA—When the balance is not in use, the pillar supporting the beam is lowered, so that the beam rests on another support and the under-surfaces of the pans just touch the base board. Thus the knife-edge at the centre of the beam does not always rest on the agate plate and so its sharpness is preserved.

At each end of the beam there is a nut working in a screw. By displacing the position of the nut the effective weight on each side can be altered through a small range. The base of the instrument is provided with levelling screws, whereby the pillar is made vertical. The plumb line L indicates the correct adjustment. *Weight Boxes* with standard weights are supplied for weighing. A delicate balance for finer weighing has various other parts and is always kept within a glass case to prevent blast of air causing disturbances during an operation of weighing.

After a preliminary checking of the balance the beam is placed on its support by the key. The body, whose mass is to be found, is placed on the left pan and some 'weight' from the balance box is taken up *by the forceps* and placed on the right pan to counterpoise the body. The beam is raised up and the inclination of the pointer is observed. If the 'weight' appears to be too heavy, *the beam is lowered* and a smaller 'weight' is replaced. Again the beam is raised up and inclination of the pointer is marked. By repeated trials, certain 'weights' are placed on the pan, when on raising the balance beam, the pointer oscillate *equally* about the zero position of the scale or it rests vertically downwards. The mass of the 'weights' placed on the pan is then *equal to the mass* of the body. [For further details regarding a balance and weighing, *vide* Chap. IV].

12. Measurement of Time—The earliest method of reckoning the hours of the day was done by means of the **sun-dial.** *Water-clocks* and *sand-glasses* were in use long ago. As such time measurers are out of date, these are only of historical importance.

In 1583 Galileo discovered that the time of oscillation of a pendulum of a given length is always constant (Chap. IV) and in 1656, Huygens first used the pendulum to regulate the motion of clocks. The measurement of time is now generally done by watches and clocks. The first use of the coiled hair-spring in watches is credited to Robert Hooke in 1660. A *chronometer* is a watch specially constructed so as to keep time with perfect regularity. **A stop-clock** is a type of watch which can measure seconds accurately and which can be started or stopped whenever desired. A *stop-watch*, constructed on the same principle, is a better precision instrument (Fig. 20) capable of measuring one-fifth or one-tenth part of a second. The **metronome** is a mechanism, run by clock-work and is used to mark time at regular intervals. (*Vide* J. Chatterjee's *Intermediate Practical Physics.*)

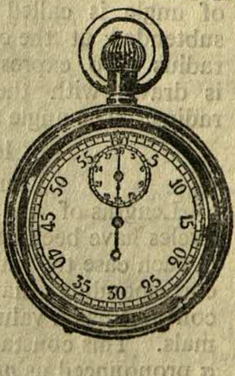


Fig. 20 – Stop-watch

A reed or thin metal strip can vibrate a large number of times per second and the period of a single vibration is quite uniform. The vibration of a reed can be utilised to measure an interval as small as one-hundredth or one-thousandth part of a second. In recent times scientists have found that even one-thousandth part of a second is quite a large interval in comparison to the quickness of action of a few electrical apparatus such as a radio, radar, etc. To keep pace with such apparatus, electrical instruments have been devised which can measure one-millionth part of a second with sufficient accuracy. (*Vide* part on Electricity.)

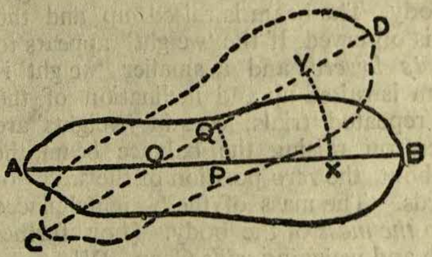


Fig. 21

13. Units of Angle—The idea of an angle is associated with the rotation of a straight line about an axis. Let a straight line AB (Fig. 21) rotate about the point O on the plane of the paper and occupy the position CD. Then any two points P and X on the straight line AB would trace out two circular arcs PQ and XY about the common centre O. If the lengths of the arcs PQ and XY as well as the distances OP and OX are actually measured, then it will be found that

$$\frac{\text{arc PQ}}{\text{PO}}=\frac{\text{arc XY}}{\text{XO}}.$$

This shows that for a given rotation of the straight line, any arc made by a point on it divided by the distance of that point from the centre of rotation is constant. Therefore, the ratio of the arc and its corresponding distance from the centre is a measure of the angle. This system of measuring an angle is called a **circular measure**. According to this system, the unit of angle is called a **radian**, which is defined to be the angle subtended at the centre by a circular arc of length equal to the radius. To express an angle in terms of radian, a circular arc is drawn with the apex of the angle as centre and with any radius. The angle in *radian* is given by the quantity

$$\frac{\text{length of the arc}}{\text{length of the radius}}.$$

Lengths of the circumferences and diameters of various circles have been carefully measured and it has been found that in each case the ratio of the length of the circumference to the corresponding diameter of any circle is a constant. This constant has a value of 3·1415 correct to fourth place of decimals. This constant is popularly designated by the Greek letter π pronounced as *pie*.

$$\text{Thus, }\frac{\text{circumference of a circle}}{\text{diameter of the circle}}=\pi=3{\cdot}1415...$$

$$\text{or, }\frac{\text{circumference}}{\text{radius}}=2\pi \quad \therefore \text{ circumference}=2\pi\times\text{radius}.$$

When a straight line makes one *complete rotation* about a point on it on the plane of the paper, any other point on the straight line moves over a complete circle with the fixed point as the centre. The number of radians for one complete rotation of the straight line must then be,

$$=\frac{\text{length of the circumference}}{\text{length of the radius}}=2\pi=6{\cdot}2830...$$

If the circumference of a circle be divided into 360 equal parts, the angle subtended at the centre by any arc is called a **degree**. In this measure the angle subtended at the centre by a semicircle contains 180 degrees and the angle by any quarter of a circle contains 90 degrees. The multiple and submultiple of a degree are the following. 1 right angle=90 degrees (90°); 1 degree (1°)=60 minutes (60′); 1 minute (1′)=60 seconds (60″)

Since the circumference of a circle subtends an angle of 2π radians at the circle and also 360°, 2π radians$=2\pi^{\circ}=360^{\circ}$

$$\therefore\quad 1\text{ radian}=\frac{360^{\circ}}{2\pi}=\frac{360^{\circ}}{6{\cdot}2830}=57^{\circ}17'44{\cdot}8''.$$

14. Measurement of Angle—The apparatus most commonly employed to measure an angle is called a protractor. It consists of a semicircular piece of metal, divided on its circumference into 180 equal parts, each called a degree (Fig. 22). The scale is graduated in both directions. The lower straight edge of the protractor forms the diameter of the semicircle having the centre at O. Protractors supplied in ordinary mathematical instrument boxes are graduated with divisions of a degree; but there are more precision types of protractors having $\frac{1}{2}$ or $\frac{1}{3}$ degree as the smallest division.

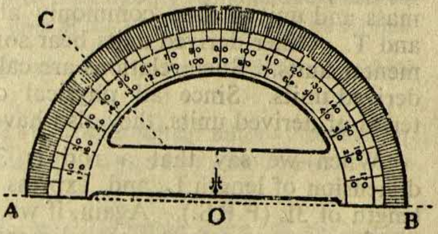


Fig. 22

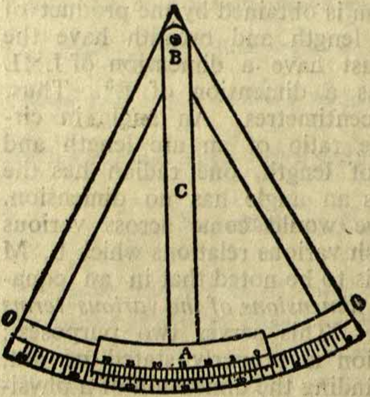


Fig. 23

In order to measure the angle between two straight lines OA and OC, place the protractor with its straight edge just to touch the line OA and the angular point to coincide with O. Now read the outer circular scale just at the point through which OC emerges out. This gives the angle AOC in degrees. If the angle BOC is to be measured, read the inner scale which starts from the right. In the particular figure the ∠ AOC is 45°.

Where greater accuracy is needed to measure an angle, an angular vernier is used in conjunction with a degree scale (Fig. 23). The apparatus consists of a degree scale OO with its centre at B. Just along the side of this scale, another auxiliary vernier scale A slides about the common centre. This vernier scale is graduated in such a way that a certain number n divisions of this scale are equal to $(n - 1)$ divisions of the main scale. In such a case the vernier is capable of measuring $1/n$ part of the main scale. If the main scale is graduated in degrees, then this angular vernier can read up to $1/n$ of a degree. A sensitive angular vernier may be as accurate as to read 1/360th part of a degree. (For details, *vide* J. Chatterjee's *Intermediate Practical Physics.*)

15. **Dimensions of Physical Quantities**—It has been stated (Art. 4), that there are three fundamental units, apparently independent of each other. These are unit of length, unit of mass and unit of time, commonly abbreviated by letters L, M and T. Other derived units bear some relations to the fundamental units. Such relations are called the *dimensions* of those derived units. Since all physical quantities are expressed in terms of derived units, they also have certain dimensions.

When we say that a rod is 3 feet long, we refer to the dimension of length L and express it in F.P.S. system as a length of 3L (F.P.S.). Again, if we say that a body has mass of 15 grams, we refer to the dimension of mass M which is equivalent to 15 M (C.G.S.). Similarly, 20 T means 20 seconds on both the systems.

The unit of area in any system is obtained by the product of length and breadth. Since both length and breadth have the dimensions of length, area must have a dimension of L×L or L^2. Similarly, a volume has a dimension of L^3. Thus, $35L^3$ (C. G. S.) means 35 cubic centimetres. An angle in circular measure is expressed by the ratio of an arc length and radius. Both being in units of length, one radian has the dimension of $L \div L$ or L^0. Thus an angle has no dimension. As in subsequent Chapters, we would come across various derived units, we should establish various relations which L, M and T bear with such units. It is to be noted that in an equation between two quantities, the *dimensions of the various terms in the equation must be the same*. This serves two purposes : (*i*) it indicates whether an equation is properly stated or not, and (*ii*) it furnishes a means of finding the dimension of a physical quantity whose relation to the fundamental quantity is not clearly known.

[Students are required to remember the following conversion factors so as to work out problems dealing with reduction of units.]

British into Metric Conversion Factors

British	Metric	British	Metric
1 inch	=2·538 cm.	1 cu. ft.	=28·3153 litres
1 yard	=·9144 metre	1 gallon	=4·546 litres
1 mile	=1·6093 km.	1 pound (av.)	=453·56 gm.
1 sq. in.	=6·451 sq. cm.	1 grain	=·0648 gm.
1 sq. yd.	=83·61 × 10^{-6} Hectare	1 ton	=1016 kgm.
1 cu. in.	=16·386 c. c.	1 Metric Tonne	=1000 kgm.

Examples :

1. Taking the shape of the Earth to be exactly round and its diameter to be 7900 miles, find the surface area of the Earth. Find also its volume.

Ans. The surface area of a sphere $=4\pi r^2$. Now $r=7900/2$ miles $=3950$ miles.

$\therefore$ Surface area of Earth $=4\times3{\cdot}1415\times3950^2$ sq. miles $=196061015$ sq. miles. Also the volume of Earth $=\frac{4}{3}\pi r^3=\frac{4}{3}\times3{\cdot}1415\times3950^3$ cubic miles $=258147003057$ cubic miles.

2. Find the angle subtended by an arc of length 1 cm. at the centre of a circle radius of 10 ft. Express your result in radian and in degrees of an arc.

Ans. We know 1 foot $=30{\cdot}479$ cm. $\therefore$ 10 ft. $=304{\cdot}79$ cm.

$\therefore$ angle $=\dfrac{1}{304{\cdot}79}$ radian $={\cdot}0033$ radian

Since $3{\cdot}14$ radian $=180^\circ$, 1 radian $=\dfrac{180^\circ}{3{\cdot}14}$

Thus the required angle $={\cdot}0033$ radian $=\dfrac{180\times{\cdot}0033}{3{\cdot}14}$ degree $=0{\cdot}19$ degree.

EXERCISES ON CHAPTER I

Reference

1. Name the system of units commonly used for the measurement of physical quantities. Which of them is more convenient and why ? Explain with examples what you mean by fundamental units, derived units and practical units. Art. 4

2. Define mass, volume and density ; state the relation that exists between them. (C. U.—1940 ; Dac. U.—1949) Arts. 5 & 6

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3. What do you mean by an *apparent solar day* and *mean solar day* ? Art. 7

4. Give the principle of a vernier and explain its working. Art. 8
Each division of a main scale is 0·5 mm. 9 divisions of the main scale are equal to 10 divisions of the vernier. Length of a cylinder is measured. The readings are : 78 divisions of the main scale; and the 6th division of the vernier coincides with a division of the main scale. Calculate the length of the cylinder. (Del. H. S.—1953)

Ans. 3·93 cm.

5. Derive the formula, $R=\frac{a^2}{2h}+\frac{h}{2}$ for a spherometer. Art. 8

The legs of a spherometer occupy the corners of an equilateral triangle of side 4 cm. When placed on a plane surface, the reading is 4 mm. and on a concave surface, it is 2 mm. Calculate the radius of curvature of the surface.
(Anna. U.—1948, '53)

Ans. 13·4 cm.

6. Describe a spherometer and explain carefully its use for measuring the radius of curvature of a spherical surface. Art. 8
(U. P. B.—1948, '53)

7. How will you find the volume of a solid of irregular shape. (C. U.—1947, '49 ; Dac. U.—1942) Art. 10

8. What is meant by the dimension of a physical quantity ? What is its utility ? What are the dimensions of density and velocity ? Art. 15

CHAPTER II

MOTION

16. Position and Reference Co-ordinates—In our everyday life we get an idea of the position of any object by its direction and distance from some other object of reference. For this purpose, we choose some unit of distance and the four standard directions east, west, north and south. The object of reference may be anything apparently fixed. When we say that the school building B is situated at a distance of half-a-mile to the north-east of the Post-Office O, (Fig. 24), the Post-Office stands for the object of reference, north-east gives the direction and a half-a-mile is the distance. Thus, we specify the position B of the school building by its distance OB, which is half-a-mile in this case, and also by the north-easterly direction, with

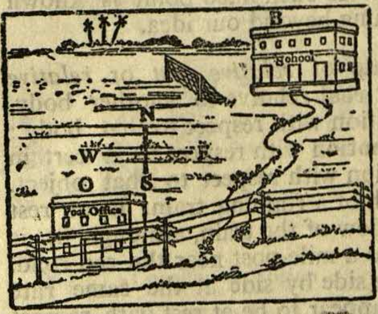

Fig. 24 – Cardinal Directions

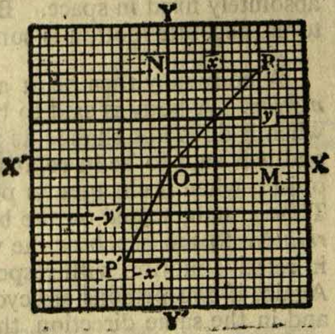

Fig. 25 – Rectangular Co-ordinates

reference to the Post-Office. If we know the distance OB and the north-easterly direction, we can easily determine the position of the school building. The position of an object is, therefore, determined by its distance along a definite direction at any time from some apparently fixed point in space.

The position of a point on a graph paper is determined exactly in a similar manner. Two straight lines are taken at right angles on the graph paper, which we call reference lines XOX′ and YOY′ like the standard directions (Fig. 25). The intersection of the two lines is the *origin* O of the co-ordinates. To find the co-ordinates of any point P on the graph paper, the distances of this point from the two lines are measured in terms of the units chosen along the abscissa OX and the ordinate OY. These co-ordinates, which are x and y say, determine the distance of the point from the reference origin O. On measuring x and y, the *actual distance* OP *of the point* from the origin is known, which is $\sqrt{x^2+y^2}$; further we can also measure the angle POX or POY and determine *in which direction the point* P *lies with respect to the axes.*

17. Rest and Motion—A body is said to be at *rest*, when it does not change its position with respect to some point fixed in space. It is said to be in *motion* when it changes its position with time with respect to a point. So, *motion* is *a change of position with time* with respect to some reference point. *Absolute rest* is the complete absence of motion. We never get a body at absolute rest, since the earth and all other planets are continually revolving about their respective axes and also since each moves round the sun in its own orbit, while the sun itself with all the planets is moving through space among the stars.

The stars are found to move away with respect to one another. *Absolute motion* of a body is its motion with respect to a point absolutely fixed in space. But as no such fixed point is known to us, absolute motion is something beyond our idea.

A body may then be in a state of *relative rest* or *relative motion.* A body is said to be at **rest** relative to another body, when it does not change its position with respect to that body; and it is said to be in **relative motion** with respect to a certain object, when it changes its position with respect to that object. Thus a man, seated on the bench of a running train, is at rest *relative* to the bench or the window of the train, although he is in relative motion with respect to a mile-post near the rail-road. Again, if two persons are cycling side by side at the same rate and in the same direction, they appear to be at rest with respect to each other, although each is in motion with respect to a man standing near by. We usually refer the motion of all terrestrial bodies with respect to any *point* fixed on the earth.

Nature of Motion—The motion of a body may be either **translatory** or **rotatory** or both. Translatory motion may again be subdivided into *rectilinear* and *curvilinear* motions.

A body is said to possess a **translatory motion** when a line joining any two points on the body *remains always parallel* for all positions of the body. Fig. 26 represents a body moving in a direction shown by the arrow head. Any two points, say *a* and *c*, are taken on the body at its initial position. In the displaced position the corresponding points are *x* and *z*. If the straight line joining the points *a* and *c* is parallel to the straight line joining *x* and *z* in the displaced position of the body and if the path of motion of any one point of the body be straight, the type of motion is called a *rectilinear* motion. A stone falling down from a height or a train running on a straight track is an example of rectilinear motion.

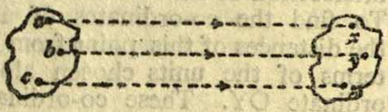

Fig. 26—Rectilinear Motion

When the motion of the body takes place along a curved path in such a way that any line drawn on the body remains always parallel to its original direction, it is called a *curvilinear* motion. In Fig. 27, a body is moving in the direction of the arrow-head. The path from E to F is rectilinear; but the path FGHI is on a curve. The vertical line MP remains always vertical irrespective of its position along the direction of motion. Expect for a little swing, the motion of the armchairs of the vertical merry-go-round in fun fairs is an example of a curvilinear motion.

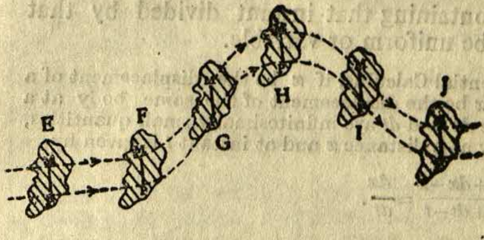

Fig. 27 – Curvilinear Motion

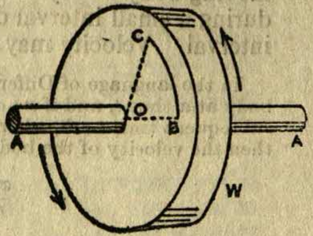

Fig. 28 – Rotatory Motion

A body is said to have a *rotatory motion*, when any straight line taken on it moves about a fixed point or axis. Fig. 28 represents a solid wheel rotating about its axis AA. If we consider a point B on the wheel at any instant, it will be found to occupy a position C at a later instant. By joining the successive positions of the point in between B and C, we get an arc of a circle subtending an angle BOC at the centre of rotation. The same angle is subtended at the centre by an arc made by any point on the radial line OB in the same interval of time. Therefore, in a rotatory motion, a line joining any two points on the body in the displaced state would make an angle to the original line in the initial state of the body.

The motion of a body may often be complex in nature, being a combination of a rotation and translation ; thus the motion of a cart wheel or a ball rolling on the ground or a planet round the sun comprises both kind of motion.

18. Terms relating to Motion—The displacement of a moving particle is its change of position in a given direction and so it is the distance between its initial and final positions irrespective of its intermediate positions. The displacement of the point P from the origin O (Fig. 25) is the distance OP. Displacement has thus a *direction* as well as a *magnitude*. Any physical quantity, which possesses both *magnitude and direction*, is called a **vector** quantity (Ref. Art. 21). So displacement is a vector quantity.

The **speed** of a body is the rate of change of its position and is measured by the distance passed over by the body in unit time. The speed does not refer to any direction. Any physical quantity, which has *magnitude only but no direction*, is called a **scalar** quantity. So speed is a scalar quantity.

The **velocity** of a moving body is its rate of change of displacement with time. Since velocity has got some direction and magnitude, it may be represented by a straight line. Evidently speed in a definite direction is velocity. The *velocity* of a moving body *at any instant* is measured by the distance traversed during a small interval containing that instant divided by that interval. Velocity may be uniform or variable.

In the language of Differential Calculus, if x be the displacement of a body at a time t, and if $x+dx$ be the displacement of the same body at a subsequent time $t+dt$, where dx and dt are infinitesimally small quantities, then the velocity of the body at a distance x and at instant t is given by

$$\frac{x+dx-x}{t+dt-t}=\frac{dx}{dt}.$$

Uniform Velocity—The velocity of moving body is said to be *uniform*, when it moves in a definite direction over equal distances in equal intervals of times, however small these intervals may be. When *uniform*, the velocity is measured by the distance traversed in a unit of time*.

Figure 29 illustrates a uniform velocity of a body ; *a*, *b*, *c*, and *d* represent its various positions at intervals of one minute. Its rate of displacement is found to be 120 ft. in every minute. Therefore, its velocity is 2 ft. per second. Thus if a body, moving with a uniform velocity *v*, passes over *s* ft. in *t* seconds, then $v=s/t$ ft. per sec.

When a body does not move through equal distances in equal intervals of time, it possesses a *variable* velocity. In this case velocity may be measured at any given instant or an average velocity may be found during a given interval. The **average or mean velocity** of a moving body during a given interval is measured by the total distance passed over by the body divided by that interval.

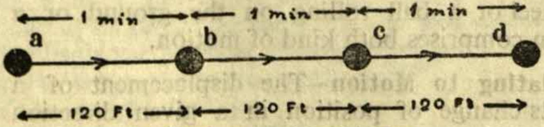

Fig. 29—Uniform Velocity

If x be some measurable distance traversed by the body in a known interval t, the average or mean velocity during the interval t is given by x/t.

The distance may be expressed in any unit of length, such as miles, yards, feet, metres, centimetres, etc., and the corresponding time may also be expressed in any unit such as hours, minutes, seconds, etc. For example, if a body covers l miles in h hours, we may denote its average velocity as l/h miles per hour or even as $\frac{1\times 1760}{h\times 60}$ yards per minute, etc. But since the unit of length is accepted as a foot or a centimetre and the unit of time is a second, it is customary to express the velocity in feet per second or in centimetres per second. Thus a velocity of 30 miles an hour is equivalent to a velocity of 44 ft. per second.

---

*When velocity is uniform, dx/dt is constant which means that wherever in its path or whenever this ratio is measured, the value does not change.

The *unit of velocity* may be defined as the velocity of a moving body which traverses unit length in a *given direction* in unit time. Hence, in the F.P.S. system, the unit of velocity may be taken as *one foot per second* and in the C.G.S. system, it is *one centimetre per second* sometimes abbreviated as *cm.* sec^{-1}.

Acceleration—The acceleration of a body is the *rate of change* of its velocity, *i.e.*, the change of velocity per unit time. As a velocity has both magnitude and direction, the change may take place either in both or in one only. In the simplest case, the magnitude of velocity may change, the direction remaining the same. Acceleration has thus a direction as well as magnitude. It can, therefore, be represented by a straight line with an arrow-head and it is a **vector** quantity.

For example, let a body, with a *continuously increasing velocity*, possess a velocity of 7·5 ft. per second at the position *a* (Fig. 30). After one second let it move to *b* and let its velocity at that position be 12·5 ft./sec. After the lapse of one second, let its velocity be 17·5 ft./sec. at *c*. In this manner after the lapse of each second when it is at *d* and *e* let the velocities be 22·5 ft./sec. and 27·5 ft./sec. respectively. So the velocity of this body is increasing at the rate of 5 ft. per second. Thus we can say that the acceleration of the body is its rate of change of 5 ft./sec of its initial velocity of 7·5 ft./sec. Since the term per second occurs twice in stating the quality of an acceleration, the acceleration of this body is stated as 5 ft. per second per second or 5 ft./sec^2.

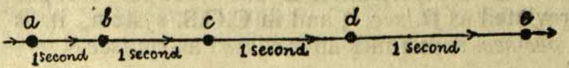

Fig. 30—Acceleration showing Time—Velocity relation.

We can also look to an acceleration of a body from its displacements after *successive equal intervals* of time. Let the body pass over a distance of 10 ft. in one second from *a* to *b* (Fig. 31). Then its *average velocity* during the first interval of 10 ft. per sec. If again it passes over a distance of 15 ft.

during the second interval, its *average velocity* is 15 ft. per sec. during the next second. Thus the average velocity has been increased by 5 ft. per second. If this increase of velocity is maintained constant, it would pass over distances of 20 ft. in the third second, 25 ft. in the fourth second and so on. Thus the rate of change is 5 ft. per second of an average initial velocity of 10 ft. per second. So its acceleration is 5 ft. per sec.^{2} or 5 ft/sec.^{2}. Here if the acceleration is *uniform*, it can be proved that at the position 'a' the body possesses an initial velocity of 7·5 ft. per second and at 'b' of 12·5 ft. per second, so that the average velocity between 'a' and 'b' is 10 ft./sec.

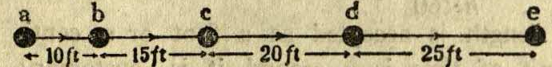

Fig. 31 – Accelerated Motion showing Time – Displacement Relation.

Thus we find that for a body moving with a uniform acceleration, the velocities at the ends of successive seconds increase by a fixed amount. In general, if u and v are the velocities at the beginning and at the end of an interval t, the acceleration f is given by,

$$f=\frac{v-u}{t} \qquad \cdots (18, 1)$$

If however, the rate of increase or decrease of velocity changes with time, the acceleration is said to be variable. The acceleration may also be expressed in any unit depending upon the units of velocity and time. A body is said to have a *unit acceleration*, when its velocity changes by unity in unit time. In the F.P.S. system, the unit of acceleration is *one foot per sec. per sec.* usually abbreviated as *ft./sec.*^{2}, and in C.G.S. system, it is *one cm. per sec. per sec.* sometimes abbreviated as *cm./sec.*^{2}

In the language of Differential Calculus if v be the velocity of a body at an instant t and if $v+dv$ be its velocity at the subsequent instant $t+dt$, then the acceleration of the body at the instant t is given by the differential coefficient

$$\frac{v+dv-v}{t+dt-t}=\frac{dv}{dt}.$$

Retardation—When the velocity of a moving body decreases continuously the motion is said to be *retarded*. Mathematically speaking, *retardation* is considered as a *negative acceleration* and is expressed by putting a negative sign before the change in velocity taking place per unit of time.

19. Measurement of Velocity—In order to measure the average velocity of a body, we are to determine how far the body travels in a known interval of time. Suppose that we are to measure the speed of a railway train in which we are travelling and we are provided with a watch capable of measuring seconds. When our compartment just passes by a mile post we record the time in our watch and wait till the compartment passes by the next mile post. The difference of the two readings in the watch measures the interval required for the train to pass over a distance of one mile. The distance divided by time gives the *mean speed* of the train over a distance of one mile.

Measurement of Acceleration—It is now evident that when a body moves with an accelerated or retarded speed, the distances passed over by the body in successive intervals of time either increase or decrease continuously. Hence, to measure an acceleration, the distances passed over by the body at equal intervals are to be measured. Take the case of a railway train when it starts off from a station. The positions of the train are marked at intervals of, say, half a minute. It will be found that the distance moved through during the first interval is less than that during the second interval. The average velocities during the two successive intervals are measured. The difference of these two velocities gives the increase of velocity during half a minute. Hence the acceleration is determined.

20. Equation of Motion—In the case of a body moving on a straight line, the relations between the displacement, time, velocity and acceleration can be represented by some simple equations. These equations were first worked out by Galileo.

(*i*) *Motion with Uniform Velocity*—If the motion is uniform, which means that the body possesses no acceleration, and if the body passes over u units of length in every unit of time, the total space s described in an interval t is given by*

$$s = ut. \qquad \cdots \qquad (20, 1)$$

(*ii*) *Motion with Uniform Acceleration*—Let the initial velocity of the body be u. As it is subjected to a uniform acceleration f, the velocity of the body is increased in *each unit of time* by f and the total increase in velocity in time t will be

---

* Following the present notation and according to Art. 18.

$\frac{ds}{dt}$ = velocity $= u$, a constant.

$\therefore \; ds = udt$. Now on integrating both sides, we have $\int ds = u \int dt$

or, $s = ut + a$, a being a constant...(i)

If now time is measured when there is no displacement, $t = 0$ when $s = 0$.

Then on substituting in (i) we have,

$0 = 0 + a$, whence $a = 0$. Thus eqn., (1) transforms into $s = ut$.

$f \times t$. Hence, the final velocity v acquired by the body after an interval t is given by**

$$v=u+ft, \text{ whence } f=\frac{v-u}{t} \qquad \dots \quad (20,2)$$

$\therefore$ **acceleration=increase in velocity÷time.**

If the body possesses a negative acceleration, f would be negative and the expression would become

$$v=u-ft. \qquad \dots \quad (20,2A)$$

(*iii*) *To find the space described with a uniform acceleration—*

Velocity at the end of the first second$=u+f$.

" 1 second earlier than the last second,

i.e., at $(t-1)$th second$=v-f$.

$\therefore$ average velocity in each of these two seconds

$$=\frac{(u+f)+(v-f)}{2}=\frac{u+v}{2}$$

Velocity after 2 seconds from the beginning$=u+2f$.

Velocity at 2 seconds earlier than the last second$=v-2f$.

$\therefore$ average velocity during each of these two seconds

$$=\frac{(u+2f)+(v-2f)}{2}=\frac{u+v}{2}.$$

Thus, taking intervals in pairs equidistant from the beginning and the end, the average velocity in each second is found to be *half* of the sum of u and v. As the acceleration is uniform,

---

** In case of uniform acceleration $\frac{dv}{dt}=\text{constant}=f$, say, and velocity

$v=\frac{ds}{dl}$, so that $\frac{d}{dt}\left(\frac{ds}{dt}\right)=$ $\dots(i)$

By transposition, $d\left(\frac{ds}{dt}\right)=fdt$ and by integration

$$\int d_{\circ}\left(\frac{ds}{dt}\right)=\int dft \text{ or, } \frac{ds}{dt}=ft+b\dots(ii)$$

where $\frac{ds}{dt}$ represents the velocity for any value of t during the interval.

If u be the initial velocity, u represents the velocity when $t=0$. Substituting in eqn. (*ii*) we get

$$\left(\frac{ds}{dt}\right)_{t=0}=0+b=u, \text{ whence } u=b.$$

If v be the final velocity, we find $\left(\frac{ds}{dt}\right)_{t=t}=ft+b=v.$

Substituting these values in eqn. (*ii*) we get $v=u+ft$.

the body may be supposed to be moving with this *average velocity* during the whole period t. The space described by the body is then by*

$$s=\frac{u+v}{2}\times t=\frac{u+(u+ft)}{2}\times t$$

$$\mathbf{s=ut+\tfrac{1}{2}ft.^2} \qquad \cdots \quad (20,3)$$

If the body has a retardation f, eqn., (20,3) takes the form

$$s=ut-\tfrac{1}{2}ft.^2 \qquad \cdots \quad (20,3a)$$

From (20,2), $v^2=(u+ft)^2=u^2+2uft+f^2t^2$

$$=u^2+2f(ut+\tfrac{1}{2}ft^2)$$

$\therefore$ from (20,3) $\quad \mathbf{v^2=u^2+2fs.} \qquad \cdots \quad (20,4)$

For a retardation f, eqn., (20,4) takes the form $v^2=u^2-2fs.$

(*iv*) *Space described in any particular second.*—Let the space described in the tth. second be denoted by st

Then s_t=space passed in t sec.—space passed in $(t-1)$ sec.

$$=(ut+\tfrac{1}{2}ft^2)-\{u(t-1)+\tfrac{1}{2}f(t-1)^2\},$$

whence $\quad s_t=u+\dfrac{2t-1}{2}\times f. \qquad \cdots \quad (20,5)$

Special Cases—

(*i*) *If the body starts from rest, $u=0$.*

Then $v=ft$; $s=\frac{1}{2}ft^2$; $v^2=2fs$; $s_t=\dfrac{2t-1}{2}f.$

(*ii*) *If the body has retardation f* (*i.e.*, acceleration$=-f$).

then $v=u-ft$; $s=ut-\frac{1}{2}ft^2$; $v^2=u^2-2fs$; $s_t=u-\dfrac{2t-1}{2}f.$

Examples :

1. A body starts with a velocity of 45 miles per hour and moves under a constant retardation of 11 ft. per sec.² Find (*i*) its velocity after 2 secs. (*ii*) after what time it will stop, and (*iii*) what distance it moves before coming to rest.

---

* We know that

$$\frac{d}{dt}\left(\frac{ds}{dt}\right)=f \text{ whence by integration } \frac{ds}{dt}=v=ft+u.$$

By transposition we have $ds=f\,tdt+udt$.

Again, by integration we have,

$$\int ds=\int ftdt+\int udt \text{ or, } s=\tfrac{1}{2}ft^2+ut+c\ldots\ldots(1)$$

where c is constant. If now time is measured from the instant when displacement is zero, we have when $t=0$, $s=0$ whence $c=0$.

$$\text{So } s=ut+\tfrac{1}{2}ft^2.$$

Ans. Here $u=45$ miles per hour $=66$ ft. per second, and $f=-11$ ft. per sec.^{2}

(*i*) From $v=u-ft.$, $\therefore$ $v=(66-11\times 2)$ ft./sec. $=44$ ft. per second.

(*ii*) Let it stop after t seconds. Here $0=66-11\times t$ $\therefore$ $t=6$ seconds.

(*iii*) Let the body move through s ft.; then $v=0, u=66$ and then $0^2=66^2-2\times 11\times s$ when $s=198$ ft.

2. A body moving with uniform acceleration and with an initial velocity of 14 ft. per sec. passes over 8 ft. and then its velocity is 18 ft. per sec. What is the acceleration ?

Ans. Let $f=$ acceleration of the body. Its initial velocity is 14 ft./sec. and after passing over 8 ft, its final velocity is 18 ft./sec. Then from the eqn., $v^2=u^2+2fs$ follows.

$18^2=14^2+2f\times 8$, whence $f=8$ ft./sec.^{2}

3. A bullet moving at the rate of 200 ft. per second is fired into the trunk of a tree into which it penetrates 9 inches. If the bullet moving with the same velocity were fired into a similar piece of wood 5 inches thick, with what velocity would it emerge, supposing the resistance to be uniform ? [Raj. U. – 1953]

Ans. The striking velocity of the bullet is its initial velocity and its final velocity is zero after moving through a distance of 9 in. $=\frac{3}{4}$ ft. through the trunk. Hence, the retardation f of the bullet is obtained from the relation $v^2=u^2-2fs$.

$$\therefore\quad 0^2=200^2-2f\times\tfrac{3}{4},\ \text{where } f=\frac{8\times 10^4}{3}=26666{\cdot}66\ \text{ft./sec.}^2.$$

Retardation being the same in a similar piece of wood 5 in. or $\frac{5}{12}$ ft. thick, for its final velocity a similar equation will hold. Thus

$$v^2=200^2-2\times\frac{8\times 10^4}{3}\times\tfrac{5}{12},\ \text{whence}\ v=133\ \text{ft./sec. approximately.}$$

4. A body starting from rest travels 150 ft. in the 8th second. Calculate the acceleration assuming it to be uniform.

Ans. Space moved over in 8th second = space moved over in 8 seconds – that in 7 seconds. Now the space covered in 8 seconds (assuming f to be the acceleration) is given by

$$s_1=0+\tfrac{1}{2}f\times 8^2=32f.$$

Similarly, the space covered in 7 sec. is given by,

$$s_2=0+\tfrac{1}{2}f\times 7^2=\tfrac{49}{2}f.$$

But $s_1-s_2=150$ or, $32f-\frac{49}{2}f=150$, whence $f=20$ ft. per sec.^{2}.

5. A point mass moving with uniform acceleration, describes in its last second of motion $\frac{9}{25}$th of the whole distance. If it started from rest, how long was it in motion and through what distance did it move, if it describes 6 inches in the first second ? [Utkal U. – 1954]

Ans. When it describes 6 inches or ·5 ft. in the first second starting from rest, then applying eqn. (20.3) and putting $u=0$ and $t=1$, we get,

$${\cdot}5\ \text{ft.}=\tfrac{1}{2}f,\ \text{whence}\ f=1\ \text{ft. per sec.}^2.$$

Let the whole distance be s ft. described by the body in time t sec. Then $s=\frac{1}{2}ft^2=\frac{1}{2}t^2$. ...(1)

Again, since it travels $\frac{9}{25}$ of s ft. in the tth sec.

$$\therefore\ \tfrac{9}{25}s=\frac{2t-1}{2}\times f \text{ or, } t-\tfrac{1}{2}=\tfrac{9}{25}s \qquad \text{...(2)}$$

Multiplying eqn. (1) by $\frac{9}{25}$ and we derive from (2)

$$\tfrac{9}{25}s=\tfrac{9}{50}t^2=t-\tfrac{1}{2}, \text{ whence } 9t^2-50t+25=0. \qquad \text{...(3)}$$

Solving the quadratic equation in t, we get two values t_1 and t_2,

where $t_1=\dfrac{50+\sqrt{2500-900}}{18}=5$ seconds and $t_2=\dfrac{50-\sqrt{2500-900}}{18}=\frac{5}{9}$ second.

Substituting in eqn. (1) when $t_1=5$ sec., $s_1=12{\cdot}5$ ft.;

and when $t_2=\frac{5}{9}$ sec., $s_2=0{\cdot}15$ ft.

The second pair of values t_2 and s_2 are improbable and to be rejected.

21. Vector and Scalar Quantities—Any physical quantity, which has got a *magnitude* only and is subject to ordinary arithmetical laws, is called a **scalar quantity**. Thus, if a man works for 5 hours in the morning and again for 4 hours in the afternoon, he has worked for 9 hours during the day. The total period is obtained by the arithmetical sum, showing that *time is a scalar quantity.* For a similar reason mass, density, distance and volume are *scalar quantities.*

Any physical quantity, which has a directional property in addition to its magnitude, is called a **vector quantity**; so, for the specification of a vector, its direction must be mentioned along with its magnitude. Thus, when we say that a man has moved 20 yards from a certain point, he may be anywhere on the circumference of a circle of radius 20 yds. about the initial position. But if it is stated that the man has moved 20 yds. due *east*, his position can at once be determined. Hence, to state a *displacement* completely, both its *magnitude* and *direction* must be mentioned. Or in other words, *displacement is a vector quantity.* Similarly, velocity, force, etc., are vectors.

Any vector quantity can be represented by a straight line, whose *length represents the magnitude and whose direction gives the direction of the vector.* A single vector, whose effect is the same as the combined effect of two or more vectors, is called the **resultant** of these vectors or simply the vector sum. The process of finding the resultant is called a *vector addition* or the *composition of vectors.*

22. Graphical Representation of Motion—A motion may be represented graphically by indicating the variation of displacement of the body with time. Let OX and OY be two rectangular axes of co-ordinates with O as the origin (Fig. 32). The time is measured along OX and the displacement along OY. The values of the units on the graph paper may be suitably chosen.

Suppose now that particle has a displacement of 15 ft. in the 1st sec., 30 ft. in the first two seconds and 45 ft. in 3 secs. The corresponding points on the graph paper are A, B and C lying on a straight line passing through the origin. Thus a straight line on the time-displacement graph represents equal displacements in equal times, which is a uniform velocity. It is evident that the velocity V represented by the straight line OC is given by

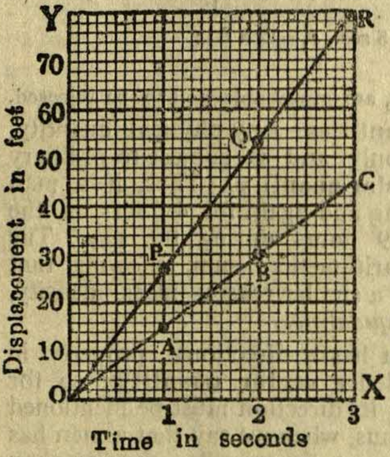

Fig. 32 – Graph of Uniform Velocity

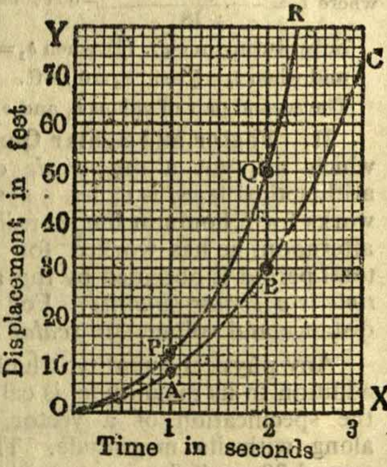

Fig. 33 – Uniform Acceleration

$V=\frac{CX}{OX}=\tan\angle COX=15$ ft. per sec. according to units chosen.

Hence the tangent of the angle made by the straight line with the time axis represents the velocity. The value of the tangent of an (acute) angle increases continuously with the angle, and so the line OPQR which makes a larger angle with the time axis represents a higher velocity than that of the line OABC. As shown in the graph, the line OPQR represents a velocity of 25·5 ft./sec. Any line coinciding with or parallel to OX makes no angle with OX, and so this line represents zero velocity, whereas a line coinciding with or parallel to OY makes an angle of 90° with OX, and so it represents an infinite velocity. A uniform velocity is given by the equation $v=s/t$ or, $s=vt$. Therefore, a relation of s with t for a given value of v is similar to the equation $y=mx+c$, where m is a constant and $c=0$. Thus $s=vt$ represents a straight line passing through the origin.

Again consider that a body starting from rest is moving with a uniform acceleration. During first second, let the body traverse a distance of 8 ft. During the first two seconds let the displacement be 32 ft. and in first three seconds let the displacement be 72 ft. Evidently, the body is moving with a *constant* acceleration 16 ft./sec.^{2}. Now on the graph paper plot time as the abscissa and displacement as the corresponding ordinate, shown by the points A, B and C (Fig. 33). Pass a free-hand smooth curve through these points ; the curve is a parabola. The more steepy parabola as that given by the curve OPQR signifies a higher constant acceleration, as it represents larger displacements than that given by the curve OABC in any given interval.

We know that $s=ut+\frac{1}{2}ft^2$ represents an equation of accelerated motion. If $u=0$, then $s=\frac{1}{2}ft^2$ which is similar to the equation $y=4ax^2$ representing a parabola. Therefore, the relation of s and t in a uniformly accelerated motion is a parabola. Even if there is some initial velocity u of the body, the graphical relation between the displacement s and the time t is parabolic in nature.

23. Angular Velocity—We have already seen in Art. 16 that when a rigid body moves *round* any straight line as its *axis of rotation*, all the particles of the body move in circles about the axis as the central line. The angular velocity is said to be *uniform* when equal angles are swept over in equal intervals of time. The **angular velocity** of the body is measured by the angle swept out in unit time by a line joining any point of the body to the centre of rotation, provided that *the motion is uniform*. Let a particle of the body move through an arc MN in one second (Fig. 34). The angle at the centre O subtended by the arc MN is equal for all points of the body and is known as its angular velocity. It is generally expressed in radians per second and is usually denoted by ω.

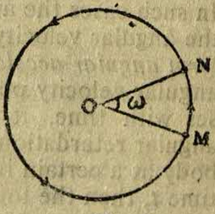

Fig. 34

Let a body, moving with a uniform angular velocity ω, describe angle θ in time t.* Then $\theta=\omega t$, or $\omega=\theta/t$. Again, the angle described in one complete revolution is 4 right angles or

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* Let θ be the angular rotation of a body at *the end of* an interval of time t and let $\theta+d\theta$ be the angular rotation of the body at the end of an interval of time $t+dt$, where $d\theta$ and dt are both infinitesimally small as compared to θ and t. Then the angular velocity at time t is denoted by $\frac{\theta+d\theta-\theta}{t+dt-t}=\frac{d\theta}{dt}$. When uniform, the coefficient $\frac{d\theta}{dt}$ is constant for all values of θ and t. When variable, this differential co-efficient varies with time.

2π radians. If the time for one complete revolution be T and if the number of revolutions per second be n, then—

$$\omega T = 2\pi, \text{ whence } \omega = \frac{2\pi}{T} = 2\pi n. \qquad \cdots \quad (23, 1)$$

Here T is called the period of rotation and n is known as the frequency of rotation in units of cycles per sec.

Relation between Angular and Linear Velocities—Let the length of the arc described by the particle in unit time be v and let the radius of the circular path be r. Then the angle subtended by this arc at the centre is v/r radians ;

$$\therefore \text{ angular velocity } \omega = \frac{v}{r} \text{ radians per second}$$

$$\text{whence} \qquad v = \omega r. \qquad \cdots \quad (23, 2)$$

Hence the speed of rotation of any particle of the body is proportional to its distance from the axis of rotation and is given by the product of this distance and the angular velocity. Thus in the case of a rotating wheel, a point on the rim has greater speed than that of any point nearer the centre.

Angular Acceleration—The speed of the revolution of a body may often change ; for example, when the flywheel of an engine is in a state of acquiring speed or when slowing down. In such cases the angular velocity also changes with time. If the angular velocity increases at constant rate, we call it a *uniform angular acceleration*, and it is measured by the change of angular velocity per second. When the angular velocity decreases with time, it is called a *negative angular acceleration* or angular retardation. If ω_1 be the angular velocity of a rotating body at a certain instant and ω_2 be its angular velocity after a time t, then the total change in angular velocity during an interval t is $\omega_2 - \omega_1$. If the angular acceleration is uniform, then the *angular acceleration* during the interval is $(\omega_2 - \omega_1)/t$. But if the angular acceleration is variable, the *average angular acceleration* during the interval is given by the same expression.*

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* In general, if $d\omega$ represents an infinitesimal increase of angular velocity in an infinitesimal interval dt at the end of time t, then the angular acceleration at that instant is represented by $\frac{d\omega}{dt}$. If the angular acceleration is constant, then $\frac{d\omega}{dt} = k$ (constant) or, $d\omega = k\,dt$.

Integrating between two limiting values of ω_1 and ω_2 corresponding to the times t_1 and t_2, we get

$$\int_{\omega_1}^{\omega_1} d\omega = k \int_{t_1}^{t_2} dt, \text{ or, } \omega_2 - \omega_1 = k(t_2 - t_1)$$

whence $\frac{\omega_2 - \omega_1}{t_2 - t_1}$ = constant angular acceleration between the given limits.

Example :

A flywheel rotates about a fixed axis at the rate of 150 revolutions per minute ; find the angular velocity of any point on the wheel. What is the velocity, if the radius of the wheel is $3\frac{1}{2}$ ft. ?

Ans. The angle described per minute $=(150 \times 2\pi)$ or 300π radians.

So the angular velocity $\omega = \frac{300\pi}{60}$, or, 5π radians per sec.

Hence the required linear (speed) velocity $\omega r = (3\frac{1}{2} \times 5\pi)$ or 55 ft. per sec.

24. Composition of Velocities—If a body possesses simultaneously more than one velocity, it may, due to the combined effect of the velocities, be at rest or in motion with a definite single velocity. In the latter case, the effect of the single velocity is the same as the combined effect of the separate velocities. The single velocity is called the **resultant velocity** and the several velocities are called its **components.** The process of finding the resultant when the components are given is known as **composition of velocities.**

To find the resultant of velocities along the same straight line :

(*i*) If a body has two or more velocities along the same line and in the *same direction*, the *resultant* is a single velocity in that direction and is of magnitude equal to the *algebraic sum* of the components.

(*ii*) If a body has two velocities exactly in *opposite directions*, the resultant is a single velocity in the direction of the greater velocity and its magnitude is equal to the *difference* of the two velocities.

For example, suppose that a railway train is running uniformly with a velocity of 30 miles an hour. A passenger, sitting in a comparment, has also the same velocity impressed on him. Now if he walks inside the compartment at the rate of 4 miles an hour *in the direction of the motion of the train*, two velocities are impressed upon him along the same direction and his resultant velocity is 34 miles per hour. But when he moves in the opposite direction with the same velocity, his resultant velocity becomes 26 miles per hour along the direction of motion of the train.

To find the resultant velocity when components are inclined :

If a body has two velocities not along the same straight line but inclined at an angle to each other, the magnitude and direction of the *resultant* is obtained by the **parallelogram law.**

Parallelogram of Velocities—*If a particle simultaneously possesses two velocities represented in magnitudes and directions by the two adjacent sides of a parallelogram drawn from a point, their* **resultant** *is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.*

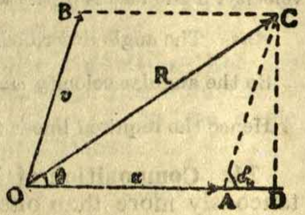

Fig. 35—Parallelogram Law

Let the sides OA and OB of a parallelogram (Fig. 35) represent in magnitudes and directions the velocities u and v respectively acting upon a particle at O and let the directions of the velocities be inclined to each other at an angle α. Complete the parallelogram OACB and join the diagonal OC. Then the resultant velocity R acting upon the particle is represented in magnitude and direction by the diagonal OC.

Produce OA and from C draw CD perpendicular to it. Let the direction of the resultant OC make an angle θ with the direction of the component u, so that $\angle AOC=\theta$. Now the $\triangle ODC$ being right-angled at D,—

$$
\begin{aligned}
OC^2 &= OD^2+CD^2=(OA+AD)^2+CD^2 \\
&= OA^2+2OA.AD+AD^2+CD^2 \\
&= OA^2+AC^2+2OA.AC\cos\alpha.
\end{aligned}
$$

Since, $AD^2+CD^2=AC^2$ and $AD=AC\cos\alpha$.

$$\therefore\quad R^2=u^2+v^2+2uv\cos\alpha. \qquad \cdots \quad (24,1)$$

$$\text{Also}\quad \tan\theta=\frac{CD}{OD}=\frac{CD}{OA+AD}=\frac{v\sin\alpha}{u+v\cos\alpha} \qquad \cdots \quad (24,2)$$

Thus, knowing u, v and α, the values of R and $\tan\theta$ may be calculated. Further referring to the Table of tangents, the value of θ in degrees can be found. In general, the resultant velocity is inclined at an angle $\tan^{-1}\dfrac{v\sin\alpha}{u+v\cos\alpha}$ to the direction of the velocity u.

25. Composition of Vector quantities—Since the displacement also is a vector quantity, it has a magnitude and direction, and the parallelogram law can be applied to a body under the action of two displacements acting simultaneously or successively. We can state the *parallelogram of displacements thus :*

If two displacements, represented in magnitudes and directions by the adjacent sides of parallelogram, act upon a body either simultaneously or successively, their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point. The magnitude and direction of resultant displacement may be found exactly in the same way as in the parallelogram law of velocities.

It is evident from figure 35 that vector of length OA, representing a velocity or displacement along with a vector OB of another velocity or displacement produce a resultant vector OC. Since OB is equal and parallel to AC, we may also state that a vector OA and a vector AC produce a resultant vector OC. Thus instead of drawing a parallelogram, the resultant may be found by completing the triangle with the two vectors as its two sides as shown in figure 36. If OA and AC represent two velocities or displacements acting upon a particle, their resultant is represented by the third side of the triangle taken in the opposite direction. In terms of **vectors** we can write the equation in the form,—

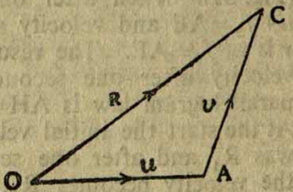

Fig. 36—Vector sum

$$\overrightarrow{OA}+\overrightarrow{AC}=\overrightarrow{OC} \qquad \cdots \quad (25,1)$$

It is to be remarked that the algebraic sum of the length of OA and AC of a triangle is always greater than the third side OC. Such an algebraic summation is called a *scalar* summation. So OA and AC without any arrow-head signs are called the *scalar values* of the corresponding vectors. But the two vectors $\overrightarrow{OA}$ and $\overrightarrow{AC}$ represented in magnitudes and directions, would combine to make up a vector $\overrightarrow{OC}$. This is called a *vector summation* being represented by the equation (25,1).

Parallelogram of Accelerations—The parallelogram law applies also in the case of two or more **accelerations** imposed on a body. Suppose a body simultaneously possesses two accelerations f_1 and f_2 represented magnitudes and directions by OA and OB respectively (Fig. 35) acting at an angle α with each other.

The resultant acceleration is given by,

$$f=\sqrt{f_1{}^2+f_2{}^2+2f_1f_2\cos\alpha} \text{ and } \tan\theta=\frac{f_2\sin\alpha}{f_1+f_2\cos\alpha} \qquad \cdots \quad (25,2)$$

Since an acceleration has got some magnitude and direction, it can be treated as a vector. So f_1, f_2 and f can be expressed as vectors

$\vec{f_1}, \vec{f_2}$ and $\vec{f}$. Then $\vec{f_1}+\vec{f_2}=\vec{f}$ ... (25,3)

Geometrical Proof.—Let u and v represent the components of an initial velocity R, resolved along accelerations f_1 and f_2 (Fig. 37). Then after one second the velocity along u will be $u+f_1=$AE and velocity along v is $v+f_2=$AF. The resultant velocity after one second by parallelogram law is AH$=R_2$. At the start the initial velocity was R_1 and after one second the velocity becomes R_2. By triangle of velocities the resultant of AD and DH is AH. Therefore the difference of velocities between R_2 and R_1 is DH$=f$. So f represents the resultant acceleration due to the accelerations f_1 and f_2. It is clear from the diagram that GDKH is a parallelogram with sides DG$=f_2$ and DK$=f_1$. This proves the theorem on parallelogram of accelerations.

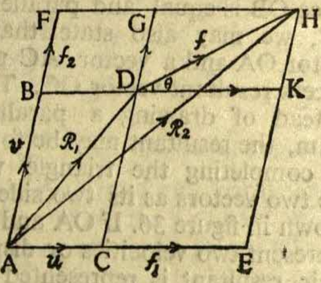

Fig. 37

26. Triangle of Velocities—This is a theorem of explaining how three velocities acting upon a particle produce no motion, which is known as a condition of equilibrium. The theorem states that if a body simultaneously possesses *three* velocities represented in magnitudes and directions by the sides of a triangle **taken in order**, the resultant is zero, *i.e.*, the body remains at rest. In figure 38, three velocities represented by AB, BC and CA of the triangle ABC act upon a particle. By applying the parallelogram law of velocities, the resultant of any two of these velocities, *e.g.*, AB and BC, which are the same as AB and AD, is found to be AC. Consequently, three velocities AB, BC and CA combine to become two velocities AC and CA. Now these two velocities being equal and

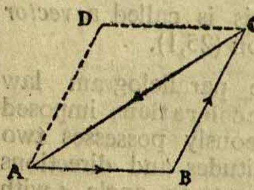

Fig. 38

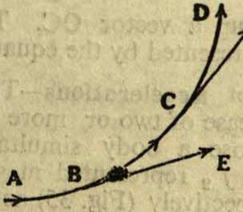

Fig. 39

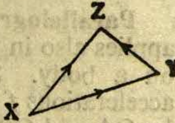

Fig. 40

opposite, the *resultant* of the three velocities AB, BC and CA becomes nil. This is also a *composition of vectors*.

The principle of the triangle of velocities may also be applied to find the change in velocities *during an interval* when a body is moving over a curved path. Imagine a particle to be moving along a path ABCD (Fig. 39). Let its velocity at B be represented in magnitude and direction by the straight line BE, and that at C by the line CF. It is evident that there would be a continuous change of velocity as the particle moves from B to C, the initial velocity being BE and final velocity being CF.

Draw a line XY equal and parallel to BE and another line XZ equal and parallel to CF (Fig. 40). Complete the triangle by joining YZ. Then by the parallelogram law or vector summation,—

$$\overrightarrow{XY}+\overrightarrow{YZ}=\overrightarrow{XZ} \quad \text{or,} \quad \overrightarrow{XY}-\overrightarrow{XZ}=\overrightarrow{YZ}.$$

Thus the change of velocities *during the interval* is represented in magnitude and direction by the line YZ.

Examples :

1. Velocities of 12 ft. per sec. and 15 ft. per sec. act upon a particle and are inclined to each other at an angle of 30°. Find the magnitude and direction of the resultant velocity.

Ans. Let $u=12$ ft. per sec. and $v=15$ ft. per sec.

Then, $R^2=12^2+15^2+2\times12\times15\cos30^\circ$

$=369+360\times{\cdot}866=680{\cdot}76$, whence $R=26{\cdot}08$ ft. per sec.

Again, $\tan\theta=\dfrac{15\sin30^\circ}{12+15\cos30^\circ}=\dfrac{7{\cdot}5}{12+12{\cdot}99}=\dfrac{7{\cdot}5}{24{\cdot}99}={\cdot}3.$

With reference to the table of tangent, $\theta=16^\circ42'$ with respect to the direction of the velocity of 12 ft. per sec.,

or, $\theta=\tan^{-1}{\cdot}3$.

2. The wind blows from a point intermediate between north and east. The southernly component of velocity is 5 m.p.h. and the westerly component is 12 m.p.h. What is the velocity with which the wind blows ? [Del. U.—1948]

Ans. Since directions south and west are at right angles to each other, the two component velocities also are at right angles to each other. Hence, if R be the resultant velocity of the wind, then

$\therefore R^2=5^2+12^2+2\times5\times12\times\cos90^\circ=169.$

i.e., the resultant velocity $=\sqrt{169}=13$ miles per hour.

3. A particle is found to move with a velocity of 10 ft. per sec. due east at a certain instant. After 2 minutes it is found to move with the same velocity due north. Find the change in velocity during the interval.

Ans. Although the velocities, at the initial and final stages, remain the same there is change in velocity. Let BC represents the velocity due east and BA the velocity due north, both being equal to 10 ft. per sec. (Fig. 41). Then from the triangle of velocities, velocities BC and CA combine to produce BA. Hence the change in velocity is CA. Now if D be the middle point of AC,

$$DB = CD = BC \sin 45^\circ = 10 \times \cdot 707.$$

$$CA = 2DB = 14\cdot 14.$$

Thus during the interval the change in velocity is 14·14 ft./sec. due north-west.

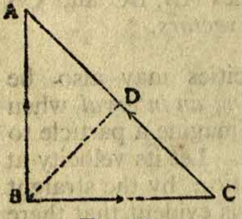

Fig. 41

27. Resolution of Velocity—Let AC represent in magnitude and direction a velocity which is to be resolved into two components (Fig. 42). Draw a parallelogram ABCD with AC as diagonal. Then the adjacent sides of the parallelogram AB and AD will represent in magnitudes and directions the two required components. But since any number of parallelograms can be drawn with a given line as diagonal, a velocity such as AC can be resolved into an *infinite number* of pairs of components. The adjacent sides of any parallelogram AGCH or AECF, for example, drawn with AC as the diagonal, may as well represent the components.

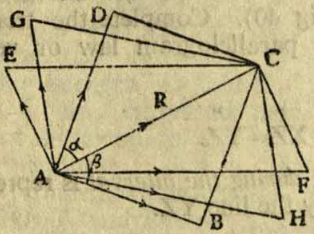

Fig. 42

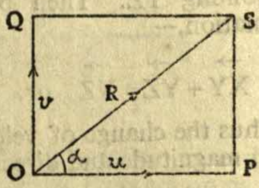

Fig. 43

Let ABCD be a parallelogram of which the angles made by AD and AB with the diagonal AC are α and β. Since $\angle DAC = \angle ACB$, and $\angle DAB = \alpha + \beta$, and consequently $\angle ABC = 180^\circ - (\alpha + \beta)$. Hence from the (trigonometrical) law of sines of a triangle, if AC=R, it follows that,—

$$\frac{AB}{\sin\alpha} = \frac{BC}{\sin\beta} = \frac{AC}{\sin\,[180^\circ - (\alpha+\beta)]}$$

$$\therefore\quad AB = R\frac{\sin\alpha}{\sin\,(\alpha+\beta)} \text{ and } BC = \frac{\sin\beta}{\sin\,(\alpha+\beta)} \quad \cdots (27{,}1)$$

The components are very often resolved at right angles to each other. In this case each component is said to be a **resolute** or the *resolved part* in that particular direction.

Let OS (Fig. 43) represents in magnitude and direction a given velocity R, which is to be resolved into two mutually perpendicular directions OP and OQ. Let the angle POS be denoted by α. From O draw OQ perpendicular to OP. From S draw SP $\perp$ to OP and SQ $\perp$ to OQ. Then by the parallelogram law, OP and OQ represent the required components u and v respectively.

Since OP=OS cos α and OQ=OS sin α, we get

$$u = R\cos\alpha \text{ and } v = R\sin\alpha. \qquad \cdots \quad (27,2)$$

Evidently, if α be given which means that if the direction of one of the components be specified, then both the components become definite in magnitudes and directions.

Resolution of an Acceleration—In a similar way, it can be proved that if OS (Fig. 43) represents a given acceleration f in magnitude and direction, the *resolved parts* of the acceleration along the directions OP and OQ will be represented by $f\cos\alpha$ and $f\sin\alpha$ respectively.

28. Relative Velocity—The relative velocity is defined to be the apparent motion of one body *relative* to another, which may either be at rest or in motion. In this case, if the distance between the two is found to change from time to time, each is said to have a velocity *relative* to the other. Thus, the **relative velocity** of a particle A with respect to a second particle B is the rate at which A changes its position relative to B.

When both the bodies move in the *same direction with the same velocity*, the relative velocity of each with respect to the other is zero. For example, if two trains are running on parallel lines along the same direction with the same velocity, then to a passenger in one train, it would appear that the other train is at rest.

If two railway trains A and B run on parallel lines in the same direction but if A has a greater velocity than B, then to a passenger in A, B will appear to move *backwards*, while to a passenger in B, A will appear to move *forwards*. If the two trains move in opposite directions, to a passenger in one, the other would appear to be moving away faster than its own

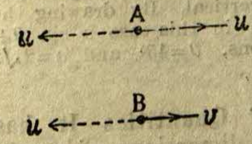

Fig. 44

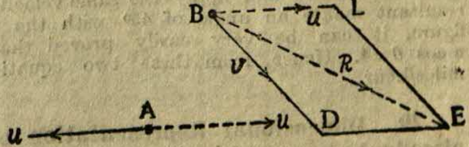

Fig. 45

speed. For a similar reason, when a man is running in a shower of rain, the rain-drops, though falling vertically, appear to strike his face in a slanting direction.

In general, the *relative velocity* of any body B with respect to another body A, is obtained by taking A to be at rest and compounding with the velocity of B a velocity equal and opposite to that of A. The resultant thus obtained gives the required *relative velocity* of B in magnitude and direction. For example, imagine that two bodies A and B are moving parallel to each other in the same direction with velocities u and v respectively (Fig. 44). To find the relative velocity of B with respect to A, impose an equal and opposite velocity u on A and B. Thereby A is brought to rest and B moves with a velocity $u-v$. The relative velocity of two bodies moving with different velocities in different directions is shown in figure 45. Here A is moving backwards with velocity u, and B is moving with velocity v along BD. To find the relative velocity of B with respect to A, the latter is brought to rest by imposing an opposite velocity in the forward direction. An equal and parallel velocity u is imposed on B along BL and their resultant R is obtained. B then appears to be moving along the resultant direction BE when observed from A. It is found that *laws of mechanics are the same on two bodies* having a relative uniform motion with respect to each other.

Example :

To a man walking at the rate of 2 miles an hour, the rain appears to fall vertically. When he increases his speed to 4 miles an hour, it appears to meet him at an angle of 45° ; find the real direction and speed of rain.
[Pat. U.—1951 ; Utkal U.—1953]

Ans. Let u=velocity of the rain in miles per hour and θ=the direction of the rain with vertical in degrees. Suppose that the man is walking towards east. To bring him to rest, impose a velocity on him of 2 miles/hour towards west and also apply the same velocity on the falling rain drops. The resultant velocity of the rain drops according to the problem has a vertical direction. Thus $u \sin \theta = 2$. Again when he walks 4 miles/hour towards east, bring him to rest by imposing an opposite velocity and apply the same velocity on the rain drops. The resultant makes an angle of 45° with the vertical. By drawing the figure, it can be very easily proved that in this case $u \sin \theta + u \cos \theta = 4$. Hence, from these two equations, $\theta = 45°$ and $u = 2\sqrt{2}$ miles/hour.

29. Dimensional Representation of Quantities—It has already been stated in Art. 13, that the dimensions of length, mass and time are represented respectively by the letters L, M and T. Since velocity is the distance covered by a body in unit time, it can be obtained by the quotient of total length covered and total time. Hence the dimension of velocity is L/T or LT^{-1}. The same is the dimension of variable or uniform speed. If a body possesses a velocity of 10 cms./sec., we may as well express it as $10LT^{-1}$ C.G.S. units. Acceleration is the rate of change of velocity and hence it has the dimension of velocity/time or LT^{-1}/T, *i.e.*, LT^{-2}. For example, an acceleration of 3 ft./sec.^{2} may be expressed as $3LT^{-2}$ F.P.S units. A retardation has a negative sign before it.

An angle expressed in radians is the quotient of arc and radius. The dimension of an angle is therefore L/L or unity. Hence an angle has no dimension. An angular velocity is expressed as an angle divided by time. Hence its dimension is $L^{\circ}T^{-1}$.

It is interesting and instructive to note that in an equation involving physical quantities, dimensions on both sides of the equation ought to be identical. In fact the identity of dimension is a true test of the validity of an equation. Thus, taking the equation, $s=ut+\frac{1}{2}ft^2$, the left hand side term has the dimension of L ; ut has the dimension of $LT^{-1}\times T=L$ and ft^2 has $LT^{-2}\times T^2=L$. Thus the equation is true.

[To facilitate quick calculations in mathematical problems students are required to remember the following equivalences :

30 miles per hour=44 ft. per sec.

1 mile per hour=44·703 cms. per sec.

π=3·1415 ; 3·1415 radians=180°

1 cm. per sec.=·0328 ft. per sec.]

EXERCISES ON CHAPTER II

Reference

Art. 20 1. An object needs 3 seconds to fall down vertically from rest from a height of 144 feet. Find its acceleration. [W.B.H.S—1971]

Ans. 32ft./sec.^{2}.

Art. 20 2. Derive the relation $s=ut+\frac{1}{2}ft^2$. (Del. H. S.—1949 ; Anna. U.–1950)

Art. 20 3. A train, starting from rest, travels with uniform acceleration and acquires a velocity of 60 miles per hour in 2 minutes. Find the acceleration and the distance passed over in the time. (C. U.—1957)

Ans. $f=\frac{11}{15}$*ft.* per sec.^{2} ; 5280 ft.

Art. 20 4. A train travelling at the rate of 30 miles per hour is stopped in 1 minute by putting a brake. What is the retardation and for what distance was the break applied ?

Ans. $\frac{11}{15}$ ft./sec.^{2} ; $\frac{1}{4}$ of mile.

Art. 20 5. A body moving with uniform acceleration describes 76 ft. in the 5th second and 116 ft. in the 10th second. Find the distance it passes over in the 15th second.

Ans. 156 ft.

Reference

6. A train starts from rest with an acceleration of 0·5 ft./sec.^{2} and at the same time a dog starts off and runs parallel to it with a constant velocity of 16 miles an hour. How far will the dog have gone before the train overtakes him and what time will have elapsed ? Art. 20

Ans. 2203·2 ft. ; 93·86 seconds.

7. Two trains leave the same station on parallel lines. The first starts from rest with a uniform acceleration of ·5 ft./sec.^{2} and attains a maximum velocity of 15 miles per hour which is kept constant. The other starts from rest 40 seconds after the first with a uniform acceleration of 1 ft./sec.^{2} and attains a maximum and constant velocity of 30 miles per hour. At what distance from the station and how long after leaving will the second pass the first ? Art. 20

Ans. 1760 ft. ; 62 secs., when both the trains will have uniform speeds.

8. The distance between the sun and the earth is 93×10^6 miles. Assuming that the earth moves round the sun in a circular orbit once in 365 days, find its angular velocity in radian per sec. and linear velocity in miles per sec. Art. 23

Ans. $1{\cdot}99\times10^{-7}$ radians/sec. ; 18·5 miles/sec.

9. State and prove the law of parallelogram of velocities. (Utkal U.—1955 ; Del. U.—1958) Art. 24

10. A river has got a current of 5 miles an hour and a boat is rowed across it with a speed of 4 ft./sec. at an angle of 60° with the bank down the current. Find the resultant speed. Calculate the time to cross the river, if its width is half a mile. Art. 24

Ans. 9·9 ft./sec. ; 12 min. 41 secs..

11. A body moving with a velocity of 8 cms. per sec. has an acceleration of 6 cms. per sec.^{2} applied to it in a direction making an angle of 90° with its original direction of motion. Calculate its final velocity and its displacement after 1 second. Art. 24

Ans. 10 cms. per sec. ; 8·5 cms.

12. Rain drops are falling to the earth vertically with a speed of 4 ft. per second. What should be the speed of a cyclist on a level road at which the rain drops appear to strike him at an angle of 30° to the vertical ? Art. 28

Ans. 2·3 ft./sec.

13. To a motorist going north at 30 miles per hour, the wind appears to blow from north-west. Actually it is blowing from west. What is its true velocity and what is its apparent velocity ? Art. 28

From what direction and with what velocity would the wind appear to blow to a motorist going south at 15 m. p. h. Illustrate your explanation by diagrams.

Ans. 30 m.p.h. ; 42·4 m.p.h. ; 26°34′ S of W ; 33·5 m.p.h.

14. Explain what you mean by 'relative velocity' and how it can be measured. (C. U.—1960 ; Utkal U.—1954) Art. 28

CHAPTER III

FORCE : NEWTON'S LAWS OF MOTION

30. **Inertia and Force**—It is an inherent property of a material body that without any external agency it cannot change its state of rest or of motion. If the body is found to be at rest, it continues to be at rest for all time to come. If again, it is found to be moving, then in the absence of any retarding agency it keeps on moving for all time to come. This property of a material body is called its **inertia**. Whenever a change of state is brought about, it must be due to some external cause, which is termed a **force**. Hence a **force** *is that which acting on a body changes or tends to change the state of of rest or of uniform motion of a body*. It means that any change in *the state of intertia of a material body* is brought about by a force. A force must have its (*i*) *point of application*, that is, the point at which the force acts, (*ii*) *direction* and (*iii*) *magnitude*. A force is thus a vector.

31. **Newton's Laws of Motion**—The three laws of motion, first enunciated by Sir Issac Newton (1643—1727), are based on general observations of motion of material bodies and form the basis of Dynamics. The following laws were first published in his famous book ''Philophiae Naturalis Principia Mathematica" in 1686. The three laws are stated thus :

First Law—*Every body continues in the state of rest or of uniform motion in a straight line, except in so far as it is compelled by some external impressed force to change that state.*

Second Law—*The rate of change of momentum of a body is proportional to the impressed force and takes place in the direction of the straight line along which the force acts.*

Third Law—*To every action there is an equal and opposite reaction.*

32. **Explanation of the First Law**—This law consists of two parts. The first of these implies that a material body can not of itself change its state of rest or of uniform motion. It also states that a moving body, when not acted upon by an external force, will have **free motion**, that is, *uniform motion in a straight line*. Previous to Newton, Galelio Galilei (1564—1642) made several experiments on the motion of balls on an inclined plane and gave a preliminary idea that the motion of a body over a perfectly smooth horizontal surface would never stop. It is not possible to verify this experimentally, for we cannot make a body perfectly free from external forces. Thus, owing to the resistance of the ground and of air, a very smooth

ball, rolled on the ground, soon comes to rest. If the surface were more smooth, the ball would roll on for a longer time and distance. The nearest approach to a free motion is the motion of a glass ball over a smooth sheet of ice or on a marble floor in the absence of wind.

The second part of the law furnishes us with an idea regarding the function of a force. Thus (*i*) a force changes or tends to change the state of rest of a body ; and (*ii*) a force changes the state of motion of a body, by increasing the speed or by decreasing it. The law also implies that the force, which is to produce a change in the state of rest or motion of a body, must be externally *impressed*, that is acting *from outside* on the body and not *stresses* or internal forces between the parts of the body. It is then evident that the effect of inertia may be of two kinds, namely :

(*i*) *inertia of rest*, which means that a body at rest has a tendency to remain at rest ;

(*ii*) *inertia of motion*, which indicates that a body in motion tends to preserve its motion.

Illusrtations of Inertia of Rest—To show the effect of inertia of rest of a body, take a ball B and place it on a rectangular card C just above a hollow cup fixed on a vertical stand S (Fig. 46). The base is provided with a metal spring R which can be swung aside and fixed to a clamp L on one side. Now fix the spring to the clamp as shown in the figure. On releasing the clamp, the spring jumps back and strikes the card which is thrown off, while the ball owing to its inertia of rest does not move forward but slips down into the cup.

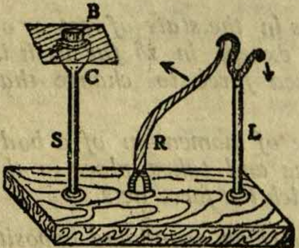

Fig. 46

A bullet fired against a window pane makes a clean hole and not cracks. The glass surface, near about the place where the bullet strikes, cannot share a quick motion of the bullet and remains at rest. When a motor car or a tram car starts and rapidly picks up speed, a passenger standing or sitting loosely in the car falls backwards. The lower part of the body, being in contact with the car, shares the motion but the upper part tends to maintain a state of rest. In beating the dust off a coat by a stick, the coat is suddenly set into motion, while the loose dust particles, tending to remain at rest, fall off.

the rope and its other end is coiled over a wheel. The top of the crane (Fig. 62) is in equilibrium under the action of *three* forces, *viz.*, (1) the load W hanging vertically downwards, (2) the thrust R on the jib, and (3) the pull P of the tie-rope. Let *ab* represent W in magnitude and direction. Draw *bc* parallel to the tie-rope and *ac* parallel to the jib, meeting each other at *c*. Then *bc* and *ac* represent P and R respectively. Thus the three forces, being represented in magnitudes and directions by the sides of the triangle *abc*, taken in order, are in equilibrium. In this condition a *little more* pull by the tie-rope will lift the load with a slow motion. It is evident that the heavier is the load pulled by the tie-rope, the greater is the thrust on the jib. So the jib rod must be very rigid and the base of the crane must be very stout.

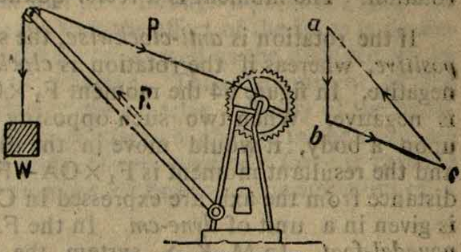

Fig. 62. Simple Crane

42. Moment of a Force—Let a solid wheel be fixed about an axle AB about which it can rotate freely (Fig. 63). To give it a rotation the easiest way is perhaps to hold its rim at any point and to pull it tangentially. If we pull in any direction other than the tangent, then we have to exert a greater force to give it the same amount of motion. If the wheel is provided with a number of slots to which nails can be fixed we can fix up a nail at any slot and tie a piece of cord to the nail, and by pulling the cord in any direction we can set the wheel in motion. By fixing up nails at dtfferent slots and pulling the cord in different directions. We find that the **rotatory effect** of a force in such a case depends on (*i*) the magnitude of the force, and (*ii*) the line of application of the force with respect to the axis or, more properly, the perpendicular distance of the line of action of the force from the *axis of rotation.*

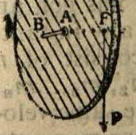

Fig. 63

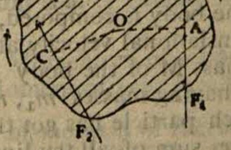

Fig. 64

The tendency of a force to produce rotation of a body is measured by a term 'moment' of the force. The **moment of a force** about an axis is measured by the product of the force and the perpendicular distance of its line of action from the axis. Since the rotating body may be of any shape and size whatsoever, we may generalise that if a body, whose axis of rotation is at O, be acted upon by a force F_1 and if OA be the perpendicular distance of its line of action from O, then the moment of the force is $F_1 \times OA$. The perpendicular distance is called the *arm of the force.* Thus it becomes easier to open or to close a door by turning about its hinges, if the force is applied perpendicular to the surface of the door and as far away from the hinges as possible. Again the moment of a force vanishes, if (*i*) the force itself is zero or (*ii*) its line of action passes through the axis of rotation. The moment is a *vector* quantity.

If the rotation is *anti-clockwise*, the sense of motion is taken to be *positive*, whereas if the rotation is *clockwise*, the sense of motion is negative. In figure 64 the moment $F_1 \times OA$ is positive, while $F_2 \times OB$ is negative. When two such opposing moments simultaneously act upon a body, it would move in the direction of the larger moment and the resultant moment is $F_1 \times OA - F_2 \times OB$. If the force and the distance from the axis are expressed in C.G.S. units, then the moment is given in a unit of *dyne-cm*. In the F.P.S. system it is expressed as *poundal-foot*. In M. K. S. system the unit is newton-metre. The dimension of moment is the product of a force and distance which is $MLT^{-2} \times L = ML^2T^{-2}$.

So, if a rigid body capable of turning about a fixed point is acted on by two forces, such that their moments about the point are equal in magnitude but opposite in sense, the body remains at rest. If a body, under the action of any number of coplanar forces, be at rest, then the *algebraic* sum of moments of all forces about *any* point in their plane is zero.

43 Moment of Inertia and Angular Momentum—It has already been explained in Arts. 31 and 33 that the inherent property of a material body is to maintain its state of rest or its state of motion which is called its *inertia*. Due to the inherent property the body it requires an external force to change it's state of rest or of motion and the force is, by Newton's Second Law, proportional to the acceleration of the body. The constant of proportionality between the force applied and the acceleration generated is called the *mass*. So inertia is not exactly the mass but a property associated with it which tends to conserve its state of rest or of uniform motion. But it is an observed fact that if mass is increased, the force required to give it an equal acceleration increases proportionately and this is due to the increased inertia of the mass.

When a material body possesses a rectilinear motion (Art. 17), each elementary particle, of which the body is composed, has got an equal translational velocity at any instant. Let the mass M of the body consist of n particles having masses $m_1, m_2, m_3, \ldots m_n$. Since each particle has got the same velocity v, the sum of all the linear momenta of all the particles of the body is given by $m_1v + m_2v + \ldots + m_nv = (m_1 + m_2 + \ldots m_n)v = Mv$. ... (43,1)

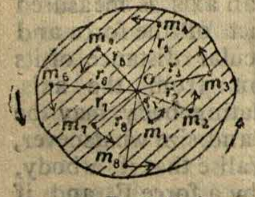

Fig. 65

To change this linear momentum a force is necessary and this is due to its property of inertia.

If now this body revolves uniformly round a fixed axis through O with an angular velocity ω, its *linear momentum* is zero, since the body as a whole has no motion of translation (Fig. 65). But to increase or decrease the angular velocity, a force would have to be applied on the body whose *line of action will pass through a point not lying on the axis* of rotation. This means that the effect of the force is dependent upon the magnitude of the force as well as the distance of its line of action from the axis of rotation. Thus the moment of the force can only produce a change in the angular velocity of the body. It is evident that, while a force produces a change in the instantaneous linear momentum of a body, the *moment of a force* is capable of producing a change in the moment of instaneous momentum of a rotating body.

Consider now the particles $m_1, m_2, \ldots m_n$ at distances $r_1, r_2, \ldots\ldots r_n$ respectively from the axis of rotation at O (Fig. 65). Each particle is having a uniform circular motion with an angular velocity ω. Let $v_1, v_2, \ldots, v_n$ be their tangential velocities at any point in their paths. Then the linear momenta of the particles at any instant are given by $m_1v_1, m_1v_2, \ldots\ldots, m_nv_n$. Now taking moments of all these momenta about the axis of rotation and adding them, we get the expression.

$$m_1v_1r_1 + m_2v_2r_2 + \ldots + m_nv_nr_n.$$

But we know $v_1 = r_1\omega, v_2 = r_2\omega, \ldots\ldots, v_n = r_n\omega$. Thus by substituting these values, we get the following condition.—

Moments of instantaneous momenta of the particles composing the body $= (m_1r_1^2 + m_2r_2^2 + \ldots m_nr_n^2)\omega$

$$= \omega\Sigma mr^2 \qquad \ldots(43{,}2)$$

where as usual Σ denotes the summation of the product of all the masses of the particles and the corresponding squares of their distances from the axis of rotation.

The term Σmr^2 is called the Moment of Inertia of the body about the given axis of rotation and is generally denoted by I. The value of the moment of inertia depends upon the mass and shape of the body as well as on the position of the axis of rotation. The moment of momentum is also called the *angular momentum*. Thus we can write :

Angular Momentum of a body $= \mathrm{I}\omega$
$=$ Moment of Inertia × Angular Velocity.

The unit of moment of inertia in the C.G.S. system is gram × cm.^{2} and its dimension is $[\mathrm{ML}^2]$. If M be the mass of the body, then we can write of $\Sigma mr^2 = \mathrm{M}k^2$, where k has the dimension of a length. The value of k depends upon the shape and the position of the axis of rotation and is called the *radius of gyration* of the body about the given axis of rotation. For evaluation of moment of inertia in cases of regularly shaped bodies reference is made to the Supplementary Appendix C to Intermediate Physics.

44. Parallel Forces—Forces, whose lines of action are parallel, are termed *parallel forces*. Two parallel forces acting along the same direction are said to be **like**, while two parallel forces acting in opposite parallel directions are termed (anti-parallel) **unlike** parallel forces.

Two Like Parallel Forces.—Let p and q be two *like parallel forces* acting at two points A and B of a body (Fig. 66). Since these two parallel forces act along the same direction their resultant is evidently $p+q$. To find the *point of application* of the resultant, we can make the following construction. Join A and B by a straight line. Impose two *equal and opposite forces 's' at A and B*. These two equal forces will not disturb the action on the body. Now compound force $p=$ BD with force $s=$ BE ; their resultant is BF. Again compound force $q=$ AC with $s=$ AG and get the resultant AH.

Now produce HA and FB backwards to meet at a point P. Then P is the point where there is a combination of *four* forces ; p and q acting along PO drawn parallel to BD or AC, also forces ss acting in opposite directions. Hence PO is the direction of the resultant

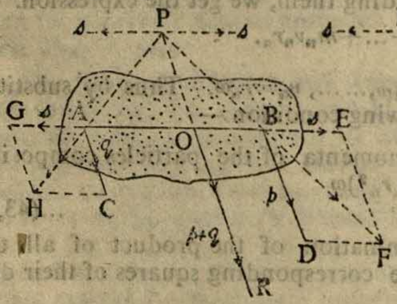

Fig. 66

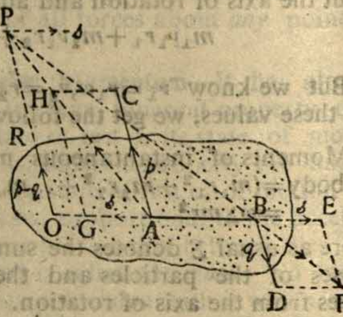

Fig. 67

meeting AB at O If OR of length $p+q$ is drawn parallel to any one force, then OR is the resultant and O is the point of application. To find the position of O, we see that from similar Δs, HCA and AOP,

$$\frac{PO}{AO}=\frac{q}{CH}=\frac{q}{s}.$$

Also, from similar Δ^s BOP and FDB,

$$\frac{PO}{OB}=\frac{p}{DE}=\frac{p}{s}.$$

$$\therefore \quad PO=AO.\frac{q}{s}=OB.\frac{p}{s} \text{ whence } AO\times q=BO\times p \text{ or, } \frac{p}{q}=\frac{AO}{BO}$$

So the point O divides AB *internally* in two *arms* or segments in he *inverse ratio* of two respective forces.

Two Unlike Parallel Forces.—Here p=AC and q=BD (fig. 67). As before impose two equal and opposite forces s and find the resultants. These two resultants AH and FB meet at P. At P then there are two equal and opposite forces s s and two unequal forces p and q acting along OP drawn parallel to any force p or q. If p is greater than q, then the resultant is $p-q$. To find the point of application O, it can be shown from similarity of triangles, that

$$\frac{OP}{p}=\frac{OA}{s} \text{ and also } \frac{OP}{q}=\frac{OB}{s} \quad \therefore \quad p\times OA=q\times OB \text{ or, } \frac{p}{q}=\frac{OB}{OA}$$

So the line BA is produced externally at a point A, so that this point divides BA externally into two arms or segments in *inverse* ratio of the forces. The resultant is the *difference* of the two forces p and q.

44A. Couple—Two unlike parallel forces of equal magnitude constitute what is called a *couple* (Fig. 68). The effect of a couple upon a body is to rotate it and the rotational tendency depends on (*i*) the magnitude of the forces constituting the couple and (*ii*) the perpendicular distance between the two forces. The perpendicular distance is called the **arm** of the couple.

Hence the **moment of a couple** is measured by the product of one of the forces forming the couple and the perpendicular distance between their lines of action. The moment of a couple is also called the *Torque* which is a vector. The moment of a couple in the anti-clockwise direction is taken as *positive*, and that in the clockwise direction as *negative*. A couple can be balanced by another couple when they act in the same plane or in parallel planes and when the moment of one is equal and opposite to that of the other.

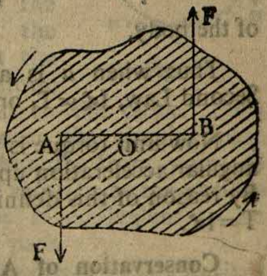

Fig. 68

***45. Application of Newton's Laws in Circular Motion**—Newton's laws of motion can be applied to bodies in circular motion. The laws may be stated thus.—

I. Everybody continues in its state of rest or of uniform circular motion about an axis through its centre of mass, except in so far as it is compelled by some external impressed torque to change that state.

II. The rate of change of angular momentum is proportional to the applied torque and takes place in the same direction of rotation in which the torque acts.

II. To every rotational action there is an equal and opposite reaction.

Explanation—The first law states that a material body of a definite shape and volume cannot of itself change its state of rest or of *uniform circular motion.* If it is found to revolve, its speed of revolution persists. But if the speed of revolution is found to change, its mass, shape and volume remaining constant, then it must have been due to an *external* torque acting upon it. Illustration of this law is obtained in continuous and uniform rotational motion of heavenly bodies. The speed of diurnal revolution of the earth will remain the same at all times until or unless an external torque acts upon it.

The second law gives a quantitative definition of a torque or the turning effect of a couple. If I be the moment of inertia of a body with reference to its axis of rotation and ω be its angular velocity, then the angular momentum of the body is measured by the product of the moment of inertia and angular velocity. Thus the angular momentum is I ω.

Let a body having an initial angular velocity ω_1 be acted upon by a torque T for t seconds and let its final angular velocity be ω_2. Then the change of angular momentum in t secs. $= I\omega_2 - I\omega_1$.

$\therefore$ the rate of change of angular momentum

$$= \frac{I\omega_1 - I\omega_2}{t} = I . \frac{\omega_2 - \omega_1}{t} = I\alpha,$$ where $\alpha =$ the angular acceleration of the body.

Thus when a is a constant to be suitably chosen then by the Second Law, $I\alpha \propto T$, or, $T = aI\alpha$ ... (45, 1)

Now unit torque is now defined to be that which produces a unit angular acceleration upon a body possessing unit moment of inertia. By reason of this definition, the constant a becomes unity.* Hence $T = I\alpha$.

Conservation of Angular Momentum ; Third Law illustrates the principle of conservation of angular momentum. Thus, if a revolving body of angular momentum $I_1\omega_1$ be coupled with another revolving body of angular momentum $I_2\omega_2$, the *total angular momenta of the system*, which is the sum of the two angular momenta, *i.e.*, $I_1\omega_1 + I_2\omega$, *remains the same in any particular direction of rotation.*

Example

A fly-wheel, having a moment of inertia of 100 lb. ft^2 and revolving at the rate of 400 r. p. m. is stopped by a brake in course of 5 seconds. Find the controlling torque. Calculate the force of friction on the brake when it is plaeed on the rim of the fly-wheel, having a radius of 6 inches

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* If instantaneous angular velocity ω be represented by $\frac{d\theta}{dt}$, then the angular acceleration at the end of interval t is $\frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2} = \alpha$. Then we get torque $T = I\frac{d^2\theta}{dt^2}$. This is an well known equation in Dynamics.

Ans. The initial speed$=400$ revolutions per minute$=\frac{20}{3}$ revolutions per second$=n$, say.

$\therefore$ the initial angular velocity$=2\pi n=\frac{2\times 3{\cdot}14\times 20}{3}=41{\cdot}86$ radian/sec.

Now the final angular velocity$=0$ and the duration of interval $t=5$ secs.

$\therefore$ the angular acceleration $\alpha=\frac{41{\cdot}86}{5}=8{\cdot}37$ radians per sec. per sec.

Thus torque $T=100\times 8{\cdot}37=837$ foot-poundals.

Again, force$=$torque$\div$perpendicular distance$=\frac{837}{0{\cdot}5}=1674$ poundals.

46. Uniform Motion in a Circle—Imagine a material particle to be moving uniformly in a circular path. At any point in its path the particle owing to its inertia has a tendency to move along the direction of the tangent at that point. For example, at the points A, B, C, and D in its path of motion, it possesses tangential velocities represented by AM, BN, CP and DR respectively (Fig. 69). Illustrations of tangential motion are found in the following cases: Pieces of mud sticking to the rims of the wheel of a carriage are thrown off tangentially when the wheels rotate rapidly. The effect necessitates the use of *mud-guards* in vehicles. A piece of stone is tied at the end of a string and is whirled round over the head in a circle by holding the other end, in the hand. If the string is suddenly released, the stone is found to fly off in a straight line tangential to the circle. Such cases illustrate the fact that even if the speed over the circular path remains constant, the direction of motion is changing from point to point. Hence *there is a continuous change of velocity*, as the particle moves over the circle. The change of velocity is equivalent to an acceleration towards the centre of the circular path.

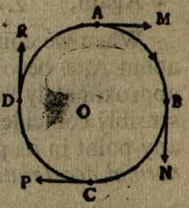


Fig. 69

Normal Acceleration in a Uniform Circular Motion—Let a particle move with uniform speed v along the circumference of a circle of radius r and with centre at O. Let the particle describe a *small* arc AB (Fig. 70) in a *short* interval of time t.

At the point A draw a tangent AD representing the direction of motion of the particle. Similarly draw another tangent BE at B. Let the two tangents meet at C. Cut off two equal lengths CD and CE from two tangents and let these represent velocities at A and B. Then the change in velocities during the period t, in which the particle moves from A to B, is by the triangle of velocities represented in magnitude and direction by the side DE.

Let $\angle AOB$ be θ. Elementary geometry gives $\angle AOB=\angle DCE$.

From C draw a perpendicular CF on DE. Since $\triangle$ DCE is isosceles and perpendicular is dropped from the vertex C,

$$\angle DCF=\angle ECF=\frac{\theta}{2}.$$

Now $DE=2DF=2CD\sin DCF=2v\sin\frac{\theta}{2}$.

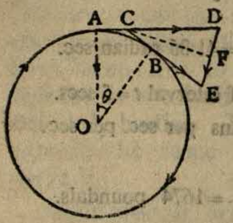


Fig. 70

This value of DE represents the total change of velocity as it moves from A to B. If now we assume that the arc AB grows smaller and smaller until it becomes a *very small arc* at A, the value of DE would then represent the change in velocity at A. Now as the arc becomes smaller, θ also assumes a smaller value and for an infinitely small arc, $\sin\theta=\theta$ in circular measure. Thus

the change in velocity at $A=2v\sin\frac{\theta}{2}=2v\frac{\theta}{2}=v\theta$

Thus the acceleration at A is $=\frac{\text{Change in velocity}}{\text{time}}=\frac{v\theta}{t}=\omega v$ ...(46,1)

where ω is the angular velocity of the particle.

Again, $\angle ADF=180^\circ-\angle DFC\sim\angle DCF=90^\circ-\frac{\theta}{2}$. ... (46,2)

When we consider the acceleration ωv over a *very short segment* about A, θ becomes infinitely small, $\angle ADF$ from equ (46,2) is very approximately 90°. This means that the direction of acceleration sensibly coincides with AO. Hence the acceleration of the particle at any point in its path is towards the centre of rotation. This is called *normal acceleration.* Now since $v=\omega r$, the

normal acceleration $\omega v=\frac{v^2}{r}=\omega^2 r$ ...(46,3)

47. Centripetal Force—Whenever there is an acceleration of a body in a certain direction, it must be assumed from Newton's Second Law that a force acts on the body *in that direction.* Hence, a force must be acting on the body *in a direction towards the centre* of the path of motion to generate its normal acceleration. The force, which acting on a body in a direction towards the centre of the circle makes it move in the circular path, is known as the **centripetal force** (Lat, *peto*, to seek), because the force always seeks or acts towards the centre. If the body be of mass m, the magnitude of the centripetal force is then its mass × acceleration, which is mv^2/r. In the experiment with the stone and the string this force is exerted by the hand, placed at the centre of rotation on the revolving body, through the string in form of tension towards the centre (Fig. 71).

It is to be carefully borne in mind that in all cases where a material body revolves in a circular path. the centripetal force is always to be supplied to the *revolving body* in a direction towards the centre. The *active* force may be any one of the following types, namely,—mechanical tension or thrust, gravitational attraction, electric or magnetic force.

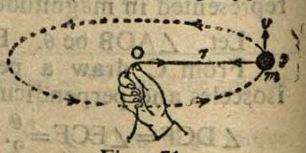


Fig. 71

Since every action has an equal and opposite reaction, the particle moving uniformly in a circle exerts a force, directed away *from the centre* which is called the **centrifugal force** (Lat., *fugio*, to fly). The force is exerted by the rotating body at the centre of the circular path. In the experiment of the stone and the string, this force is exerted by the stone on the finger at O through the string, whereby the string is put under tension. It is to be noted that the centrifugal force is a *reaction* due to the centripetal force. With the stoppage of rotation, the centripetal force vanishes and so the centrifugal force at once cases to act. and so the centrifugal force is sometimes called a *pseudo force.* The unit and dimension of a centrifugal force are similar to those of an ordinary force.

On simple assumptions of a tangential uniform speed and a normal acceleration, the result of a circular trajectory can be established in the following manner. Let δs be the *short length* of an arc passed over by the point mass in a time δt.

The velocity $v=\frac{\delta s}{\delta t}$ and normal acceleration$=\frac{v^2}{r}=f=\frac{1}{r}\left(\frac{\delta s}{\delta t}\right)^2$.

Where r is the instantaneous distance from the attracting origin, Due to normal attraction in a short interval δt it tends to move towards the origin through a small distance δr given by dynamical equation,

$$\delta r=0+\tfrac{1}{2}f\,\delta t^2=\frac{\frac{1}{2}v^2}{r}\,\delta t^2 \text{ also } \delta t=\frac{\delta s}{v}$$

$$\therefore\quad \delta r=\tfrac{1}{2}\frac{v^2}{r}\frac{\delta s^2}{v^2}=\frac{1}{2r}\,\delta s^2=\frac{1}{2r}\,(dx^2+dy^2) \text{ in rectangular co-ordinates.}$$

or, $2r\delta r=\delta x^2+\delta y^2$ or, by integration $2\int r\delta r=\int \delta x^2+\int \delta y^2$
whence $r^2=x^2+y^2$ representing a circle.

Examples :

1. A ball of mass 0·1 lb. at the end of a string is whirled at constant speed in a horizontal plane. If the radius of the circle is 4 ft. and the speed of the ball is 10 ft. per sec., calculate the magnitude of radical acceleration and tension of the string. [C.U.]

Ans. The tangential velocity $v=10$ ft./sec.

$\therefore$ the normal acceleration$=\frac{v^2}{r}=\frac{100}{4}$ ft./sec.$^2=25$ ft./sec.^{2}.

The tension of the spring$=\frac{mv^2}{r}=0{\cdot}1\times 25$ poundals$=2{\cdot}5$ poundals.

2. A bucket containing water is tied to one end of a rope 8 ft. long and rotated about the other end in a vertical circle. Find the minimum number of rotations per minute in order that water in the bucket may not spill.
$(g=32$ ft./sec.$)^2$. [Pat. U.]

Ans. In course of rotation when the bucket is at the topmost position, it has its mouth downwards and water has a chance to spill ; but if the weight of water be equal to the centripetal force necessary for rotation, there would be no resultant force on water and it would not spill. If m be the mass of water, then $m\times 32=m\times 8\times\omega^2=m\times 8\times 4\pi^2n^2$, when $n=$frequency of rotation per second ; whence $\pi^2n^2=1$, or, $n=\frac{1}{\pi}$ per sec.$=\frac{60}{3{\cdot}14}$ per min.$=19{\cdot}05$ rotations per minute.

48. Some Practical Illustrations of Centripetal Forces—The following are a few cases of equilibrium in circular motion :

Motion of Planets—It is well-known that the planets revolve round the sun in definite orbits. For simplicity, let us assume that the orbit is circular and that the sun forming the central mass is fixed in space. Let the mass of the sun be M, and that of a planet be m revolving with a constant speed v in a circular orbit of radius r.

In order that the planet may execute uniform motion on a circle.

centripetal force mv^2/r must have to act on the planet in a direction towards the sun which means that *the sun has got to attract the planet with such an amount of force.* An investigation regarding the origin of this force led Newton to discover the universal law of gravitation, according to which there is mutual force of attraction between the sun and the planet, the force being $G \times Mm/r^2$ (Chap. IV), where G is a constant. This force is equal to the centripetal force mv^2/r acting on the planet.

$$\therefore \quad \frac{GMm}{r^2}=\frac{mv^2}{r}, \text{ whence } M=\frac{v^2r}{G}=\frac{v^2}{r^2}\times\frac{r^3}{G}=\frac{\omega^2r^3}{G}, \qquad \ldots \quad (48,1)$$

where ω is the angular velocity of the planet. The angular velocity of a planet can be measured by astronomical observations. Thus knowing r and G, the mass of the sun can be calculated. The illustration given above is one in which the centripetal force is of gravitational origin.

The picture of an atom is almost a miniature solar system. Within an atom minute electrically charged particles, known as *electrons*, keep continuously revolving round a central positively charged body known as *nucleus*. In such cases the centripetal force of attraction supplied to the revolving electrons is of electrical origin.

All the above cases are illustrations of *dynamic* equilibrium ; the system as a whole does not change its configuration due to motion within it. If the motion stops, there is no centripetal force acting on the revolving body and the dynamic equilibrium is lost. The revolving body, therefore comes directly to the central mass due to the mutual attractive force.

Some very instructive conclusions may be derived from the kind of equilibrium stated above. If the body while rotating in a circular orbit, experiences an attracting force towards the centre greater than that required to keep it revolving in the particular orbit, the equilibrium would be lost and the body would be urged toward the centre with a force equal to the *difference of the attracting force and the centripetal force in that orbit.* The revolving body, therefore, having a constant tangential velocity throughout would come nearer the central mass in a path of gradually decreasing radius somewhat like a spiral. But during this process of advance as the distance r diminishes, the centripetal force mv^2/r gradually increases. Ultimately when this force is again equal to the attracting force, a fresh equilibrium sets up and the body continues to revolve in a circular orbit. Therefore, an increase in the force of attraction would make the body revolve in a circular orbit of a smaller radius, while a decrease of force would make it move in an orbit of a larger radius.

Bodies on the Surface of the Earth—Let m be the mass of a body at any point on the equatorial region P on the surface of the earth. Let M be the mass of the earth and let r be its radius in this plane (Fig. 72). Let the attractive force on the body exerted by the earth be w, when both are supposed to be rest. This is the true weight of the body at that locality. But as the earth revolves on its axis the body shares an equal motion with it in a circular orbit of radius r with a speed v, say.

In order that m may revolve in a circular orbit, a centripetal force of magnitude mv^2/r would have to act on m towards the centre. There being no other force present, this force is *derived from the attractive force* w. Thus the apparent weight w' of the body on the equatorial section of the revolving earth is $w-\frac{mv^2}{r}$. Therefore the loss in true weight of m depends upon the speed of rotation of m which in turn depends upon the geographical latitude of the locality. It may be argued *from a mathematical standpoint* that this reduction is due to a force mv^2/r always acting on m opposite to the direction of w, due to its circular motion. But this is not actually true from the physical point of view.

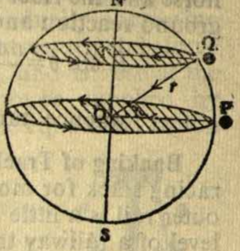


Fig. 72

For a discussion of apparent weight of the body at a region other than equatorial surface, the reader is referred to Art. 54, Chap. IV. If the body is placed at any higher latitude such as R, the body revolves round a smaller circle ; and consequently the tangential velocity, to which it is subjected, is smaller. Hence the part of the true gravitational force contribution to the centripetal force becomes less. At the poles this centripetal force vanishes and there is no reduction in the weight of the body.

Motion of a Cyclist on a Circular Path—A cyclist while taking a bend along a circular path automatically leans his body out of the vertical towards the centre of the circular path. If the cycle and the rider be taken as one body of mass m, their weight mg acts vertically downwards (Fig. 73) through the centre of gravity C (described later) of the system. Let mg be equal to a length CD. The only other force is the *reaction* R of the ground whose line of action passes through the track G of the wheels and centre of gravity of the system. Let the reactive force R be CA and mg be AB. The resultant of these two forces, which is CB in direction and magnitude, is the necessary centripetal force generated. For simplicity, we can also assume that the ground reaction can be resolved into two components ; one, acting vertically upwards and the other acting horizontally towards the centre of curvature of the circular track. The vertical component balances the weight of the system and the horizontal component gives the necessary centripetal force to the system to revolve in the circular path.

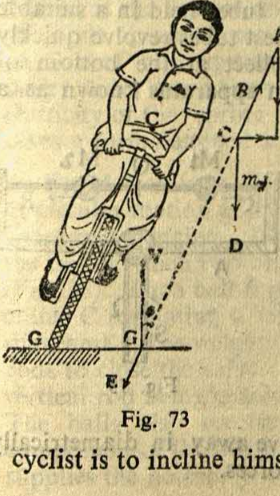


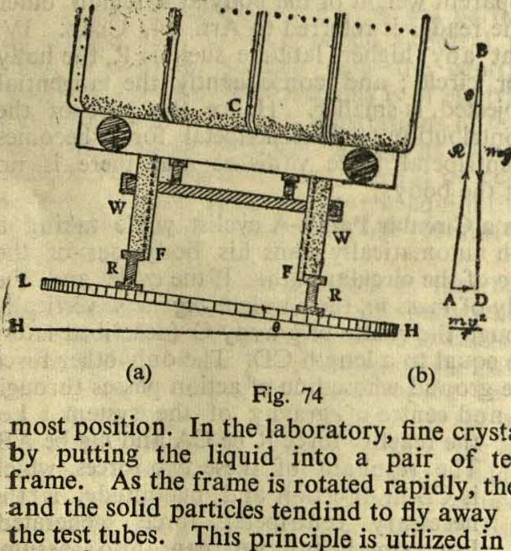
Fig. 73

Thus the greater is the speed or curvature of the path, the greater is the centripetal force and the further the cyclist is to incline himself. If he leans too much the ground reaction may not be sufficient and there will be a side slip. Similarly, in a circus show when a horse is running along a circular path, both the horse and the rider are to incline their bodies inwards. If R be the ground reaction and θ the inclination of the cyclist to the vertical, then

$$R \sin \theta = \frac{mv^2}{r} \text{ and } R \cos \theta = mg$$

$$\text{whence } \tan \theta = \frac{v^2}{rg} \qquad \dots \quad (48,1)$$

Banking of Tracks—The level at a bend on a railroad or in a racing track for motor car is constructed a bit inclined, so that the outer rail is a little higher than the inner one. The line HL is the level of a railway track at an inclination θ with the horizontal plane HH (Fig. 74*a*). The reaction R of the rail on the wheel may be resolved into a vertical component DB and a horizontal component AD (Fig. 74*b*). The vertical component supports the weight *mg* of the carriage, while the horizontal component supplies the necessary centripetal force mv^2/r. There is always a *caution for maximum speed limit* at every bend of a railway track.



(a) (b)

Fig. 74

If a small can filled with water is rapidly swung in vertical circle, the water will not fall down, even when the bottom is at the uppermost position. In the laboratory, fine crystals in a liquid are collected by putting the liquid into a pair of test tubes held in a suitable frame. As the frame is rotated rapidly, the test tubes revolve quickly and the solid particles tendind to fly away collect at the bottom of the test tubes. This principle is utilized in an apparatus known as a *centrifuge*.

A very simple apparatus for demonstrating centrifugal force consists of a uniform highly polished rod R of round cross-section placed horizontally being fixed by two cross-bars at its ends (Fig. 75). Two metal balls M_1 and M_2 having diametrical holes can slide over the rod with very small friction. The rod is fitted to a vertical shaft S fixed to a turn table. The balls are brought to the central position of the rod. If now the frame is rapidly rotated the balls would move away in diametrically opposite ends of the rod due to centrifugal forces.

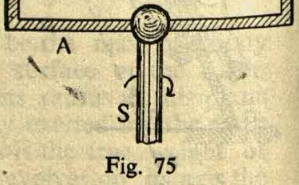


Fig. 75

Flattening of the Earth at the Poles—The earth in its infancy consisted of a mass of molten matter. As the earth continued to revolve about its axis, every portion of matter composing the earth revolved—the tangential velocity in rotation gradually increasing from the poles to the equator. We know that centripetal force necessary for rotation is greatest at the equator and is zero at the poles. The necessary centripetal force could not be supplied by gravitational and other forces of attraction at the equatorial region at the semi-solid state, and as a result that portion of matter in tending to move away tangentially selected a greater distance from the polar axis, such that the reduced centripetal force could be equal to the resultant attractive forces. Here a fresh equilibrium was set up. Consequently, the earth became bulged at the equator taking more matter from polar end, which flattened the poles. The outer layers of the earth, due to gradual cooling have become solidified in course of time and have taken permanent flattened shape.

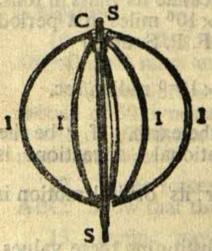


Fig. 76

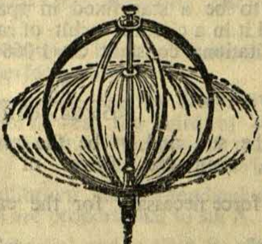


Fig. 77

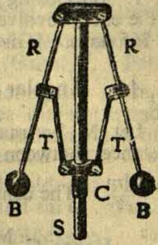


Fig. 78

The *model*, as shown in Figure 76, shows the nature of flattening of the earth due to rotation. It consists of a central rod SS and a circular hoop with some pieces of thin strips I attached to it. The hoop is fixed at the bottom and at the top it is attached to a collar C, which can slide up and down the rod. When put into rapid rotation, each particle of the strip tries to move outwards, as a result of which the collar slides down the rod. The extent of the sliding depends on the elasticity of the spring and the speed of rotation. Thus the hoop takes a form flattened at the ends and bulged in the middle (Fig. 77).

Speed Governor—This is a device which is based on the principle of circular motion and is used to regulate *automatically* the maximum speed of an engine. It consists of a vertical central spindle S, at the top of which two other rods RR carrying balls BB are hinged (Fig. 78). Each ball is partially supported by a rod T resting upon a collar C regulating a safety valve of the engine (not shown in the diagram). The weight of the balls supplies necessary thrust on the rods to close the valve. As the speed of the engine increases, the vertical rod being geared with it revolves more and more speedily. The balls also revolve about the rod as axis. The horizontal component of the thrust on the oblique rods resting upon the valve supplies the necessary centripetal force for the circular motion of the balls. When the speed increases to such a limit that this thrust is insufficient to keep the balls revolving in the particular orbit, the balls go farther apart, raising the collar and thereby opening the valve partially. The gas pressure is thus partly released and the speed of the engine diminishes. Such automatic governor are also fitted with the revolving system of gramophone or casset turn tables.

Examples :

1. Supposing the sun to be a star fixed in space, calculate its mass in tons, if the earth revolves round it in a circular orblt of radis 93×10^6 miles in a period of 365 days. [Given gravitational constant $G=1{\cdot}066\times10^{-9}$ F. P. S. unit.]

Ans. Angular velocity of earth $\omega=\frac{2\pi}{365\times24\times60\times60}={\cdot}2\times10^{-6}$ radian/sec.

Let M be mass of the sun and m be the mass of the earth. If r be the distance between them, then the force of gravitational attraction is $G.\frac{Mm}{r^2}$. The centripetal force necessary for the earth for its orbital motion is $\frac{mv^2}{r}$. Hence $G\,\frac{Mm}{r^2}=\frac{mv^2}{r}$ and since $v=\omega r$, $M=\frac{w^2r^3}{G}$. Substituting these values, we get,—

$$\therefore \text{ the mass of the sun } M=\frac{w^2r^3}{G}=\frac{{\cdot}04\times10^{-12}\times93^3\times10^{18}+5280^3}{1{\cdot}066\times10^{-9}}\text{ lbs.}$$

$$=4443\times10^{27}\text{ lbs.}=1{\cdot}9\times10^{27}\text{ tons.}$$

2. Calculate the inclination of a railway track at a bend of radius of curvature of 1000 ft. when the speed limit at the bend is assigned to be 45 miles per hour. [Given $g=32$ F.P.S. units.]

Now 45 miles/hour=66 ft./sec.

Ans, We have $\tan\theta=\frac{v^2}{rg}=\frac{66^2}{1000\times32}={\cdot}136$ (approx),

Hence $\theta=\tan^{-1}{\cdot}136=7°48'$ approximately (from tangent table).

EXERCISES ON CHAPTER III

1. State and explain the Laws of Motion.
(W. B. H. S.—1971, '72 ; Dac. U. ; U. P. B. ; Pat. U. ; Del. H. S.)

2. A 10 gm. bullet is shot from a 5 kg. gun with a speed of 400 metres per second. What is the backward speed of the gun ? *Ans.* 80 cm. per sec

3. A force of 100 dynes acts on a mass of 10 gm. for 5 sec. Calculate the change of momentum of the body. *Ans.* 500 C.G.S. (Del. U. ; P. U.)

4. A constant force acts for 3 seconds on a mass of 16 lb. initially at rest and then ceases to act. During next 3 seconds the body describes 81 ft. Find the magnitude of force in poundals. (Dac U.) *Ans.* 144 poundals.

5. A motor car of mass 400 lb. is moving with a velocity of 30 miles per hour. By the application of brakes it is brought to rest in a distance of 10 ft. Find the average force resisting the motion. *Ans.* 38720 poundals.

6. State Newton's Second Law of Motion and show that $P=mf$. Define a unit of force. (Dac. U. ; W.B.H.S.—1969 ; Nag. U. ; Vis. U. : Pat. U. ; P. U.)

7. State Newton‘s Third Law of Motion and explain it carefully. Show how this leads to the principle of conservation of momentum.

(W.B.H.S—1980 ; C. U. ; Pat. U.)

8. An one ounce rifle bullet leaves the rifle with a velocity of 100 ft. per second, the rifle tending to recoil with a velocity of 2 ft. per second. Find the mass of the rifle. Explain the principle employed. *Ans.* 3·125 lbs.

9. Define impulsive force and explain how the impulse of an impulsive force is measured. (Nag. U. ; Utkal U.)

10. Show that two spheres on collision will interchange their velocities, if they are of equal mass and are perfectly elastic. (Utkal U. ; Pat. U.)

11. Four forces act at a point. The first is of 500 dynes acting due south, the second of 50 dynes acting due to west, the third of 400 dynes due north and the fourth of 100 dynes acting due east. What are the magnitude and direction of the resultant force ? *Ans.* 112 dynes in a direction 60°26′ S. of E.

12. Three forces in equilibrium act perpendicularly to the sides of a triangle ABC. Show that the forces are proportional to sin A, sin B and sin C. (And.U.)

13. Define 'moment of inertia' and explain its physical significance.

A disc and a block start from rest from the top of an inclined plane. The disc rolls down, while the block slides down the same distance without friction. Compare their terminal velocities. (Poo. U.)

14. A uniform bar AB is 4 ft. long and has weights of 10 lb. and 20 lb., hanging at 1 ft. and 3 ft. from the end A. If it balances at a point 1 ft. 9 in. from the end B, what is the weight of a bar ? (P. U.) *Ans.* 10 lb.

15. A stone weighing 200 gm. is whirled round in a horizontal circle at the end of a string 60 cm. long with a speed of 150 revolutirns per minute. Calculate the tension in the string. (P. U.) *Ans.* $2{\cdot}96 \times 10^5$ dynes.

16. A body of mass 77 gm . travels in a circular path of radius 70 cm. with a uniform speed of 23 cm. per sec. What is the force that must act on the body to maintain the motion ? *Ans.* 581·9 dynes.

17. What are the centripetal and centrifugal forces and what are their relations with a body moving on a circular orbit ? (Del. H. S. ; Del. U. ; P. U ; Pat. U. ; Cal. U. ; And. U.)

18. A stone weighing 6 lb. is rotated in a circle of radius 1 yd. with a speed of 10 ft./sec. Calculate the centripetal force in pounds weight. (P. U.)

Ans. 6·25 lbs.-wt.

19. (*n*) State Newton's Second law of Motion. Show how this law will give us a unit of force when the unit of mass is 1 kg. and the unit of acceleration is 1 m/sec^2. What is the name of this unit of force ?
Ans. 1 newton. (W. B. H. S.—1978)

(*b*) Explain what are meant by centripetal and centrifugal forces ? Find an expression for centripetal force.

A cyclist is moving on a circular path of 20 m. radius at a speed of 18 km/hour. What is his inclination to the vertical. *Ans.* $\theta=\tan^{-1}0{\cdot}13$ (W. B. H. S.—1978)

20. (*a*) Why is centripetal force called a real force and centrifugal force is called a pseudo force. Briefly explain.

(*b*) How is the weight of a body affected when it is placed on equitorial plane of the earth ?

(*c*) A body of mass 70 gm. tied with a string of length 20 cm. is rotated with hand with an angular frequency of 10 times per sec. Calculate the centripetal force ? *Ans.* $56\pi^2 10^4$ dynes (W. B. H. S.—1980)

21. Why is a centripetal force necessary to rotate a body in a uniform circular motion ?

Show that when a cyclist is going round a curved path of radius r with a uniform speed v, his body must be inclined the vertical by an ange θ where $\tan \theta = v^2/rg$, g being the acceleration due to gravity. (W. B. H. S.—1982)

22. A stone of mass 200 gm. is tied to a thin string of length 50 cm. and is whirled in a horizontal circle with its centre fixed. It is found to execute 5 rounds in every 2 seconds. Find the normal acceleration and the tension of the string.

Ans. 12337·5 cm./sec.^{2} ; $2467 \cdot 5 \times 10^3$ dynes.

23. A railway track has its centre of gravity at a height of 3 ft. above the rails, which are 4 ft. apart. What would be the maximum safe speed at which it could travel round an unbanked curve of radius 108 ft. ? (Gau. U.) *Ans.* 48 ft./sec.

24. A motor racing track is in the form of a circle of diameter 400 yards and is banked, so that the reaction between the wheels and the ground is normal to the track when the cars are travelling at 60 miles an hour. What is the slope of the track ? If a car is of mass of 1 ton, what is the centrifugal force on it, when going round this track at 60 miles per hour ? *Ans.* $\tan^{-1} \cdot 403$; $2 \cdot 89 \times 10^4$ poundals.

25. A cyclist riding at a speed of 15 miles and hour takes a turn round a circular bend of radius 44 ft. What is the inclination to the vertical ? ($g = 32$ ft./sec.^{2}) (Pat. U.) *Ans.* $\tan^{-1} \cdot \frac{11}{32}$.

26. Calculate the angle which the bicycle and its rider must take with the vertical when going round a curve of 22 ft. radius at 10 miles an hour. What is the least value of the coefficient of friction between the tyres and the road which will prevent slipping ? If the bicycle and the rider have a mass of 200 lb. what frictional force must be ground exert on the wheels ? (Gau. U.)

Ans. $\tan^{-1} \cdot \frac{11}{36}$, $61\frac{1}{9}$ lb.

CHAPTER IV

GRAVITATION AND GRAVITY

49. Science in Ancient India—During the period from the second century to the twelfth century A. D. there was a marked progress in India of the observational astronomy and its allied calculations. Although there is no systematic record of the observational data, the results of the calculations reveal that the people of ancient India were very advanced in Astronomy, Medicine and Mathematics.

Of those who came to the forefront in Astronomy, the names of Aryya Bhatta, Brahma Gupta and Bhaskar Acharyya are the most prominent. Aryya Bhatta, a Hindu astronomer, observed in his Tantra sometime in the fifth century A. D. that the planet revolve round the sun in elliptic orbits. He calculated the periods of revolutions of the planet such as Mercury, Earth, Mars, Jupiter and Saturn. His results have been found very nearly correct by modern astronomers. He also discovered that that earth is round and that the day and night effect on the earth are the due to its diurnal motion. His findings were later on verified by telescopic observations by Kepler early in 17th century.

The great exponent of Hindus astronomy, Bhaskar Acharya, flourished during the 12th century. He compiled one book 'Siddhanta Shiromani', which deals with various aspects of Astronomy and Mathematics. Here he calculates the possible height of the atmosphere which came out to be about 12 jojans (about 55 miles). The most astounding of all is his diecovery of gravity and gravitation, which explains in following lines.*

"Whenever an object appears to fall to the earth, it is due to the attraction of the earth on it. It is this mutual attraction between heavenly bodies which does not allow any one to leave its respective position or orbit." This was subsequently independently discovered by Newton in the later part of the 17th century.

Such was the ancient glory of India, which faded gradually from the 13th century and became nearly extinct within the next four hundred years. This dwindling of scientific culture was due to lack of the support of the State and co-rodinate enterpises to establish schools of thought.

50. Law of Gravitation—Johaan Kepler (1571-1630) after a continuous study of the motion of planets round the sun, came the conclusion that the orbit of the planets is not circular but is slightly elliptical with the sun at the focus. He formulated the laws of planetory motion. Newton in an attempt to explain the laws discovered the *Universal Law of Gravitation in 1666*, as stated below :

Every particle of matter in the universe attracts every other particle at any distance along the line joining them, with a force which varies directly as the product of their masses and inversely as the square of the distance between them.

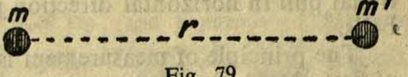


Fig. 79

If F be the force of gravitational attraction between two bodies of masses m and m' placed at a distance r from each other (Fig. 79),

$$\text{then } F \propto mm' \text{ ; also } F \propto \frac{1}{r^2}.$$

$$\therefore \quad F \propto \frac{mm'}{r^2}, \quad \text{or,} \quad F = G\frac{mm}{r^2} \qquad \dots \quad (50, 1)$$

where G is called the **gravitational constant**, which is independent of the nature of bodies. If we consider two bodies each of unit mass and placed at a unit distance apart, the force of attraction between these bodies would obey the inverse square law. Then according to equ. (50,1), if $m = m' = 1$ and also $r = 1$, then $G = F$ in magnitude.

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*आकृष्टि शक्तिश्च मही तया यत् । खस्थ गुरू स्वाभिमुखं स्वशक्त्या ।
आकृष्यते तत् पततीव भाति । समे समन्तात् क्व पतत्विदं खे ॥

In other words, the gravitational constant G is *numerically* aqual to the force of attraction between two bodies, each of unit mass and placed at the unit distance apart. The gravitational constant has been experimentally measured by Cavendish, Boys and others. The determination has been later on carried to a still higher degree precision by Heyl. The present accepted value of G is $6{\cdot}6576\times10^{-8}$ C.G.S. unit. This means that a body of mass 1 gram placed at a distance of 1 cm. from another body of mass 1 gram will *attract each other* with a force of $6{\cdot}6576\times10^{-8}$ dyne.

***51. Determination of Gravitational Constant**—The first direct determination of G was made by Henry Cavendish(1731-1810)in 1798. The apparatus consisted of a thin and light rod R of length of about8 ft. from the ends of which two small identical lead ball *bb*, two inches in diameter, were suspended (Fig. 80). The rod was also suspended by a torsion wire FC from a support. Two large lead balls BB each 12 inches in diameter, could be placed on opposite sides of the smaller balls at equal distance, so that the centres of all the balls were on a horizontal plane. When the bigger balls were so placed as shown in the figure, each smaller ball was attracted by the nearer bigger ball due to gravitational pull in horizontal direction joining the centres of the respective balls.

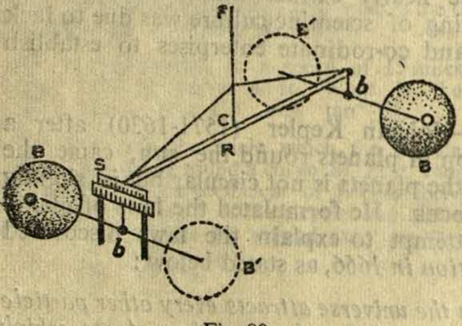

Fig. 80

The principle of measurement is as follows : The bigger balls are at first placed far away so as not to effect the smaller ones and the position of the suspended rod is read with reference to the scales S attached to a vernier scale. Then the large balls are placed in position at equal distances from the corresponding small balls. Due to the attraction on the two small balls, a couple is exerted on the rod and this couple moves the rod through a small angle twisting the suspension wire. The angle of rotation can be meaiured by knowing the length of the rod and the amount of its movement on the scale S.

If *d* be the distance between the centres of a large ball and the corresponding smaller one, and M and *m* the masses of the ball, the attracting force at each end of the rod is $\frac{GmM}{d^2}$. The two equal, parallel and oppositely directed forces constitute the deflecting couple on the rod whose moment is $\frac{GmM}{d^2}\times l$, where l represents the length of the rod. If the rod is deflected through an angle θ, then the restoring torque due to twisting of the wire is $C\theta$, where C is the torque per unit twist. Thus for equilibrium,

$$G\frac{Mm}{d^2}l=C\theta, \text{ whence } G=\frac{C\theta d^2}{Mml}. \qquad \ldots \qquad (51, 1)$$

Thus, knowing all the terms on the right-hand side of the equation, the value of G may be found.

The value of the torque per unit twist C is found by giving a small oscillation of the rod when the balls are removed and counting the period of oscillation. If T be the period of oscillation, I is the moment of inertia of the rod, then $T=2\pi\sqrt{\frac{I}{C}}$

Examples :

1. In one of Cavendish experimonts each of the larger lead balls had a mass of 30 kgm. and was placed 40 cm. from the corresponding small ball, each of mass 200 gm. The length of the suspension rod was 3 metres. The deflection on the scale was found to be 1·5 mm. If the restoring torque of the suspension fibre be 75 dynes per radian, calculate the gravitational constant.

Ans. If θ be the angular deflection of the suspension rod, then by the condition of the experiments $\tan\theta=\frac{\text{linear deflection of one end}}{\text{half the length of the rod}}=\frac{\cdot 15}{150}=10^{-3}$.

The value being so small, $\tan\theta=\theta=10^{-3}$ radian.

Then by (51, 1) we have

$$G=\frac{C\theta d^2}{Mml}=\frac{75\times10^{-3}\times40^2}{30\times10^3\times200\times300}=\frac{75\times16\times10^{-1}}{18\times10^8}=6.66\times10^{-8} \text{ C. G. S. unit.}$$

2. Calculate the gravitational force in poundals between two masses, each 1 lb., placed 1 foot apart, given that $G=6{\cdot}6576\times10^{-8}$ C. G. S. Two persons, each of mass 140 lb., are sitting side by side with their centres of mass 2 ft. apart. Calculate the gravitational pull between them.

Ans As 1 lb.$=453{\cdot}59$ gm., 1 ft.$=30.48$ cm., and 1 poundal$=13{,}825$ dynes.

$$\therefore\ F=6{\cdot}6576\times10^{-8}\,\frac{453{\cdot}59\times453{\cdot}59}{30{\cdot}48\times30{\cdot}40}\text{ dynes}=\frac{6{\cdot}6576\times10^{-8}}{30{,}825}\times\frac{453{\cdot}59^2}{30{\cdot}48^2}\text{ poundal}$$

$=1{\cdot}0665\times10^{-9}$ poundal.

Thus, when masses are expressed in lb. and distanee in feet, the value of $G=1{\cdot}0665\times10^{-9}$ F. P. S. unit.

Now $m_1=m_2=140$ lb. and $r=2$ ft.

$$\therefore\ F=1{\cdot}0665\times10^{-9}\times\frac{140^2}{2^2}\text{ poundal}=5{\cdot}2\times10^{-6}\text{ poundal.}$$

52. Gravity—Gravitation is a general term indicating the force of attraction between any two material bodies. In the particular case, the gravitational attraction betweed the earth and any other body, either terrestrial or extra-terrestrial, is called *gravity*. Due to the force of gravity the earth and any terrestrial object on or near the surface of the earth attract each other equally, but as the mass of the earth is very very large in comparison to the mass of the body under gravitational force, it is the body which is dragged towards the earth by the law of conservation of momentum. This pulling of the body towards the earth's surface is called the vertical fall of the body under gravity. Actually the body falls in a direction towards the centre of the earth.* If the mass of the earth be M, the mass of an object on or near the surface of the earth be m, and the distance between the centres of the earth and of the body be d,

$$\text{the } \textit{force of gravity } F = G\frac{Mm}{d^2} \quad \dots \quad (52, 1)$$

It is due to this force of attraction that all bodies possess their weights and tend to fall downwards. For a similar reason a plumb line, which consists of a long piece of thread with a bob at one end, when suspended, is attracted *towards* the centre of the earth and hangs in a perfectly vertical line.

Acceleration due to Gravity—We know that a force acting on a body produces acceleration on it. So the force of *gravity* produces acceleration on all bodies falling towards the surface of the earth.

From equ. (52, 1) the force of gravity F on a mass $m = \frac{GMm}{d^2}$

$$\therefore \text{ acceleration produced on the mass} = g = \frac{F}{m} = \frac{GM}{d^2} \quad \dots \quad (52,2)$$

The *acceleration due to gravity* is usually represented by the letter 'g'. As G and M are constants, the value of 'g' remains constant, so long as d does not change. Remembering that d is the distance between the body and the centre of the earth, *the value of 'g' is constant at all equidistant places from the centre of the earth.* The value of 'g' at any place can be determined by Atwood's machine or more accurately by a pendulum.

Variation of 'g'—It is evident from (52,2) that 'g' at one place varies inversely as the square of its distance from the centre of the earth. Hence the value of 'g' depends upon the height above the sea-level. But the *variation* in 'g' for ordinary differences in height is inappreciable. Again, owing to the fact that the earth is not perfectly round, all points on its surface are not at equal distance from its centre ; the *equatorial radius* is the largest, while the *polar radius* is the shortest. Hence the value of 'g' varies *slightly* with the *latitude* of the place on the surface of the earth. It is least at the equator and gradually increasing becomes maximum at either of the poles. This was first verified by Edward Halley, (1656-1742) a British astronomer, with a pendulum in 1677, during his voyage from London to St. Helena. The variation over the earth's surface, however, is very small ; the value of 'g' at the equator is about 978 cm. per sec.^{2}, and that at the poles it is about 983 cm. per sec.^{2}. The *mean value of 'g'* is taken to be 981 cm. per sec.^{2} or 32·19 feet per sec.^{2} at latitude 45° and at sea-level, which is accepted as the standard of reference.

53. Weight of a Body—The weight of a body is due to the attractive force on the body by the earth. If the mass of the body is increared, it is evident from equ. (52,1) that the force of attraction would increase, in the same proportion. Hence *the weight of a body at a*

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* In considering the gravitational effect of the earth upon bodies lying on or outside its surface, the whole mass of the earth may be supposed to be concentrated at its centre.

place is proportional to its mass. Again, force=mass × acceleration : so that the weight of W of a body of mass m is given by

$$W = mg. \qquad \ldots \qquad (53,1)$$

Mass and Weight—The word 'weight' is sometimes loosely used to denote mass of a body. But *mass* of a body is the quantity of matter contained in it, while *weight* is the force with which it is attracted towards the centre of the earth. The *mass* of a body remains constant at all places, but with the variation of 'g' its *weight* varies *very slightly* from place to place on the earth's surface. The variation in the weight of a body is dealt with elaborately in the next Article.

Gravitational Unit of Force—This is the force with which a unit mass is attracted by the earth and is equal to the weight of a unit mass.

Hence, in the F.P.S. system, the gravitational unit of force
=the weight of 1 lb. $=1\times g=(1\times 32{\cdot}19)$ or 32·2 poundals.

In the C.G.S. system, the gravitational unit of force
=weight of 1 gm. $=1\times g=(1\times 981)$ or 981 dynes.

Further, 1 dyne=weight of 1 milligram (nearly) ;

1 lb. wt. $=g$ poundals. Also 1 gm.wt. $=g$ dynes, or, we can say that a gravitational unit of force $=g\times$ absolute unit of force.

So m lbs. wt. $=mg$ poundals and m gm. wt. $=mg$ dynes.

Again, 1 *Poundal* $=\dfrac{1}{32{\cdot}2}\times$ wt. of 1 lb. = wt. of $\dfrac{16}{32{\cdot}2}$ oz.

=*wt. of half an ounce* (nearly).

As the value of 'g' varies on the surface of the earth, the gravitational unit of force is *not* constant at all places.

Example :

A car of mass 1 ton is moving with a speed of 30 miles per hour. A constant force of 28 lb. opposes its motion. Calculate (*a*) the time required to bring the car to rest, and (*b*) the distance it will travel in stopping. [Mys. U.]

Ans. Mass of 1 ton $=(28\times 4\times 20)$ lb. and a force of 28 lb. is $=28\times 32{\cdot}2$ poundals.

∴ retardation f of the car is obtained from the relation $P=mf$,

or, $28\times 32{\cdot}2=28\times 4\times 20\times f$, whence $f=\dfrac{32{\cdot}2}{4\times 20}$ ft. per sec.^{2}.

(*a*) Initial velocity of the car=30 m.p.h.=44 ft. per sec. When at rest, the final velocity is clearly zero.

Hence, $0=44-\dfrac{32{\cdot}2}{80}t$, whence $t=109{\cdot}3$ seconds.

(*b*) If s ft. be the required distance, $0^2=44^2-2\times\dfrac{32{\cdot}2}{80}\times s$, ∴ $s=2405$ ft.

54. Variation in the Weight of a Body—As the weight of a body is proportional to 'g' it varies from place to place on the surface of the earth and at different altitudes.

(1) *Weight of a Body at a Height above the Surface of the Earth* If—the mean radius of the earth be r and its average density be ρ the

mass M of the earth is equal to the product of its volume and density ; That is, $M=\frac{4}{3}\pi r^3 \rho$.

Let a mass m be placed at a height h above the surface S of the earth (Fig. 81). The force of attraction due to gravity on the mass, which is the weight of the body, is given by

$$F=G\frac{Mm}{(r+h)^2}=G\times\frac{4}{3}r^3\rho\times\frac{m}{(r+h)^2} \quad \dots \quad (54,1)$$

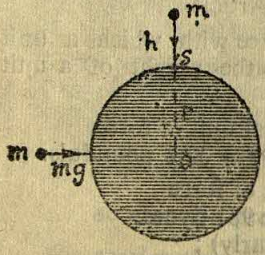


Fig. 81

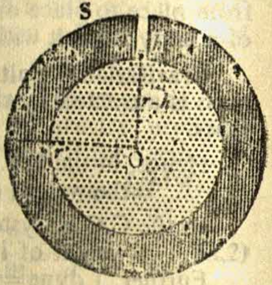


Fig. 82

Let the acceleration due to gravity at a height h be g.

$$\text{Then,} \quad g=\frac{F}{m}=G\times\frac{M}{(r+h)^2} \quad \dots \quad (54,2)$$

Hence the acceleration due to gravity and consequently the weight of the body decreases with height from the surface of the earth, being inversely proportional to the square of the distance of the body from the *centre* of the earth. Thus at a height of 4000 miles above the surface of the earth a body would weight $\frac{1}{4}$ of its normal weight. At the top of a mountain 4 miles high, a mass of 1000 gm. would weight about 998 gm. It may be of interest to know that at a distance of about 1,80,000 miles from the earth along a line joining the earth and the moon, the attraction by the earth is counterbalanced by the attraction of the moon. So any body placed there will be *weightless* although having some mass.

(2) *Weight of a Body at a Depth below the Surface of the Earth* —Take a body of mass m, say to the bottom of a mine at a depth d below the surface of the earth (Fig. 82). Let r be the mean radius of the earth. Imagine a sphere, concentric with the earth, to be drawn with a radius equal to $r-h$. The earth may now be considered to be divided into two portions—one an inner sphere of radius $(r-h)$ and the other, a hollow outer shell of thickness h. Then the given body lies on the surface of this inner sphere but just inside the hollow outer shell.

As the gravitational force inside a hollow shell is zero, the force of attraction due to this portion of the earth is nil. Hence the force of attraction on the given body would be only due to the inner solid spher.

Now the mass of the inner sphere$=\frac{4}{3}\pi(r-h)^3\rho$.

∴ The force of attraction on m due to the sphere, which is also the weight of the body at this region, is given by

$$F=G\tfrac{4}{3}\pi\ (r-h)^3\rho\times\frac{m}{(r-h)^2}=\tfrac{4}{3}\pi G(r-h)\rho m. \quad \ldots \quad (54,4)$$

So the acceleration due to gravity g_h at a depth h is given by

$$g_h=\frac{F}{m}=\tfrac{4}{3}\pi G(r-h)\rho.$$

Hence the *acceleration due to gravity* inside the earth varies *directly* as its distance from the centre of the earth. Thus if we go farthar and farther inside the earth, the value of g gradually decreases. Hence, at the *centre of the earth*, the acceleration due to gravity is zero and there is no weight of a body.

* (3) *Variation in the weight of a body with different Latitudes on the surface of the Earth*—The weight of a body varies with latitude on the surface of the earth for two reasons.

(*i*) the peculiarity in the shape of the earth and

(*ii*) the rotation of the earth about its axis.

The radius of the earth is the longest at the equator and decreases continuously with latitude, becoming least at the poles. The polar radius is less than the equatorial radius by about 13½ miles. Consequently, the force of gravitational attraction is the greatest at the poles and least at the equator, and so the weight of a mass increases from the equator to the poles in the same proportion.

Again, owing to the diurnal rotation of the earth about its axis, everybody on its surface revolves in a circular orbit. Further, we know that for a circular motion to take place a centripetal force must have to act on the body. A *small* part of the gravitational force of the earth is utilised in supplying the necessary centripetal force. The former thereby falls short of the required amount. Therefore, the observed weight of the body is less than its true weight.

Due to the diurnal rotation of the earth about its axis, a point near the pole describes a circle of smaller radius than the one near the equator. Suppose that NS represents the polar axis of the earth (Fig. 83). Since rotation of all the parts is completed in one day, a body at a point P near the equator would move on a larger circle than a body at a point Q near a pole. So the former would move with a greater linear speed. Hence the centripetal force is the greatest at the equator and zero at the poles. Therefore, the loss of weight due to the diurnal rotation ia least at the poles and greatest at the equator and so the observed weight of a body is the greatest at the poles and least at the equator. Thus, for both the causes, *the weight of a body increases when taken from the equator to the poles.*

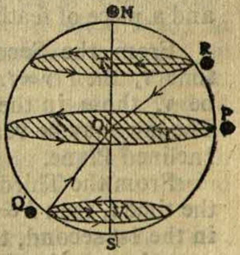


Fig. 83

55. **Laws of Falling Bodies**—When a body falls freely from rest, subject only to the action of gravity, then for all pratical purposes (the height not being too large) the acceleration due to gravity may be assumed to remain constant throughout the fall. Then the motion is governed by the following simple laws :

(*1*) *In vacuum, all bodies starting from rest fall with equal rapidity.*

(2) *The velocity, acquired by a falling body in given time, is directly proportional to the time.*

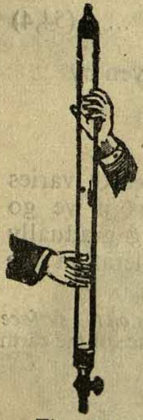


Fig. 84

(3) *The space traversed by a falling body in a given time is directly proportional to the square of time.*

Analysis of the Laws—The first law seems contrary to our common observation, as a heavy body appears to fall to the ground more quickly than a lighter one when dropped from the same height. Galileo (1564-1642) first showed in 1589 by dropping balls of different sizes and materials from the top of the leaning tower of Pisa that they reached the ground practically in the same time. He pointed out that the slight observed differences in the rate of fall between a light body and a heavy body were due to the resistance offered by air. He then argued that all bodies would fall the same rate, if the resistance due to air could be avoided and stated the laws as above. After the invention of the air-pump in 1650, the laws were conclusively verified by Newton in the well-known *guinea and feather experiment.*

Guinea and Feather Experiment—A glass tube about a metre long with a cap screwed at one end and at a stop-cock at the other is taken (Fig. 84). A small coin and a piece of paper are introduced into the tube. The air is then exhausted from the tube by an air-pump and the stop-cock is closed. On suddenly inverting the tube, the coin and paper are found to be falling together and reach the other end simultaneously. If air is now re-introduced by opening the stop-cock and the tube is inverted the two are found to fall separately, the coin reaching earlier. The experiment was originally done with a guinea and a piece of feather within the tube and hence the name was derived.

From the Second Law, if v be the velocity acquired in falling in time t, then $v \propto t$. Thus, if the velocity attained in the 1st second be v, those in the 2nd, 3rd, etc., second are $2v$, $3v$, etc., respectively. The law may be verified by experimenting with *Atwood's Machine* or Inclined Plane.

From the Third Law, if a body falls freely through a height h in the time t, then $h \propto t^2$. Thus, if the body falls through a height x ft. in the 1st second, the height fallen through in the 2nd, 3rd, etc., seconds are $4x$ ft., $9x$ ft. etc., respectively. These laws also follow from the Equation of Motion given in Chapter II.

56. Vertical Motion of Bodies due to Gravity—When a body falls vertically downwards, then provided the distance fallen through is not *very large*, the acceleration 'g' due to gravity may be supposed to be uniform.

Bodies projected downwards—If a body is projected *downwards* with an initial velocity u, then

$$\left.\begin{array}{ll}\text{The velocity } v \text{ after time } t \text{ is} & =u+gt\\ \text{and the height } h \text{ fallen through} & =ut+\tfrac{1}{2}gt^2\\ \text{and hence } v^2 & =u^2+2gh\end{array}\right\} \quad \dots \quad (56, 1)$$

If the body *starts from rest*, then $u=0$,

$$\left.\begin{array}{lll} v=gt, & i.e.,\ v\propto t & \dots\ \text{2nd Law} \\ \text{and}\quad h=\tfrac{1}{2}gt^2, & i.e.,\ h\propto t^2 & \dots\ \text{3rd Law} \end{array}\right\} \quad \dots\ (56,2)$$

Bodies projected upwards—Here the acceleration due to gravity is opposite to the direction of motion and is thus negative.

$$\text{Hence}\quad \left.\begin{array}{l} v=u-gt \\ h=ut-\tfrac{1}{2}gt^2 \\ \text{and}\quad v^2=u^2-2gh \end{array}\right\} \quad \dots\ (56,3)$$

When the body reaches the highest point, its velocity is zero.

∴ from equ. (56,3) the time to reach the highest point :

$$t=\frac{u}{g}. \quad \dots\ (56,4)$$

The time to reach the ground from the highest point is also u/g. Therefore, the total time of flight is $2u/g$.

If h is the maximum height reached by the body,

$$0=u^2-2gh, \text{ or, } h=\frac{u^2}{2g} \quad \dots\ (56,5)$$

***57. Verification of Laws of Falling Bodies**—The equations of motion as given in Art. 56 can be verified by means of an Atwood's machine, of which a simpler type has been described in *Intermediate Practical Physics* by J. Chatterjee. A more improved form is shown in Figure 85, which consists of a stout metal rod fitted vertically to a rigid support. The top of the rod carries a pulley P which can turn very freely without any friction. A specially made paper tape T runs over the pulley and carries two equal loads at its ends. Small additional loads can be put on either load to make it heavier and hence to start the motion of the system. A metal reed M carrying an inked brush at its end is clamped to a crosspiece F. Once swung through a small range, this reed alway maintains a fixed period of transverse vibration.

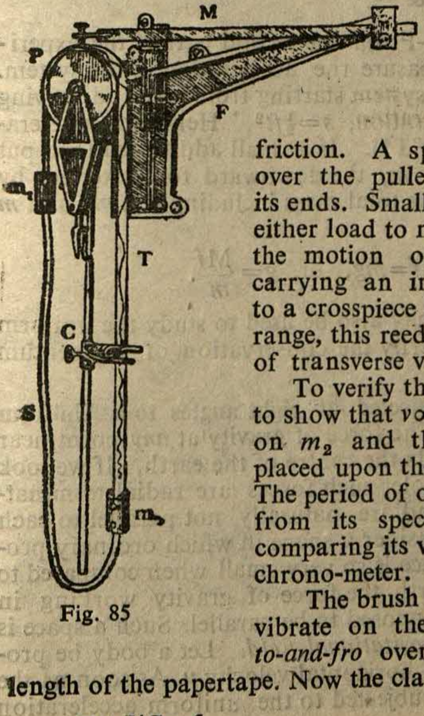


Fig. 85

To verify the law for accelerated motion, *i.e.*, to show that $v\propto t$, a small additional load is put on m_2 and the combined load is raised and placed upon the clamp C at any suitable height. The period of oscillation T of the reed is known from its specification or may be found by comparing its vibration with that of a standard chrono-meter.

The brush is inked and the reed is made to vibrate on the paper tape. The brush moves *to-and-fro* over a line at right angles to the length of the papertape. Now the clamp C is released and m_2 is found to descend, while on the tape a wavy curve as shown in Figure 86 is traced by the inked brush. The curve becomes longer and longer as the load m_2 descends more and more. This shows that the speed increases continuously. Each complete curve as that between 'a' and 'b' marks one unit of time, which is here, the time T of vibration of the reed. The tape is taken out and placed on the table. The number of such curves from the beginning up to a certain point is counted. Let it be *n*. The time *t* from the start of the load to the termination of count is *n*T. The straight length of a complete wave curve (wave length) about *any region* is measured, which is *l*, say. Then the average velocity *v* at this region is *l*/T. Take various intervals and the corresponding velocities. It would be found thar in each case,

$$\frac{l}{T} \propto nT, \text{ or, } v \propto t$$

This verifies the second law.

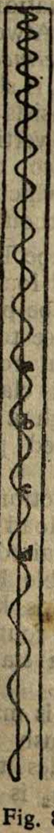


Fig. 86

To verify the law that in an accelerated motion of a body, $h \propto t^2$, he experimental procedure is the same. Measure a length *h* covering a few wavy curves from the start. Count the number of complete waves in this length ; let it be *n*. Then the time to cover this distance is *n*T. Measure a few other lengths and count the corresponding times. It would be found that in each case that

$$h \propto n^2T^2, \text{ or, } h \propto t^2.$$

To measure 'g'—From the second part of the experiment it is easy to measure the acceleration of the system. We know that for a system starting from rest and moving with uniform acceleration, $s=\frac{1}{2}ft^2$. Hence the acceleration f is $2s/t^2=2s/(n^2T^2)$. If a small additional load put upon m_2 be of mass m, the downward force exerted by this load is mg. If the total mass including m_1, m_2 and m be M: then

$$Mf=mg, \quad \text{or,} \quad g=\frac{Mf}{m}.$$

This apparatus may also be used to study the uniform motion or even to verify the conservation of momentum of two bodies.

58. A body projected at right angles to a Uniform Gravitating Force—The force of gravity at *any* point near the earth acts towards the centre of the earth. If we look to the earth as a whole, such forces are radial terminating at its centre and are naturally not parallel to each other. But if we consider a space in which ordinary projectiles work, the space is so very small when compared to the volume of the earth, the force of gravity working in this space may be supposed to be parallel. Such a space is called a *uniform gravitational field.* Let a body be projected from O with a horizontal velocity *u*. As soon as the body leaves O, it is subjected to the uniform acceleration

g due to gravity acting vertically downwards (Fig. 87). We shall assume here that the motion of the body is not retarded by air friction and that the acceleration due to gravity is uniform throughout its range of motion.

Let us trace the successive positions of the body under horizontal velocity *ignoring* gravity. Let the horizontal direction be OX. If it were moving along OX with a uniform velocity u alone, then $OP_1=P_1P_2=P_2P_3=u$. Next consider the action of the gravity alone ignoring horizontal velocity. Let the vertical direction be OY. If the body starts from rest at O and moves down only with uniform acceleration g, its positions along OY at the ends of successive seconds would be respectively at Q_1, Q_2, Q_3, such that $OQ_1=\frac{1}{2}g\times 1^2$, $OQ_2=\frac{1}{2}g\times 2^2$, $OQ_3=\frac{1}{2}g\times 3^2$, etc. Hence, due to the two motions combined, the actual position of the body at the end of the first second will be at R_1 which is the end of the diagonal OR_1 of the parallelogram $OP_1R_1Q_1$. Similarly, the actual positions of the body at the ends of successive seconds will be at R_2, R_3, etc. Taking the intervals to be very small, the actual path of the body will be along the continuous curve $OR_1R_2R_3$...

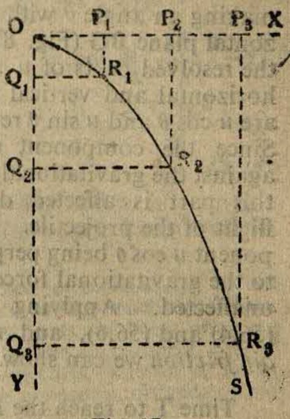


Fig. 87

Now the horizontal distance (x) travelled in time t is given by

$$x=ut \quad \ldots \quad (1)$$

The vertical distance (y) fallen through in the same time is

$$y=\tfrac{1}{2}gt^2 \quad \ldots \quad (2)$$

Eliminating t from equs. (1) and (2), $y=\frac{g}{2}\times\frac{x^2}{u^2}$ ... (58,1)

Since g and u are constants $y \propto x^2$ and the relation between y and x represents a parabola.

In fact, the earth is surrounded by a layer of atmospheric air which gets lighter and lighter as we go up. The air extends up to a height of 200 to 250 miles above the earth's surface. So a projectile, shot near the surface of the earth, experiences a *retarding force due to air friction.* Consequently the path of a projectile shot horizontally is *not* strictly parabolic.

If a projectile be raised up about 500 miles from the surface of the earth and then projected *horizontally* from that position, it moves there without air friction. For small horizontal velocities of projections from that position, the body will fall to the earth in an approximately parabolic orbit and its range of flight will increase with higher velocities of projection. When the velocity of projection is pretty high, about 16000 miles per hour, the projectile will not fall to the earth but will move round the earth continuously in a circular orbit. (*Vide* Example 6 following this article). When the velocity of horizontal projection is above this value, the projectile would revolve round the earth in an elliptical orbit, and with speed of projection increasing further, the ellipticity of the orbit increases. Ultimately, if the body could be hurled with a velocity of about

25000 miles per hour, it would begin to move in a parabolic orbit never to come back to the earth. It is sometimes known as the *escape velocity*. If the velocity of projection be further increased, it might move it a hyperbolic orbit. This is the principle of projection of saturn and lunik rockets and the latest space-lab projected from U.S.A.

58A. Body projected in any direction—Let a body be projected from any point G on the earth's surface with an initial velocity u making an angle θ with the horizontal plane BG (Fig. 88). Then the resolved parts of u along the horizontal and vertical directions are $u \cos\theta$ and $u \sin\theta$ respectively. Since the component $u \sin\theta$ is against the gravitational pull, only this part is affected during the flight of the projectile. The component $u \cos\theta$ being perpendicular to the gravitational force, remains unaffected. Applying eqations (56,4) and (56,6), and *neglecting air friction* we can show that :

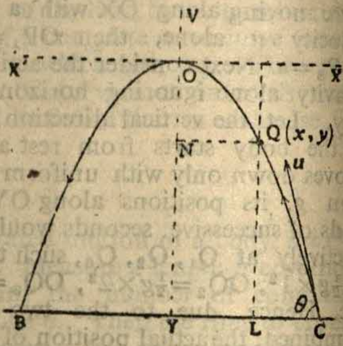


Fig. 88

Time T to reach the highest point O is$=\dfrac{u \sin\theta}{g}$.

Also, the maximum height YO(h) reached by the body is

$$=\frac{u^2 \sin^2\theta}{2g}$$

Since the time to reach O from G is equal to the time to come back from O to B, we get

Total time of flight (2T) is$=\dfrac{2u \sin\theta}{g}$. ... (58,2)

Since $u \cos\theta$ is the horizontal component of velocity, the horizontal distance GB covered by the projectile during its flight is $u \cos\theta \times 2t$. That is,

$$GB=\frac{2u^2 \sin\theta \cos\theta}{g}=\frac{u^2 \sin 2\theta}{g} \qquad \text{... (58, 3)}$$

For given values of u and g, the distance GB is maximum when $\sin 2\theta$ is greatest, that is, when $2\theta=90°$, whence θ, which is the angle of projection, becomes 45°. Thus for a given initial velocity, a projectile would reach the *farthermost point in a horizontal direction*, if the angle of projection is 45°.

To find the path of the projectile in space, let a horizontal line X′X be drawn through O and let OY be the vertical line. Let Q be the position of the projectile after a time t from its start. Draw QL and QN as vertical and horizontal lines from Q and produce LQ to meet X′X at M. If the co-ordinates of Q be (x, y), clearly QN$=x$ and ON$=y$.

Now LQ=height attained at time t is$=u \sin\theta \times t - \frac{1}{2}gt^2$.

$$\therefore \quad y = \mathrm{ON} = \mathrm{OY} - \mathrm{QL} = \frac{u^2 \sin^2 \theta}{2g} - (u \sin \theta \times t - \tfrac{1}{2} g t^2)$$

$$= \frac{g}{2}\left(\frac{u^2 \sin^2\theta}{g^2} - \frac{2u \sin \theta}{g} t + t^2\right) = \frac{g}{2}\left(\frac{u \sin \theta}{g} - t\right)^2$$

whence $\dfrac{2y}{g} = \left(\dfrac{u \sin \theta}{g} - t\right)^2.$

Again, $x = \mathrm{QN} = \mathrm{GY} - \mathrm{GL} = \dfrac{u^3 \sin \theta \cos \theta}{g} - u \cos \theta \times t =$

$$u \cos \theta \left(\frac{u \sin \theta}{g} - t\right).$$

$$\text{Thus, } x^2 = u^2 \cos^2 \theta \left(\frac{u \sin \theta}{g} - t\right)^2 = u^2 \cos^2 \theta \times \frac{2y}{g} = \frac{2u^2 \cos^2 \theta}{g} \times y \quad \ldots \quad (58,4)$$

The equation in y and x is of the form $x^2 = 4ay$ which is a parabola. Thus the path of a projectile in a *uniform* gravitational field and free from air resistance is a *parabola.*

Due to air friction projectiles (such as rifle bullets or canon shells) do not strictly follow the parabolic path but strike the earth's surface much short of the expected range. Modern projectiles carrying explosive heads and being propelled by rocket power rise high up so as to encounter very little air friction and move in an almost parabolic orbit. A class of such projectiles, known as *Inter-continental Ballistic Missile*, can be so projected as to cover a distance on the earth's surface of 4000 to 5000 miles.

Examples :

1. A stone is dropped from a tower with no initial velocity. It is observed to strike the ground after 3·5 seconds. Find the velocity of impact and the height of the tower. [Given $g = 32$ F. P. S.]

Ans. Here $u = 0$, $g = 32$ ft/sec² and $t = 3{\cdot}5$ sec.

∴ the required velocity $v = u + gt = (32 \times 3{\cdot}5)$ ft./sec. $= 112$ ft./sec.
Again, the required height $h = ut + \frac{1}{2}gt^2 = (\frac{1}{2} \times 32 \times \frac{49}{4})$ ft. $= 196$ ft.

2. A body dropped from a balloon at rest clears a tower 81 ft. high during the last quarter second of its motion. Find the height of the ballon and the velocity of the body when it reaches the ground. [And. U.]
($g = 32$ ft. per sec.²)

Ans. Let the period of fall be t seconds. Since the body falls from rest, the initial velocity $u = 0$. If s be the distance fallen through in t sec.,
$s = \frac{1}{2}gt^2 = 16t^2$. ... (1)
Again, if s_1 be the distance fallen through a quarter second earlier,

then, $s_1 = \frac{1}{2}g(t - \frac{1}{4})^2 = 16\left(t^2 - \dfrac{t}{2} + \frac{1}{16}\right) = t^2 - 8t + 1$, so that $s - s_1 = 8t - 1$ ...(2)

Hence $8t - 1 = 81$, whence $t = 10\frac{1}{4}$ seconds. Thus $s = \frac{1}{2}gt^2 = \left(16 \times \dfrac{41^2}{4^2}\right)$ ft.
$= 1681$ ft.
Finally, the final velocity $v = gt = (32 \times 10\frac{1}{4})$ ft./sec. $= 328$ ft./sec.

3. Two stones are projected vertically upwards at the same instant. One ascends 112 ft. higher than the other and returns to earth 2 seconds lates. Find the velocities of projection of the stones. ($g = 32$ ft. per sec.²) [C. U.]

Ans. Let the velocities of projection of the stones be u_1 and u_2 respectively. If h_1 and h_2 be the maximum heights attained by the stones, then

$$h_2 = \frac{u_1^2}{2g} \text{ and } h_2 = \frac{u_2^2}{2g}.$$

Now, $h_1-h_2=112$ ft. So $\frac{u_1^2}{2g}-\frac{u_2^2}{2g}=112$ or, $u_1^2-u_2^2=112\times64$. ... (1)

If t_1 and t_2 be the respective times of flight, then

$$t_1=\frac{2u_1}{g}, \text{ and } t_2=\frac{2u_2}{g}.$$

Now, $t_1-t_2=2$ sec. So $\frac{2u_1}{g}-\frac{2u_1}{g}=2$, or, $u_1-u_2=32$. ... (2)

Dividing (1) by (2), we get, $u_1+u_2=224$. ... (3)

From (2) and (3), we get $u_1=128$ ft./sec. and $u_2=96$ ft./sec.

4. An aeroplane is travelling horizontally with a velocity of 60 miles per hour and when it is at a height of 1936 ft., a bomb is dropped from it Find where the bomb strikes the ground and the time of its fall. ($g=32$ ft. per sec.)

Ans. Horizontal velocity of the plane = 60 miles per hour = 88 ft./sec.

The bomb falls vertically with an acceleration of 32 ft. per/sec.^{2} and also travels horizontally with a uniform velocity of 88 ft. per sec.

Let the time of fall of the bomb $=t$ seconds.

Then the formula $h=\frac{1}{2}gt^2$ give $\frac{1}{2}\times32\times t^2=1936$, whence $t=11$ sec.

$\therefore$ the desired distance $=(88\times11)$ ft. $=968$ ft.

Hence the bomb strikes the ground at 968 ft. from a point on the ground vertically below the point from which it was dropped.

5. Calculate the mass of the Sun, given that the distance between the Sun and the Earth is 1.49×10^{13} cm. and $G=6.66\times10^{-8}$ C.G.S. units. Take the year to consist of 365 days. [P. U.]

Ans. Let the mass of Sun be M and the mass of Earth be m.

Now the distance between the two, which is the radius of the earth's orbit, is $=1.49\times10^{13}$ cm.

$\therefore$ gravitational force of attraction $=G\frac{Mm}{r^2}=6.66\times10^{-8}\times\frac{Mm}{(1.49\times10^{13})^2}$ dynes.

Now the time of revolution of the Earth $=365$ days $=(365\times24\times60\times60)$ secs.

or, the angular velocity w of the Earth $=\frac{2\pi}{T}=\frac{2\times3.1415}{365\times24\times60\times60}$ radians/sec.

$\therefore$ the centripetal force on the Earth

$$=\frac{mv^2}{r}=mw^2r=m\times\left(\frac{2\times3.1415}{365\times24\times60\times60}\right)^2\times1.49\times10^{13} \text{ dynes.}$$

For equilibrium the centripetal force must be equal to the gravitational force. In such problems logarithmic calculations are preferable.

Hence $m\times\left(\frac{2\times3.1415}{365\times24\times60\times60}\right)^2\times1.49\times10^{13}=6.66\times10^{-8}\times\frac{Mm}{(1.49\times10^{13})^2}$

whence the required mass $M=19.67\times10^{32}$ gm.

6. A sputnik is designed to be raised up to a height 1000 miles above the surface of the earth and then given a horizontal velocity so as to revolve round the earth in a circular orbit. Calculate the velocity of projection of the sputnik as also the period of revolution, given that the acceleration due to gravity at the earth's surface is 32·19 ft./sec.^{2} and the diameter of the earth is 7980 miles. What would be the result of releasing an object from the rotating sputnik with no initial velocity.

Ans. We know $g'=G\frac{M}{(r+h)^2}=g\frac{r^2}{(r+h)^2}$ since $G=\frac{gr^2}{M}$.

$\therefore$ The value of g' at the height of 1000 miles is $=\left(32.19\times\frac{3990^2}{4990^2}\right)$ ft./sec.

$$=20.58 \text{ ft./sec.}$$

If m be the mass of sputnik, the weight of the sputnik at that height is mg'; and the centripetal force necessary to rotate it at the orbit of radius $r+h$ is $=\frac{mv^2}{r+h}$. The sputnik and any thing within it will be *weightless* while orbiting.

$$\therefore \quad \frac{mv^2}{r+h}=mg' \text{ whence } v^2=g'(r+h) \text{ or, } v=\sqrt{g'(r+h)}.$$

$$\text{or, } v=\sqrt{20{\cdot}58\times4990\times5280} \text{ ft./sec.}=23284 \text{ ft./sec.}$$

$$=\frac{23284\times60\times60}{5280} \text{ miles/hr.}=15875 \text{ miles per hour.}$$

Now the circumference of an orbit of radius (3990+1000)=4990 miles is $=2\times3{\cdot}1415\times4990$ miles=31354 miles.

$$\therefore \text{ the time period}=\frac{31354}{15875} \text{ hours}=1{\cdot}975 \text{ hr.}=1 \text{ hr. } 58 \text{ min. } 30 \text{ sec.}$$

If projected with no initial velocity any object on the sputnik would share equal velocity and remain orbiting the same path.

59. Motion of two Bodies connected by a string passing over a smooth Pulley (Atwood's Method)—Let two bodies of masses m_1 and m_2 be connected by a light inextensible string passing over a smooth pulley P, both of them hanging vertically (Fig. 89). If m_2 is larger than m_1, we are to find the resulting motion of the system and the tension of the string. As the pulley is smooth, the tension of the string is the same throughout its length. Again, as the string is inextensible, the downward motion of m_2 is equal to the upward motion of m_1; hence their acceleration is the same.

Let the common acceleration of the system be f and the tension of the string be T (in absolute units).

Now the resultant *downward* force on m_2 is$=(m_2g-\mathrm{T})$.

And the resultant *upward* force on m_1 is$=(\mathrm{T}-m_1g)$.

$$\therefore \quad m_2g-\mathrm{T}=m_2f \text{ and } \mathrm{T}-m_1g=m_1f.$$

$$\text{Adding} \qquad (m_2-m_1)g=(m_1+m_2)f,$$

$$\text{whence,} \qquad f=\frac{m_2-m_1}{m_2+m_1}g \qquad \dots (59,1)$$

$$\text{Also} \qquad \mathrm{T}=m_1(f+g)=\frac{2m_1m_2}{m_1+m_2}g \qquad \dots (59,2)$$

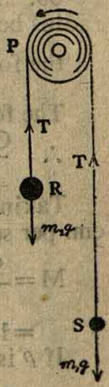


Fig. 89

Observing the acceleration f of the system and using equation (59,1), the value of g may be roughly determined.

59A. Reaction of a Moving Lift—When a man is standing on a lift, either at rest or moving up or down with a uniform velocity, the reaction at the floor of the lift is equal to the weight of the man. But when the lift is moving with an acceleration, the thrust is not equal to the weight of the man. It is, however, equal to the thrust then exerted by the man on the lift, which may thus be called the *apparent weight* of the man on the accelerating lift.

Let m be the mass of the man, R be the thrust between the man and the floor, and f be the acceleration of the system. When the lift is *accelerating up*, R must be greater than the weight of the man.

$$\therefore \quad \mathrm{R}-mg=mf \text{ or, } \mathrm{R}=mg+mf=mg\left(1+\frac{f}{g}\right),$$

i.e., the man appears to be heavier by $\frac{f}{g}$ of his weight.

It may be mentioned that in Appollo Rocket flights to the moon the acceleration f at one stage given to the rocket was about $7g$. Hence the apparent mass of the aeronaut became 8 times his normal weight. Hence a preliminary practice was necessary to withstand this unusual thrust.

When the lift is *accelerating down*, the weight must be greater than the thrust R'.

$$\therefore \quad mg-R'=mf, \text{ or, } R'=mg-mf=mg\left(1-\frac{f}{g}\right),$$

i.e., the man appears to be lighter by $\frac{f}{g}$ of his weight

If the lift falls *freely under gravity*, then $f=g$ and the reaction $R'=0$. That is the passenger within the lift will feel that he has no weight.

60. Mass of Earth—Let the mass of the earth be M gm, and its radius be R cm. Then, by the law of gravitation,—

the force of attraction on a mass m on the earth$=\frac{GMm}{R^2}$ dynes.

The force being equal to the weight of the body$=mg$ dynes.

$$\therefore \quad \frac{GMm}{R^2}=mg, \text{ whence } M=\frac{gR^2}{G}. \quad \dots \quad (60,1)$$

Taking $R=4000$ miles, $G=6{\cdot}65\times10^{-8}$ C.G.S. units, and $g=981$ cm. per sec.^{2}.

$$M=\frac{981\times(4000\times1760\times3\times30{\cdot}48)^2}{6{\cdot}65\times10^{-8}} \text{ gm.}=6{\cdot}113\times10^{27} \text{ gm.}$$

$$=1{\cdot}35\times10^{25} \text{ lb.}=60{\cdot}17\times10^{20} \text{ tons.}$$

If ρ is the mean density of the earth then $M=\frac{4}{3}\pi R^3\rho$. ... (60,2)

$$\therefore \quad \text{from (60,1) and (60,2), } \rho=\frac{3g}{4\pi GR}$$

$$\therefore \quad \frac{3\times981\times7\times10^8}{4\times22\times6{\cdot}65\times4000\times5280\times30{\cdot}48} \text{ gm./c.c.}=5{\cdot}46 \text{ gm./c.c.}$$

The value of G was first determined by Cavendish (1731-1810), whence the mass and density of the earth were calculated. It is thus said that 'Cavendish weighed the Earth in his balance'.

Examples :

1. A light string passes over a smooth pulley and has masses of 240 gm. and 250 gm. attached to its ends. Calculate the value of g, if the system starting from rest moves through a distance of 160 cm. 4 seconds. [Anna. U.]

Ans. Since the two loads are unequal, the combination would have an acceleration with the heavier load descending downwards. Let the common acceleration be f cm. per sec.^{2}

Then, $0+\frac{1}{2}f\times4^2=160$ cm. when $f=20$ cm./sec.^{2}
Let T be the tension which is throughout the same.
Then $250g-T=250\times20$ and $T-240g=240\times20$.
Adding the two equations, we get
$10g=490\times20$ whence $g=980$ cm./sec.^{2}.

2. A mass of 10 lb. is hung from a spring balance attached to a lift. The lift is (*a*) ascending with an acceleration of 4 ft. per sec. per sec., (*b*) ascending with a uniform velocity of 4 ft. per sec. Calculate how the reading of the spring balance will be affected in each case. The value of $g=32$ ft. per sec. per sec. [Pat. U.]

Ans. When the lift is ascending with an acceleration of 4 ft. per $sec.^2$, the reaction R of the mass of 10 lb. on the spring balance is given by

$$R=10\times32+10\times4, \text{ whence } R=360 \text{ poundals.}$$

Hence, the spring balance will record $(360\div32)$ lb.$=11{\cdot}25$ lb., that is 1·15 lb. more than its original reading.

Again, when the lift is moving with uniform velocity, whatever its magnitude may be, there is no change in the reaction of moving parts and so the spring balance records the same reading of 10 lb.

61. Spring Balance—A spring balance (Fig. 90) is an instrument with which weights of bodies can be determined and compared. It consists of a *spiral spring* of steel S (Fig. 90*b*), enclosed in a metallic case, with the upper end fixed to a ring at the top of the case. The lower end is attached to a straight rod R which has a hook H at the lower end, from which the body to be weighed can be hung. The front part of the case is covered by a plate with a long narrow slit C, which is graduated either in gramweight or in pound weight along the slit(Fig. 90*a*) A pointer attached to the rod passes out of the slit and rest against the scale. When a body is hung from the hook, the spring extends in proportion to the amount of pull exerted by the weight of the body. The pointer accordingly moves down and indicates the weight of a body from the reading on the scale.

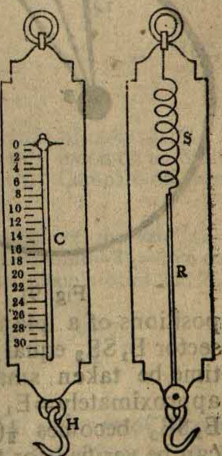


Fig. 90

As the spring extends by the pull exerted on the body by the earth, the spring balance directly gives the weight of a body at a place. If we move to a different place, the weight of the body changes and so the pull on the spring also changes. Hence from a theoretical standpoint the instrument may be calibrated so as to record the weight of a body.

Thus, in a common balance we compare different masses, while in a spring balance we compare different weights. If taken to different places, the bodies and the 'weights' used are equaliy affected by the variation of g. So it is not possible to detect any change in the weight of a body by a common balance. A spring balance, on the other hand, is differently affected due to the difference in the pulls and may indicate the variation in the weight. In practice, however, the variation of the weight of a body over the earth's surface is so small that it does not sensibly affect an ordinary spring balance.

***62. Kepler's Laws of Planetary motion**—During the last decade of the 16th century, the Danish astronomer Tycho Brahe and Johaan Kepler made a systematic observation of the nature of the orbits of the planet Mars and other planets around the Sun. After the death of Tycho Brahe, Kepler enunciated the following three laws of planetary Motion—

1, Each planet moves in an elliptic orbit with the sun at one of the foci.

2. The radius vector joining the planet to the sun describes equal areas in equal times.

3. The square of the periodic time of a planet varies as the cube of its mean distance from the sun.

Explanation of Laws—Let Fig 91 represent the elliptic orbit of a planet. Then the sun must be at either focus of the ellipse say at S. In case of the earth's orbit the First Law may be verified by a systematic measurement of the variation of the distance of the sun from the earth throughout a year. On December 31, the distance becomes minimum called the *perigee* point of the orbit. On July 1, the distance is maximum. called the *apogee* point. The eccentricity of the ellipse is very small having a value ·0168, which means that the orbit is nearly a circle.

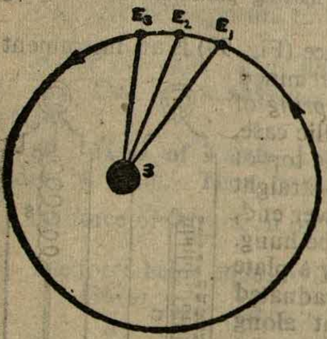


Fig. 91

The Second Law states that if E_1, E_2 and E_3 represent the positions of a planet at equal intervals of time then the area of the sector E_1SE_2 equal thal that of the sector E_2SE_3. If the interval of time be taken small, (for example, one day) the arc E_1E_2 is very approximately$=E_1S\times\theta$ where θ is the angle E_1SE_2 and the area E_1SE_3 becomes $\frac{1}{2}(E_2S)^2\theta$. Knowing E_1S and measuring θ the law can be verified for the two sectors E_1SE_2 and E_2SE_3.

Kepler's Third Law and Law of Gravitation—In order to explain the Third Law, Newton has to assume to existence of some sort of attracting force between the planet and the sun, the force varying as the product of the two masses and inversely as the square of the distance. Let M be the mass of the sun and m that of the planet at a mean distance r from the sun. For simplicity, assume that the orbit is circular.

If the tangential velocity of the planet at any point in its orbit be v, the centripetal force necessary for the rotation of the planet is mv^2/r. The origin of this centripetal force is the gravitational force. If T be the periodic time of the planet, then

$$\text{since } v=\frac{2\pi r}{T}. \quad \therefore \text{ centripetal force}=\frac{mv^2}{r}=\frac{4\pi^2 mr}{T^2}.$$

Since the centripetal force is equal to the gravitational force,

$$\frac{4\pi^2 mr}{T^2}=\frac{GMm}{r^2}, \quad \text{or, } T^2=\frac{4\pi^2 r^3}{GM} \qquad \ldots \quad (62, 1)$$

whence $T^2 \propto r^3$.

Thus, the Third Law can be proved by assuming the truth of the inverse square law of gravitational force.

Comparison of Masses of Heavenly Bodies—Kepler's Third Law gives us a clue to compare masses of heavenly bodies *which have* satellites at known distances and of known periodic times. Transforming equation (62,1), we can write

$$\frac{T^2M}{r^3}=\frac{4\pi^2}{G}=\text{a constant.}$$

If there are two heavenly bodies of mass M_1 and M_2 having two satellites of periodic times T_1 and T_2 and placed ta distances of r_1 and r_2 from their respective parent bodies, then

$$\frac{T_1{}^2M_1}{r_1{}^3}=\frac{T_2{}^2M_2}{r_2{}^3},\ \text{since each expression}=\frac{4\pi^2}{G}$$

$$\text{whence,}\ \frac{M_1}{M_2}=\frac{T_2{}^2}{r_2{}^3}\times\frac{r_1{}^3}{T_1{}^2} \quad \dots \quad (62,2)$$

Example :

A s tellite of Jupiter moves round the planet in a circle of radius $1{\cdot}161\times10^6$ miles completing one revolution in 16·6 days while Earth's moon on an orbit of radius $2{\cdot}24\times10^5$ miles completes on revolution in 27 days. Compare the masses of Jupiter and Earth.

Ans. Let the mass of Jupiter$=M_1$, The orbital radius of the satellite$=r_1$ $=1{\cdot}161\times10^6$ miles and the period of revolution of the satellite$=T_1=16{\cdot}6$ days. Let M_2 be the mass of the Earth, r_2 the radius of the moon's orbit and T_2 its period of revolution$=27$ days. Obviously, then,

$$\frac{\text{mass of Jupiter}}{\text{mass of Earth}}=\frac{27^2}{(2{\cdot}24\times10^5)^3}\times\frac{(1{\cdot}161\times10^6)^3}{(16{\cdot}6)^2}=\frac{368{\cdot}7}{1}.$$

So jupiter is about 369 times massive than earth.

***63. Journey to the Moon**—The first successful attempt of a rocket flight around the moon and back to earth was undertaken at the Spaceport of Cape Kenedy, Florida, United States of America. This was known as the Apollo—8 Mission conducted by three aeronauts, Frank Borman acting as the Commander of the mission, James Lovell and William Anders. At the start the Saturn 5 rocket at the bottom with the Apollo spacecraft at the top had an overall height of 364 feet, as high as a 36 storey building. The nature of construction and principle of flight of a rocket have already been described in Art. 38. On the launching pad the total weight of the rocket was about 3000 tons. Development of such a space craft required the combined efforts of about 350,000 scientists, engineers and technicians. The command module diameter with controlling switches, gears and panels all around it and having *just sufficient* space for the three aeronauts to a little movement.

The rocket started from the lunching pad at 7-51 A.M. U. S. A. time on December 21, 1968 from the Spaceport consuming 15 tons liquid fuel per second producing an unward thrust of 3750 tons. Within 150 seconds from the start it reached an altitude of about 100 milies from the earth's surface. This was rather a *slow upward* *acceleration* for a rocket, because in rising up through the atmosphere with a very high speed air friction will produce such a heat as to burn up the rocket and produce explosion of the fuel chamber. When the rocket reached this altitude the first fuel tank was exhausted and was dislodged mechanically from ths remaining part of the rocket.

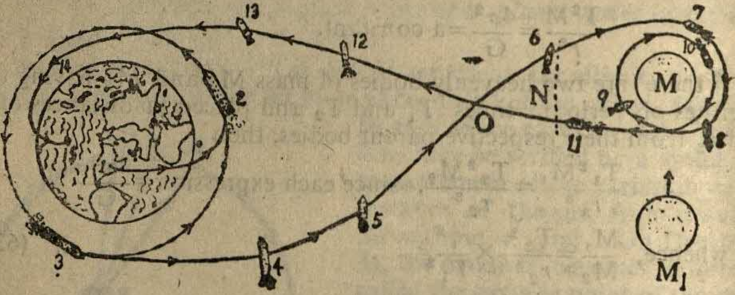


Fig. 92—Apollo-8 Mission

The next procedure was to place the rocket in an orbit around the earth, the plane of the orbit *very nearly coinciding* with the plane of moon's orbit about the earth. This plane is nearly coincident with the equatorial plane. This required steering of the rocket's course and giving it a tangential velocity in a circular course round the earth just sufficient to maintain a uniform circular motion (vide Art. 47). The speed of the rocket required to maintain it at a height of 100 miles is about 17413 miles per hour. (vide Example 7, Art. 58) and the time for one complete revolution round earth becomes 1 hr. 26·6 min. This was called the Parking Orbit. At this stage the condition within the rocket was *weightless* and the density of atmospheric air at this height was so small that resistance to rocket motion could be neglected. At this orbit station rechecking of all the apparatus containing 506 switches, 71 lights and 40 mechanical indicators was made.

Then came the direction from the earth station to recharge the second rocket engine for a journey toward moon. The engine roared and gave a speed of 24171 miles an hour to *break away from the orbit* and start its journey towards moon. This was then the greatest speed ever attained by aeronauts. At this stage the rocket was moving against the earth's gravity and so the speed gradually decreased. Nevertheless the initial speed was such as to pilot the rocket to distance within the gravitational zone of the moon. There were a number of small rocket engines to the sides of the main rocket, which helped the main rocket to place it at any angle to the course of flight. By inclining the rocket, the aeronauts could view the earth and the moon through side-windows.

As the rocket moved to a distance of about 180,000 miles from the earth towards moon, again a region of weightlessness was felt. This was the region where earth's gravitational pull was neutralised by moon's gravitational pull in the opposite direction. Then the rocket tended to fall on the moon with moon's acceleration due to gravity, which is about *one-sixth part* of that of the earth. This was a great juncture for the aeronauts, for they wanted the rocket to veer round the moon instead of falling directly on it. This was done by manouvering the craft close by the edge of the moon with *such a suitable speed* that instead of passing by the moon, it could encircle the moon by the same principle as adopted in earth circling. This could be achieved by working the rocket engine maintaining proper speed and direction. The first two were elliptical orbits around the moon and then by proper adjustments, the orbit could be made circular above 70 miles from the lunar surface with a speed of 3700 miles per hour. The time required to reach moon's orbit was about 66 hours from earth's orbit. The moon's circular orbit was completed in every two hours. The aeronauts celebrated Christmas Eve on December 24, 1968, while on the lunar orbit. They took very many photographs of the lunar surface full of craters and suggested future landing spots on the moon's surface. They took photographs of the sun and the earth. The photograph of the earth showed phases as we observe the moon from day to day. But earth did not look to be so bright as the moon because sun's rays are absorbed considerably by the earth's atmosphere while full sunlight can reach moon's surface because it has no atmosphere.

On December 25, the aeronauts restarted the engine to break away from lunar orbit with a speed of 4500 miles per hour towards the earth. On December 27 at 4-51 A.M. the craft splashed down with the help of parachutes in the South Pacific Ocean about 1000 miles south-west of Hawai islands. The total journey period was 147 hours and total mile-coverage was 595000 miles. In later period there have been another six more Apollo missions and on three occasions Apollo 11, 12 and 14 missions, the aeronauts landed on the surface of the moon, collected specimens, established small laboratories, etc. which would help geologists and other scientists to investigate the nature of formation of the Solar system.

American scientists also sent powerfull rocket Mariner IV spacecraft on July 14, 1965 which made an epic 7½ months journey and passed by planet Mars and sent tele-photographs from a distance of 7000 miles from the planet. Such are the improvements of modern technology and electro-communication techniques.

63A. An Endless Journey : In Art. 38, the principle of Rocket propulsions has been described and different types of rockets used for orbiting earth or for inter-planetory journey have been mentioned. On March 2, 1972, a small but very powerful spacecraft, Pioneer-10, was launched from Cape Canaveral, U. S. A., towards the sun's biggest planet Jupiter. During December 1973, the rocket passed by Jupiter and sent back to Earth by radio signals and colour photographs of the entire planet with its famous red spot, its striated cloud structure, a shallow region over its surface, etc. with its 9 moons. After passing Jupiter, it headed towards Saturn which it met by 1976 from a distance of 10000 miles. Its 16 revolving moons were also photographed. Then it turned towards Uranus and met it in 1979 from a distance of 8000 miles. Uranus has also rings like Saturn but not so bright. After that it met Neptune in 1984 and finally it has a chance of meeting Pluto in 1987. Then it will leave the Solar system and is likely to move towards the constallation of Taurus (Brisha). Scientists are almost sure that there is no human being on any planet of the Solar system except Earth. If the space craft survives 11 million years, it has a chance of reaching Taurus. In case Pioneer-10 meets any human society on its long and lonely journey towards infinity, it is carrying a picture with diagrams of human male and female also the directions about how to locate the earth. The scheme is almost a hopeless hope ; nevertheless it is an ever-lasting desire for inter-planetory good-will mission.

64. Requisites of a good Balance—The description of a physical balance has already been given in Chapter I. A good balance must be (1) *true*, (2) *sensitive* and (3) *stable*.

(1) A balance is said to be **true**, if the beam remains horizontal when both the pans are empty or are loaded with *equal masses*.

The *conditions* to be satisfied by a balance to be true are :—

(*i*) the two arms of the beam must be of equal length,

(*ii*) the scale pans must be of equal mass, and

(*iii*) the centre of gravity of the beam must be *vertically below the fulcrum* when the beam is horizontal.

To tesi whether a balance is true, a body is placed on one of the pans and weights are placed on the other, till the beam is horizontal. The body and the weights are then interchanged. If they still balance, *i.e.*, if the beam be still horizontal, the balance can be taken to be *true*.

(2) A balance is said to be **sensitive**, when the beam is appreciably displaced from the horizontal position, even for a very small difference in the weights of the bodies placed on the two pans. A sensitive chemical balance indicates a difference of one-tenth of a milligram.

The *conditions* to be satisfied by a *sensitive* balance are :—

(*i*) the C. G. of the beam should be very near the fulcrum,

(*ii*) the beam should be light and

(*iii*) the arms of the beam **must** be **long**. If such conditions are satisfied, even a *small excess* in the weight on one pan would produce of the weight about fulcrum sufficiently large moment to cause an appreciable inclination of the beam.

(3) A balance is said to be **stable**, if the beam after being disturbed, comes back quickly to its position of equilibrium. The *condition for stability*, requires that the C. G. *of the beam must be sufficiently below the fulcrum*. This condition is opposed to one of the conditions for sensitiveness. In other words, *stability is increased at the cost of sensitiveness*. A balance should be *rigid* ; *i.e.*, the beam should not bend when weights are placed on the pans.

64A. Method of Weighing by a Balance—Different methods are adopted to secure accurate weighing under different circumstances.

(i) **Method of Oscillation**—If the balance is *true*, the most common method of accurate weighing is the method of oscillations. With the pans empty, the balance is made free and the oscillations of the pointer are observed. The pointer should oscillate through an equal number of divisions on either side of the middle division. In Fig. 93, S represents the ivory scale over which the pointer P from the beam shows the nature of the oscillations. Here graduation 5 is the centre of the scale. Two ends of the turning are 0·5 and 9·5, so that the pointer *swings equally about the middle point.* The balance is now ready for use.

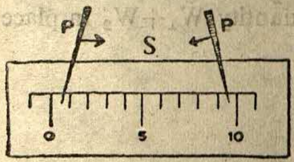


Fig. 93

If unequal oscillations are observed, the weighing pans should be cleansed with soft brush and the screw-nuts at the ends of the beam should be so adjusted that on raising the beam equal oscillations are observed. With less sensitive balances, the friction plays such a role that the oscillations rapidly die away. In such cases the **resting point** of the pointer should be calculated from a number of readings of the pointer. (For details vide J. Chatterjee's Intermediate Practical Physics, All India Edition). The body to be weighed is placed on the left pan ; weights are placed on the right pan, till on making the balance free, the pointer again oscillates through an equal number of divisions on either side. The mass of the body is then equal to the mass of the total weights placed on the pan. For accurate weighing a 'rider' of a small mass (generally 5 or 10 mg) is often used with a sensitive balance.

(ii) **Method of Double Weighing**—Even when the balance is *not true*, the mass of a body may be determined by the following method of *double weighing.*

(1) **Borda's Method** (*Method of Substitution*)—The body to be weighed is placed on one of the pans (say, the right-hand one) and is counterpoised with small shots, sand, etc. The body is then removed and standard weights are placed on that pan, till equilibrium is restored. Evidently the sum of the standard weights is equal to the weight of the given body, for whether the balance be *true* or *false*, these weights and the body, separately placed on the same pan, balance the sand, etc., on the other pan.

(2) **Gauss' Method**—This method is applied, if only the arms are of unequal lengths but the balance is otherwisee correct. The body is first placed on one of the pans (say, the left-hand one) and is exactly counterpoised in the usual way. The body is then placed on the right-hand pan is counterpoised again.

Let the arms of the balance$=a, b$; the true weight of the body $=W$; the ocserved weight with the body on one pan$=W_1$ and the observed weight with the body on the other pan$=W_2$.

Then $Wa=W_1b$ and $W_2a=Wb$, ... (64,1)
or, $W^2ab=W_1W_2ab$ whence $W=\sqrt{W_1W_2}$. ... (64,2)

Thus the true weight is the geometrtc mean of the observed weights. To fiud the ratio of the arms :—

From equation (64,1), $\frac{a}{b}=\frac{W_1}{W}$ and $\frac{a}{b}=\frac{W}{W_2}$.

Hence $\frac{a}{b}=\sqrt{\frac{W_1}{W}\times\frac{W}{W_2}}=\sqrt{\frac{W_1}{W_2}}$.

Suppose a tradesman uses a false balance with unequal arms *a* and *b*. A customer takes a quantity of some article, which appears to weight W_1 being weighed in one pan. The balance is turned and another quantity also appearing to weight W_2 is weighed in the other pan. Here the customer really gets a quantity W_1+W_2 in place of 2W.

$$\text{Now, } W_1+W_2-2W=W\frac{a}{b}+W\frac{b}{a}-2W$$

$$=W\frac{a^2+b^2-2ab}{ab}=\frac{(a-b)^2}{ab}W.$$

As *a* and *b* are unequal, $(a-b)^2>0$. Further, as *a* and *b* are both >0, the right-hand side is >0, so that $W_1+W_1>2W$,

i.e., the trades man cheats himself by the amount $\frac{(a-b)^2}{ab}W$.

Hence in a false balance it is profitable for a customer to purchase always a quantity of substance in two instalments, weighing half on one pan and the other half on the other,

Examples :

1. A tradesman sells his articles weighing equal quantities alternatively from the two pans of a balance having unequal arms. If the ratio of the lengths of the lengths of the two arms be 1·025, what is his percentage loss or gain ? [Pat. U.]

Ans. Let the true weight (mass) of the body be W and its observed weights when placed on the two pans be W_1 and W_2. Then

$$\frac{a}{b}=\sqrt{\frac{W_1}{W_2}}=1{\cdot}025.$$

When equal quantities are weighed alternatively from the two pans of the balance, the customer gets a quantity in excess of 2W by a quantity.

$$=W\frac{(a-b)^2}{ab}=W\frac{a^2+b^2-2ab}{ab}=W\left(\frac{a}{b}+\frac{b}{a}-2\right)=W(1{\cdot}025+{\cdot}9756-2)={\cdot}0006W.$$

Hence the tradesman cheats himself or loses by $\frac{{\cdot}0006W}{2W}\times100={\cdot}03\%$.

2. A body A when placed successively in the pans of a faulty balance appears to weight 8 lb. and 18 lb. Another body B when treated in the same way appears to weigh $5\frac{1}{3}$ lb. and 12 lb. In what respect is the balance false, and what are the real weights of A and B ?

Ans. When the arms of the balance are not of equal lengths, the true weight $W=\sqrt{(W_1\times W_2)}$.

For the body A, $Wa=\sqrt{(8\times18)}$ lbs. $=12$ lb.
For the body B, $Wb=\sqrt{(5\frac{1}{3}\times12)}$ lbs. $=8$ lb.

The ratio of the arms is, however, $=\sqrt{\frac{W_1}{W}}$

Illustrations of Inertia of Motion—A person, in getting down carelessly from a moving tram car, tends to fall forward. His feet, on touching the ground, come to rest but the upper part of the body due to inertia tends to move forward and so he stumbles towards the direction in which the car is moving. Again, when a car in motion rapidly stops, a passenger sitting loosely in the car swings forward. If a horse at full gallop suddenly stops, the rider unless sitting firmly on the horse back is likely to be thrown forward over its head.

Before taking a long jump a person has to run from a little distance behind, so that the inertia of motion helps his muscular effort to jump. A ball thrown vertically upwards by a person in a running train comes back to his hand. Due to inertia the ball maintains its horizontal motion which it has acquired from the car and so comes back to the hand, if the motion of the train or the position of the hands does not change in the meantime. Similarly in the circus show, a performer jumps up from the back of a running horse, and passing through a hoop again comes back on the horse.

33. Explanation of the Second Law—The Second Law states that so long as an external force is acting upon a material body, it will continuously *gain* in motion in the direction of the force. It was Galelio who first observed it and it was left for Newton to explain it with mathematics. The acceleration of the body under the action of the force is a measure of the force acting upon it. This law gives a *quantitative definition of a force.*

Momentum—The *momentum* of a body is measured by the product of the mass and velocity of the body. Hence, if the mass of the body be m and its velocity at any instant be v, the *momentum* of the body at the instant is $m \times v$. Newton termed momentum as the *amount of motion* of a material body.

The *unit* of momentum is the momentum possessed by a body of unit mass moving with a unit velocity. The F.P.S. unit of momentum is the momentum possessed by a mass of 1 lb. moving with a velocity of 1 ft. per second. It is often termed a *poundem.* The C.G.S. unit of momentum is the momentum of a mass of 1 gm. moving with a velocity of 1 cm. per second. For example, a ball of mass 150 gms. moving with a velocity of 1 metre per second has a momentum of $150 \times 100 = 15 \times 10^3$ gm. cms. per second. Momentum is a *vector* quantity.

34. Measurement of Force—Let a body of mass m having an initial velocity u, be acted upon by a constant force P for a time t. Since it is under the action of a force, its velocity will continuously increase in the direction of the force. Let its final velocity be v at the end of this period. Now the initial momentum of the body $= mu$, and the final momentum $= mv$, Hence the change of momentum in time $t = m(v-u)$.

So, the rate of change of momentum is $\frac{m(v-u)}{t}=mf.$

According to this law, the force P is proportional to the rate of change of momentum, which means that, $P \propto mf$.

or, $P=kmf$, where k is constant.

If now the unit of force be so chosen that it produces unit acceleration when acting on a unit mass, then if $m=1$ and $f=1$, then P becomes equal to 1. Accordingly k must be equal to 1. Thus with units so chosen, $\mathbf{P=mf}$. ... (34,1)

or, force=mass × acceleration.

Unit of Force—The unit of force may be defined as *that force which acting on a unit mass produces unit acceleration on it.*

The F.P.S. unit of force is that force which acting on a mass of 1 lb. produces acceleration of 1 ft. per sec. This unit is termed a **poundal.** The C.G.S. unit of force is that force which acting on a mass of 1 gm. produces acceleration of 1 cm. per second per second. This unit is called a **dyne.**

These two units are called *dynamical* or **absolute units of force**, for they are believed to remain constant throughout the universe. There is another system of units of force called the **gravitational unit.** (*Vide* Chapter IV.)

Again, $P=\frac{mv-mu}{t}$, or, $P \times t=mv-mu.$ ... (34,2)

Impulse.—When a large force acts on a body for a short period, the product (force × time) is called the **impulse** of the force during the period. During the period in which the impulsive force acts, it no doubt produces an acceleration of the body ; but the period being small, the acceleration is very large. An acceleration produces a continuous change of momentum of body. In measuring an impulse of a force, we are concerned more with the change of momentum during the period than with the acceleration produced on the body. Necessarily an impulse is also a vector.

Equation (34,2) provides another definition for the unit of force, for, if $(mv-mu)$ be unity and t be 1 second, then P becomes unity. Thus a unit force is that force which *acting on a mass for unit time produces a unit change of momentum in the direction of the force.* Let the mass taken be unity ; then a unit force acting upon a unit mass initially at rest would generate unit velocity on it. A unit mass is very often defined in this way.

Relation between a Poundal and a Dyne.—A poundal is much larger than a dyne. A force of one poundal acting on a mass of 1 lb. produces an acceleration of *one foot* per sec^2. Now 1 lb.=453·59 gm. and 1 ft.=30·479 cm.

Hence a force of 1 poundal acting on a mass of 453·59 gm. produces an acceleration of 30·479 cm. per sec. per sec.

∴ 1 poundal = 1 lb × 1 ft. $sec.^{-2} = 453{\cdot}59 \times 30{\cdot}479$ gm. cm. $sec.^{-2} = 13824{\cdot}969$ dynes = 13825 dynes (very approximately).

The momentum is a *vector* quantity, its direction at any instant being the same as that of the velocity at that instant. It can, therefore, be represented by a straight line. The momentum has the dimension of mass × velocity, which is $M \times LT^{-1} = MLT^{-1}$. The force has the dimension of mass × acceleration, which is MLT^{-2}.

Examples :

1. An automobile of mass 3500 lbs. moving with a uniform velocity of 60 miles per hour is to be stopped in 1 second by the application of a uniform retarding force. What should be the magnitude of this force ? [C.U.—1956]

Ans. We know 60 miles per hour = 88 ft. per second. If f be the *negative* acceleration necessary to bring the body to rest in one second, then in the general equation $v = u - ft.$, we set $v = 0$, $u = 88$ ft./sec. and $t = 1$ sec.

Thus $0 = 88 - f \times 1$, whence $f = 88$ ft./sec².

Again $P = mf = 3500 \times 88 = 308000$ poundals.

2. A train having a mass of 100 tons running on a level ground is kept going by the locomotive at a uniform speed of 50 miles per hour ; the steam is suddenly shut off and the train comes to rest after going 2 miles farther. Calculate the average resistance to the motion of the train. [Utkal U.—1952].

Ans. The train running with a speed of 50 miles per hour is brought to rest within 2 miles. Thus, to calculate the retardation, the formula $v^2 = u^2 - 2fs$ is to be used, where $v = 0$ and $u = 50$ miles/hr $= \frac{220}{3}$ ft./sec., and $s = 2$ miles $= 10560$ ft.

So $0 = \left(\frac{220}{3}\right)^2 - 2f \times 10560$, whence $f = \frac{605}{2376}$ ft. per sec. per sec.

The resisting force acting on the train = mass × retardation

$= \left(100 \times 20 \times 4 \times \frac{605}{28 \times 2376}\right)$ poundals $= 57037$ poundals.

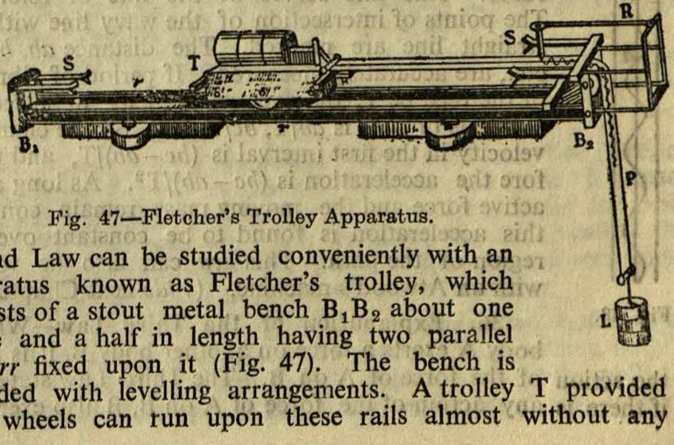


Fig. 47—Fletcher's Trolley Apparatus.

35. Verification of the Second Law of Motion—The Second Law can be studied conveniently with an apparatus known as Fletcher's trolley, which consists of a stout metal bench B_1B_2 about one metre and a half in length having two parallel rails *rr* fixed upon it (Fig. 47). The bench is provided with levelling arrangements. A trolley T provided with wheels can run upon these rails almost without any friction. A pair of springs SS, known as *friction brakes*, are fixed at the end of the bench for arresting the motion of the trolley without a direct collision. An almost frictionless pulley also a fixed at one end to facilitate the pulling of the trolley whenever required. A metal reed R capable of vibrating at right angles to the length of the bench may be clamped at either end of the bench.

The bed is made accurately horizontal with the levelling arrangements at the base, so that the trolley runs on a horizontal surface. Then the trolly is tied with a piece of thread to the end B_1 of the bench. One end of a specially made paper tape P is tied to the trolley. The tape is carried over a pulley and a scale-pan is suspended at its other end. A suitable load is put upon the pan ; this serves as the pulling force on the trolley. An inked brush is fixed at the end of the reed so as to touch the paper tape.

The reed is swung a bit at right angles to its length and then released, so that it keeps on vibrating and marking a small arc on the paper with the inked brush. The period of one vibration of the reed is determined with a stop-watch or a metronome. Then the trolley is released by cutting down the thread, when it begins to move over the rails under the action of the load. The inked brush is found to trace a wavy curve on the moving strip of paper (Fig. 48). Since time for each vibration of the reed is constant, such curves on the paper offer a convenient method of accurately measuring short intervals of time. The paper is then placed on the table and a straight line is drawn evenly through the wavy curve. This line serves as the line of reference. The points of intersection of the wavy line with this straight line are marked. The distance *ab*, *bc*, *cd*, etc., are accurately measured. If period of vibration of the reed be T, the average velocity in successive intervals of time is *ab*/T, *bc*/T, etc. So the change of velocity in the first interval is $(bc-ab)/T$, and therefore the acceleration is $(bc-ab)/T^2$. As long as the active force and the moving mass remain constant, this acceleration is found to be constant over any region of the path. This law can also be verified with an Atwood's machine. (*Vide* next Chapter).

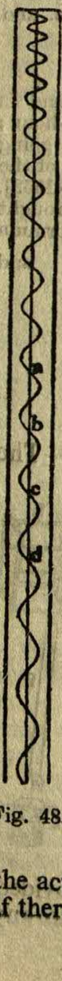


Fig. 48.

36. Explanation of the Third Law—When a body A exerts a force on another body B, it is called the action of the force of A on B. The Third Law states that if there is any action of the force of A on B, B also exerts an equal and opposite force on A, which is called the *reaction* of B on A. This law holds good whether the bodies are at rest or in motion and whether they are in contact or acting on each other from a distance. This law is, therefore, called the **law of reaction.**

Take, for example, a person standing on the ground. The weight of the person shows itself as the thrust exerted by his feet on the ground, which is called the 'action' of his weight. So long as he stands firm, the ground presses his feet *equally* in an upward direction, which is called the 'ground reaction'. This is an example of static action and reaction. But when he stands upon sandy or marshy soil, the ground resistance may not be sufficient to balance his weight. In that case his feet might be gradually submerged in the soil. Here also at any instant the ground reaction is equal and opposite to a *part of the weight of the person,* which is called the *action.* The remaining part of the weight serves as the moving force and has nothing to do with the Third Law and it is to be treated with the Second Law. Since the ground reaction is much less on sandy soil, it is rather difficult to walk fast.

This law also gives the principle of **conservation of momentum.** Thus, if a bat is moved towards an approaching ball, then on collision the force exerted by the bat on the ball is equal and opposite to that exerted by the ball on the bat. Hence the momentum given to the ball by the bat is equal to that given to the bat by the ball in the *opposite direction.* In other words, the momentum gained by one is equal to that lost by the other, so that *the total momentum of the system remains the same in any particular direction.* The principle of the conservation of momentum may be verified with an apparatus known an a Ballistic Pendulum. It can also be verified with a slight modification of Fletcher's Trolley apparatus (*Vide* J. Chatterjee's *Intermediate Practical Physics*).

37. Different Kinds of Action and Reaction—The action and reaction between any two bodies may be of the following kinds :

(*1*) *Thrust or Pressure*—This occurs when one body presses against another. Thus when a book rests on a table, it presses the table vertically *downwards* by its weight, while the table exerts a pressure on the book vertically *upwards*, called the *thrust* (or the reaction) on the book. These two forces must always be equal and opposite under static condition.

(*2*) *Pull* or *Tension*—This occurs when two bodies pull each other through a material medium. Thus when a body is kept suspended by a string, the weight of the body pulls the string *downwards*, while the string pulls the body *upwards* with an equal force called the **tension** of the string. The upward force is thus equal to the weight of the body suspended.

Horse pulling a Carriage—Let a carriage be pulled by a horse (Fig. 49). We shall now consider how the motion of the system is possible in spite of the fact that tensions along the tie-rod D are always equal and opposite.

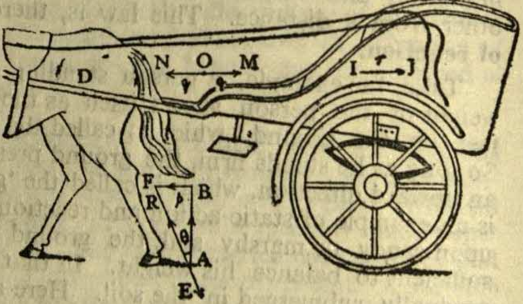


Fig. 49

To start motion, the horse presses against the ground with its hind leg in a direction with a force (say, R) along AE. Then the reaction, which is also R, acts in the opposite direction AF. Now the reaction R can be resolved into two components—(*i*) a vertical component AB supporting the weight of the horse, and (*ii*) the horizontal component BF, which tends to move the system forward. Call this horizontal component p. There is a resistance r acting on the carriage due to friction of the wheels against the ground. So long as p is less than r, there is no motion. But as the horse exerts a greater and greater force, the component p increases and ulimately when p exceeds r, the system is under the action of a resultant force $p-r$, which makes the system move in the forward direction.

To find the tension q, consider the carriage and the horse as two separate systems having common acceleration. Then the resultant force acting on the carriage is $q—r$ producing the acceleration, and the force $p—q$ produces the acceleration of the horse. On knowing their masses and acceleration, tension can be found. If the horse moves with a uniform velocity, $p—q=0=q—r$, whence $p=q=r$.

(*3*) *Attraction* and *Repulsion*—These forces occur between two bodies acting on each other at a distance. Thus two opposite magnetic poles attract each other and two similar poles repel each other with equal and opposite forces in either case.

(*4*) *Friction*—This occurs when a body tends to slide over another body and friction always acts opposite to the direction of motion. Thus when a body remains *at rest* on a *rough* inclined surface, the rough surface prevents the body from sliding down and exerts a force, called the *frictional force*, which is equal and opposite to the force causing the body to slide down. Again in walking, one presses against the ground in a *backward* direction with his feet and the frictional reaction of the ground gives him an equal *forward* force which thus sets him in motion. Hence, it is difficult to walk on a very smooth surface.

The principle applies equally in case of *bodies in motion.* When we jump on the shore from a boat, the boat is found to be pushed backward and the momentum of the boat is equal and opposite to that of the man. Again, when a bullet is fired from a rifle, the bullet is hurled forward violently by the force of the expanded gas. The bullet also exerts an equal and opposite force on the expanded gas, which pushes the rifle backward. The momentum of the rifle is equal and opposite to the momentum acquired by the bullet ; but as the rifle is much heavier than the bullet, its motion is much less. The back-push of a rifle, when fired, is a well-known fact.

The motion of a rocket is due to the burning of a fuel contained in it. During combustion a stream of gass is given off downwards with a great force and due to reaction the rocket shoots upwards. This is the principle on which a spaceship is guided. When streets are watered, water rushes out of the hose-pipe with a force, the backward reaction of which is felt by the water-man.

38. Rocket Propulsion—The motive power of any object by the rocket motor depends on the principle of the conservation of momentum. In most general case this principle is used in projecting a body high up in the atmosphere or *even beyond* it. The rocket mainly consists of three parts : (i) The tower or the nose *N* inside which there is a closed room for the aeronaut and instruments (Fig. 50), (ii) The fuel tank T containing the liquid or solid chemicals for combustion, (iii) The rocket motor M in which the fuels can be transferred in proper proportion by pressure and *rate of combustion can be controlled* from the tower chamber.

The base of the rocket engine is provided with a number of nozzles which eject out the gaseous products of combustion in a desired direction. Suppose that the rocket is to be lifted vertically upwards. In such a case the rocket is placed on a launching pad L in a vertical position (Fig. 50). On starting the engine the fuel ignites in the cylinder of the engine and hot gases are expelled *violently* through the nozzles vertically downwards. The expulsion of the gases in a downward direction gives an upward momentum to the rocket and this *change of momentum per second* is the upward force that lifts the rocket and gives it the required motion.

Theory of motion:—Let m be the mass of the rocket *including the fuels* at the start. As fuel burns explosively and is ejected out, the mass continuously diminishes. Let δm be a very small part that burns out in a small interval of time of δt. Let v_r be the downward velocity of the gas *relative to the rocket*, If v' the velocity of the gas *relative to the earth*, then so long as the rocket does not get a start. $v' = -v_r$.

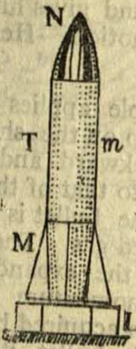


Fig. 50

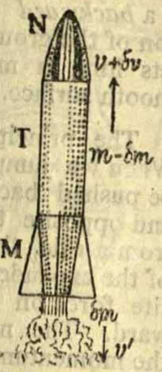


Fig. 51

The momentum of the exploded gas in time $\delta t = \delta m \times v_r$. Thus, the rate of change of momentum $= \frac{\delta m}{\delta t} \times v_r$ which is a force applied to the rocket in the upward direction. By controlling δm, a stage can be attained at which this force becomes equal to the *residual weight* of the rocket. The rocket then *begins to leave* the launching pad. Let v be the *velocity at any instant* of the rocket when it is at some position in the atmosphere (fig. 51). Then the velocity of exploded gas relative to the earth is given by $v' = v - v_r$ and its momentum is $\delta m v' = \delta m\ (v - v_r)$. At the end of the interval δt, the mass of the rocket with unburnt fuel has decreased to $m - \delta m$ and its velocity has increased to $v + \delta v$. Its momentum is therefore $(m - \delta m)(v + \delta v)$.

By Newton's Second Law,

$$\text{Force } F = \frac{\delta(mv)}{\delta t} \quad \text{whence } F\,\delta t = \delta(mv) \quad \cdots \quad (38,1)$$

If air resistance is neglected, the external force F acting upwards on the rocket is its weight mg which *acts downwards*.

Therefore $F = -mg$.

$$\begin{aligned} \therefore\ F\delta t = -mg\delta t &= [(m - \delta m)\ (v \times \delta v) + \delta m\ (v - v_r)] - mv \quad \cdots \quad (38,2) \\ &= [mv + m\delta v - v\delta m - \delta m.\delta v + v\delta m - v_r\delta m - mv] \\ &= m\delta v - v_r \delta m \quad \text{neglecting } \delta m.\delta v \text{ which is very} \end{aligned}$$

small $m\frac{\delta v}{\delta t} = v_r\,\frac{\delta m}{\delta t} - mg \quad \cdots \quad \cdots \quad \cdots \quad (38,3) \cdots$

Since $\delta v/\delta t$ denotes the acceleration, the left side of the equation represents the resultant force on the rocket. The first term on the right gives the *upward thrust* on the rocket and mg is the weight of the rocket at that instant. The upward thrust is thus proportional to the relative velocity v_r of the ejected gas and the mass of the gas ejected per unit time which is $\delta m/\delta t$.

$$\therefore \quad \text{Acceleration of the Rocket} = f = \frac{\delta v}{\delta t} = \frac{v_r \delta m}{m \delta t} - g \quad \ldots(38,4)$$

As the rocket shoots up, the value of g decreases. In the outer space far from planetory bodies, g becomes negligibly small. The values of v_r and $\delta m/\delta t$ remain approximately constant while the fuel is being consumed but m continuously decreases. Hence acceleration of the rocket increases till the last drop of fuel in it. When long flights are required the tank T as well as rocket engines are linked together in various compartments, one above the other. When fuel in the lowest tank is exhausted, it is automatically dislodged from the rocket with its ejectors. Then the upper tank with its ejector comes into operation. This one is also dislodged when its fuel is exhausted. Such an arrangement is called a **multi-stage rocket**. In Apollo excursions to the moon *three stage saturn rockets* were used by NASA of U.S.A.

Examples : A rocket during its first second of flight loses $\frac{1}{100}$ of its mass with an ejecting velocity of 7000 *ft* per second. What is the acceleration of the rocket ? Take the mean value of $g = 32 ft/sec^2$.

Answer : Here $\frac{\delta m}{m} = \frac{1}{100} . v_r = 7000 ft/\text{sec.}$ and $\delta t = 1$ sec.

$$\therefore \quad \frac{\delta v}{\delta t} = \text{acceleration} = 7000 \frac{ft}{\text{sec}} \times \frac{1}{100} \times \frac{1}{1 \text{ sec}} - 32 \frac{ft}{\text{sec}^2} = 38 \frac{ft}{\text{sec}^2}$$

In early stages, smaller rockets carrying scientific instruments to record the conditions of the upper atmosphere were usually launched. These instruments were brought to the earth's surface at a slower rate by parachutes. The first human aeronaut to go up the atmosphere was a member of the Soviet Academy in a Sputnik in 1954. Thereafter very many types of rockets such as Gemini, Saturn, Pioneer, Ranger, Mariner, Apollo, Souje, etc., have been deviced by America and Russia. These rockets could be launched and revolved round the earths in suitable orbits with aeronauts within it. Upto year 1965 about 154 artificial satellites were put around the earth in different orbits.

Examples

1. A force of 120 poundals acts for 2 seconds on a mass of 24 lbs. initially at rest and then ceases to act. Find the velocity acquired and the space moved over by the body during the time. How far will it move during the next half-a-minute ?

Ans. The equation $P = mf$ gives $120 = 24f$, whence $f = 5$. This shows that so long as the force acts upon the body, it moves with an acceleration of 5 ft./sec.^{2}. But the force acts for 2 secs., the initial velocity of the body being zero.

$\therefore$ the final velocity after 2 seconds $= u + ft = (0 + 5 \times 2) = 10$ ft./sec.

Again, the space moved over by the body in 2 seconds $= ut + \frac{1}{2} ft^2 = (0 + \frac{1}{2} \times 5 \times 4) ft. = 10$ ft, As soon as the force ceases to act on the body, there is no acceleration and the body moves with a constant velocity of 10 ft. per sec . Hence during the next half a minute, it moves through $(10 \times 30) = 300$ ft.

2. A shot of mass 40 lbs. leaves a cannon with a speed of 1000 ft./sec. If the velocity of recoil of the cannon is $4\frac{13}{28}$ ft./sec., find its mass in tons. [Utkal U. 1951]

Ans. From the law of reaction, we know that the momentum of the shot is equal and opposite to the recoil momentum of the cannon.

Hence $40 \times 1000 = M \times 4\frac{13}{28}$, whence $M = \frac{40,000 \times 28}{125}$, if the required mass be M.

$$\therefore \quad M = \frac{40,000 \times 28}{125 \times 28 \times 4 \times 20} \text{ tons} = 4 \text{ tons}.$$

3. A shell of mass 40 lbs. moving with a velocity of 12 ft./sec. explodes and bursts into two portions of masses 28 lbs. and 12 lbs. respectively. If the former be brought to rest owing to explosion, find the velocity of the latter.

Ans. Let the velocity of 12 lbs. be v ft./sec. after explosion.
The initial momentum of the entire shell $= 40 \times 12 = 480$ poundems.
The final momentum of 28 lbs. is $= 28 \times 0 = 0$.
The final momentum of 12 lbs. is $= 12 \times v = 12v$ poundems.
From the principle of conservation of momentum we get $480 = 12v + 0$, whence $v = 40$ ft. sec.

***39. Elastic Impact**—If a marble ball is allowed to fall upon a cemented floor, it rebounds to a certain height. But if the same ball is allowed to fall upon an wooden floor, it rebounds to a much smaller height. If again, instead of a marble ball, an ivory ball be dropped on the cemented floor from the same height, it rebounds to a larger height. The velocities before and after impact may be calculated from the distance fallen through and the height of rebound. As a result of experiments Newton derived the following conclusion :

For the same pair of substances undergoing elastic impact, the velocity after impact bears a constant ratio to the velocity before impact and is independent of the masses undergoing impact.

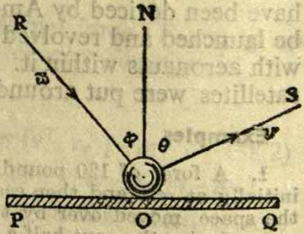


Fig. 52

This ratio, which depends on the material of the ball and the floor, is known as the **Coefficient of (elastic) Restitution**. Suppose that a ball is colliding in a direction RO against a fixed plane PQ with a velocity u making an angle ϕ with the normal ON to the surface at the point of incidence and that it has no motion of rotation (Fig. 52). After impact, let the velocity be v in a direction making an angle θ with the normal. Now resolve the incident velocity into two components, viz., $u \sin \phi$ parallel to and $u \cos \phi$ perpendicular to the colliding plane. In a similar manner resolve the velocity v along the same directions into components $v \sin \theta$ and $v \cos \theta$ respectively.

Now as the plane is supposed to be perfectly smooth, there cannot be any force acting on the ball parallel to the plane during impact and consequently the parallel component of the velocity remains unchanged. Thus $v \sin \theta = u \sin \phi$ ...(39,1)

By Newton's Law the perpendicular component is altered by a certain ratio. Let the ratio be e; then $v \cos \theta = e\,u \cos \phi$ ... (39,2)

$$\therefore \frac{v \cos\theta}{v \sin\theta} = \frac{e.u. \cos\phi}{u \sin\phi}, \text{ whence } \cot\theta = e \cot\phi \quad \cdots \quad (39,3)$$

An ideal case is one in which collision is perfectly elastic. In that case e becomes equal to unity and $\theta = \phi$. In such a case the angles with the normal before and after impact are equal.

If m be the mass of the ball, then due to a single collision the change of velocity is $u \cos \phi + ev \cos \theta$ and the change of momentum is $m\,(u \cos \phi + ev \cos \theta)$. This change of momentum taking place within a very short time acts as an impulse, handed on to the plane surface during the act of collision. If the ball collides n times per second under similar conditions, then the total change of momentum per second is $mn(u \cos \phi + ev \cos \theta)$. By the Second Law this quantity acts as the force exerted by the collisions on the plane surface. This principle is applied in the theory of gas pressure on its bounding wall due to molecular collisions on the wall.

Examples :

1. A glass marble, whose mass is one ounce, falls from a height of 25 ft. and rebounds to a height of 16 ft. ; find the impulse and the average force between the marble and the floor if the time during which they are in contact is 0·1 second, the acceleration of fall being 32 ft./sec.^{2}.

(Nag. U.—1951)

Ans. Let the velocity of the marble at the instant of colliding against the floor be v ft./sec. When falling from rest through a height of 25 ft.

$$v^2 = 0^2 + 2 \times 32 \times 25, \text{ whence } v = 40.$$

When rebounding to a height of 16ft. let the velocity of start just after collision be v_1 ft./sec. Then,

$$0^2 = v_1{}^2 - 2 \times 32 \times 16, \text{ whence } v_1 = 32.$$

The direction of velocity being reversed after collision, the change of velocity is evidently $(40+32) = 72$ ft. per second and consequently the change of momentum of the ball is $\frac{1}{16} \times 72$ ft. pound per sec. $= 4{\cdot}5$ poundems = impulse handed on by the marble to the floor during a period of 0·1 second.

Again, since force × time = impulse,
the required force = impulse ÷ time = 4·5 ÷ 0·1 = 45 poundals.

40. Composition of Forces—If a particle is simultaneously acted on by two forces in the same straight line and in the same direction, their *resultant* is equal to the sum of the two forces and acts in the same direction. If the two forces act in opposite directions, the *resultant* is equal to the difference of the two and acts in the direction of the greater of the two forces. If two forces act in directions inclined to each other, their resultant is found by the parallelogram Law enunciated below ;

Parallelogram of Forces—*If two forces, acting simultaneously at a point, be represented in magnitudes and directions by the adjacent sides of a parallelogram drawn from a point, their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.*

Thus, if a particle at O be acted upon simultaneously by two forces p and q, represented in magnitudes and directions by OA and OB respectively (Fig. 53), the diagonal OC represents the resultant in magnitude and direction. Proceeding as in Art. 21, it can be shown that the resultant R is given by the equation

$$R^2 = p^2 + q^2 + 2pq \cos \alpha \quad \cdots \ (40, 1)$$

where α is the angle (AOB) between the directions of p and q.

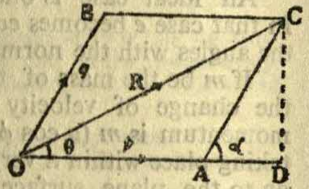


Fig. 53

If the direction of R is inclined at an angle θ to that of p, then

$$\tan \theta = \frac{q \sin \alpha}{p + q \cos \alpha} \quad \cdots \ (40.2)$$

When the two forces act at right angles to each other.

$$R^2 = p^2 + q^2, \text{ or, } R = \sqrt{p^2 + q^2} \text{ and } \tan \theta = q/p. \quad \cdots \ (40,3)$$

Since a force has some magnitude and direction, it is a vector. According to Art. 24, we can then write

$$\vec{p} + \vec{q} = \vec{R} \quad \cdots \ (40,4)$$

Experimental Verification—Take three strings knotted at a point O and attach the other ends of the strings to three spring balances L, M and N (Fig. 54). Attach the spring balances to hooks fixed on a drawing board in such a way that the strings are kept well stretched. From the readings of the balances find the forces acting along the strings and let them be represented by p, q and R respectively. Now on the paper fixed on the board mark the direction of p, q and R. Measure lengths OA and OB to represent p and q respectively. Complete the parallelogram OBCA and join OC. It is found that the length of the diagonal OC represents the resultant R in magnitude and direction in the same proportion. (*Vide* J. Chatterjee's *Intermediate Practical Physics*). The force R is

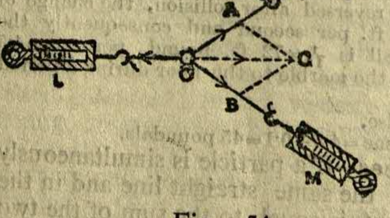


Fig. 54

called the **equilibrant** of p and q, since the three forces together keep the point O in equilibrium.

Illustrations of the Principle—A ship being towed by two tugs in two different directions will not move in the direction of either but will take an intermediate dircetion, which can be obtained by Parallelogram Law.

A bird while flying in air strikes air with wings and so air reactions act in the opposite directions (Fig. 55). If these reactions OY and OX are represented in magnitudes and directions by OA and OB respectively, the diagonal OC of the parallelogram OACB represents in magnitude and direction the force which makes the bird move forward. By varying the muscular effort of either wing the corresponding reaction may be altered and hence the direction of the resultant, *i.e.* the direction in which the bird flies, may be changed.

Resolution of a Force—A single force can be resolved into any two or more component forces. When two component forces act at right angles to each other, each component is termed a **resolute** or the *resolved part* of the given force in that particular direction. In Figure 43 a vector R is represented by OS in magnitude and direction. If u and v be the resolved parts represented by OP and OQ and $\angle SOP = \alpha$, it has been shown that $P = OS \cos \alpha$ and $Q = OS \sin \alpha$.

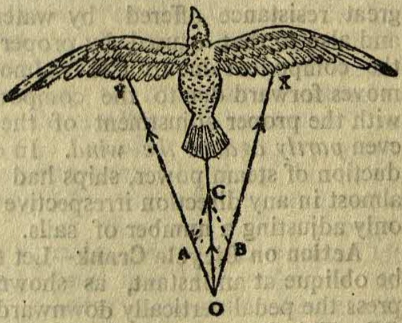


Fig. 55

Some Practical Illustrations—The following cases illustrate the resolution of forces.

Pulling of a Boat by a single Tug—Suppose a boat is pulled by a single tug attached to the mast at O and let the pulling of force be represented by OR (Fig. 56). This force can be resolved into two components, *viz.*, OA parallel to the length of the boat, and OB normal to it. The former pulls the boat ahead, while the latter tends to move it sideways. The sideway motion is less effective, since water exerts a large reaction to the entire length of the boat and the inclination of the rudder D helps to increase this reaction.

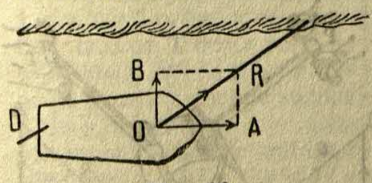


Fig. 56.

Sailing of a Boat—Let AB represent the length of the boat.

CD is the sail and WC is the direction of the wind (Fig. 57). Let CE represent the force exerted by the wind on the sail in magnitude and direction. If this force is resolved into two components, *viz.*, CF perpendicular to the surface of the sail, and CD along the surface, only the former component would be effective in exerting a pressure on the sail, while the latter component would glide past the sail. Let CF be now resolved into two components, *viz.*, CK along the length of the boat and CH at right angles to it. The component CK tends to move the boat forward, while CH tends to move it sideways. But owing to great resistance offered by water to motion in this direction and also to that caused by proper inclination of the rudder, the component due to CH is almost ineffective and the boat moves forward due to the component CK. This shows how with the proper adjustment of the sail, a boat can advance even *partly against the wind.* In days past, before the introduction of steam power, ships had to be steered on high seas almost in any direction irrespective of the motion of wind by only adjusting a number of sails.

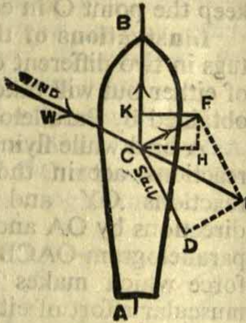


Fig. 57

Action on Bicycle Crank—Let the position of the crank AO be oblique at an instant, as shown in Figure 58. Let the foot press the pedal vertically downwards with a force represented by OR. This force can be resolved into two components ; OA which is along the crank and OC which is perpendicular to the crank. Only the component OC is effective in driving the cycle, while the component OA is entirely inoperative. Evidently then, when the pressure of the foot acts normally to the crank in its horizontal position, the most effective is the pedalling.

Moving a Lawn Roller—When *pulling* a road roller as shown by the position P, the force exerted by the hand may be represented by DE (Fig. 59 left). This can be resolved into two components:—

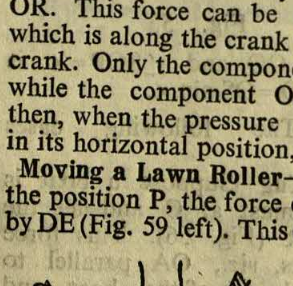


Fig. 58

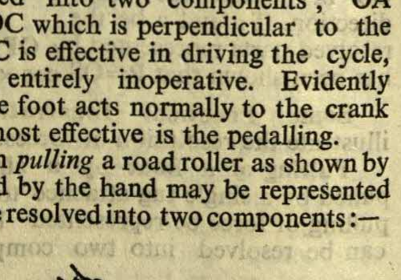


Fig. 59

DF and EF, parallel and perpendicular respectively to the surface of the ground. The former is effective in dragging the roller, while the latter acting vertically *upwards* decreases the pressure on the turf, caused by the weight of the roller. When *pushing* the roller as shown by the position T, the force applied is given by AC (Fig. 59 right) and the component BC of applied force is effective in moving the roller, while the component AB acting *downwards* adds to its weight It thus *increases* the pressure exerted by the roller on the ground and makes the job difficult to push the roller. Similarly, it is easier to pull a parambulator than to push it.

Flying of a Kite—Suppose a kite *K* is flying in air (Fig. 60). The kite is acted on simultaneously by (*i*) its own weight W acting vertically downwards, (*ii*) the tension T of the string, and (*iii*) the whole pressure on its surface due to the wind. Let the effect due to the pressure of the wind on the surface of the kite be taken to be a single force, acting at K and represented by DK. Let the weight W be represented by KA, and the tension of the string by KB. So that there are *three* forces acting on the kite at any moment. Let the resultant of the tension T and weight W be represented by KC. The wind pressure, represented by DK, can be resolved into components EK, along the face of the kite and the other along CK perpendicular to the face of the kite. The component EK glides over the kite and has little effect. If this latter component be exactly equal to and opposite to the resultant KC, the kite remains in equilibrium in air.

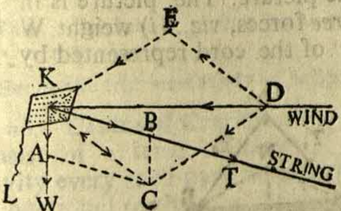


Fig. 60

If the wind pressure be higher, the normal component also increases and the kite rises up, until the resultant of T and W again becomes just equal and opposite to the normal component so as to produce equilibrium. Again, when the wind pressure decreases, the kite comes down. If the wind pressure remains the same and the string is let loose, the tension T decreases so is the pressure and hence the kite dips down. When the wind is gusty, the magnitude and direction of its pressure change suddenly and the kite moves irregularly. This is diminished by attaching a tail L to the kite, which increases the inertia and opposes any sudden movement. Near the ground, where the wind pressure is less, the man flying the kite is to run fast to produce a sufficient wind thrust upon the kite ; but when the kite rises to a height, the thrust is considerable and running may not be necessary.

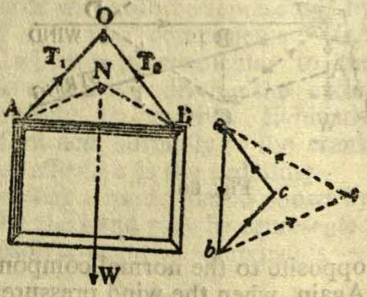
41. Triangle of Forces—If three forces, acting on a particle, be represented in magnitude and direction by the sides of a triangle, taken in order, the forces are in equilibrium. In Figure 37, the forces AB and BC acting at A can be represented by the sides AB and BC of the triangle ABC. By the parallelogram law of forces, the resultant R of these two forces is represented by AC. Hence if a third force equal and opposite to R and represented in magnitude and direction by CA be impressed on the particle, it is found to be at rest. In other words, the particle under the action of the three forces represented by the sides AB, BC and CA of the triangle ABC, taken in order, will be in equilibrium. The statment '*taken in order*' means that the direction of the three forces must go round the triangle, all in the same direction, clockwise or counter-clockwise.

The *converse of Triangle of Forces* may be stated thus.—*If three forces acting at a point are in equilibrium, they can be represented in magnitudes and directions by the sides of a triangle taken in order.*

Some Practical Illustrations—The following cases illustrate the principle of triangle of forces :

A Hanging Picture—Figure 61 (*i*) represents a picture suspended vertically by a cord passing over a single nail O and attached at the points A and B of the picture. The picture is in equilibrium under the action of *three* forces, *viz.*, (*i*) weight W of the picture, (*ii*) the tension T_1 of the cord represented by AO, and (*iii*) the tension T_2 of the cord represented by BO.

Let *ab* represent the weight W of the picture in magnitude and direction (Fig. 61 *ii*). Draw *bc* parallel to AO and *ac* parallel to OB meeting each other at *c*. Then *bc* and *ca* represent tensions T_1 and T_2 respectively, and as the same cord passes round O, tensions T_1 and T_2 are equal. If the cord is shortened, as indicated by ANB. the tension increases, for the triangle *abc* now takes the form of the dotted triangle *abc*. Evidently, if the cord is shortened too much, *it is* under higher tension and is *more likely to sap.*



(i) (ii)

Fig. 61

A Simple Crane—A simple crane consists of a stout vertical pillar to which a movable rod R is hinged at the base. A pulley is fixed at the other end over which a chain or a wire rope can run. A weight W to be lifted is tied at one end of for A, $\frac{a}{b}=\sqrt{\frac{18}{8}}=\frac{3}{2}$ and for B, $\frac{a}{b}=\sqrt{\frac{36}{16}}=\frac{3}{2}$. This ratio from either of the cases is found to be 3 : 2.

3. The turning points of the pointer of a balance were observed to be successively 13, 8, 11. With the body on the left pan and 24·82 gm. on the right pan, the turning points were 14, 9, 12. On adding 10 more milligrams to the weights, the turning points became 10, 3, 8. Calculate the correct weight of the body. The pointer scale is graduated from 0 to 20. [Mys. U.—1952]

Ans. Supposing the scale of the pointer to begin from the left, it is evident from the sets of reading of the turning points that for any set two readings were taken to the right, and one to the left. Hence, the resting point in each case is to be found thus :

(*i*) When the pans are empty and since $=\frac{13+11}{2}=12$,

$\therefore$ Resting point $=\frac{12+8}{2}=10$.

(*ii*) With the body on the left pan and 24·82 gm. on the right, since $\frac{14+12}{2}=13$. Thus the resting point $=\frac{13+9}{2}=11$.

This shows that body, which is kept on the left pan, is heavier than the weights placed (=24·82 gm.) on the right pan by a load which shifts the resting point by one division only.

(*iii*) With the addition of 10 mgm. = ·01 gm., since $\frac{10+8}{2}=9$,

Then the resting point is $\frac{9+3}{2}=6$.

Thus 5 divisions of resting point are shifted for ·01 gm.

$\therefore$ 1 division " is " for ·002 gm.

Hence the true weight of the body = (24·82 + ·002) gm. = 24·822 gm.

65. Centre of Gravity—Any material body may be supposed to consist of a large number of particles bound together. Due to gravity every such particle is attracted towards the earth with a force proportional to the mass of the particle (Fig. 94). All the component forces act towards the centre of the earth. But since the centre is at a large distance from the surface of the earth, all the forces may be considered to be parallel and *acting vertically downwards*. The resultant of these parallel forces is thus equal to the weight W of ihe body acting vertically downwards at a single point. This point is called the *centre of gravity* (written as C. G.). Thus, in Fig. 94, *a*, *b*,...*m*, *n*, etc., represent the particles of the body, whereas the arrow-heads indicate the individual weights. The point G representing the point of application of the resultant is the centre of gravity of the body.

When the body is rotated to a different position, the attractions on the particles still remain the same in magnitude and direction, being parallel to one another and so the resultant passes through the *same* point as before.

Hence the centre of gravity of a body is a point fixed relative to it, through which the resultant force due to earth's attraction on all the

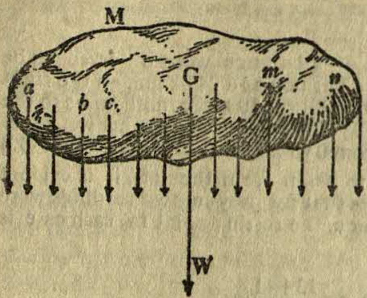


Fig. 94

particles constituting the body always passes, in whatever way the relative position of the body changes, provided the size and shape of it remain unaltered. In other words, the centre of gravity of a body is the point through which the line of action of the weight always passes.

As the line of action of the weight of the body passes through its C.G., a force equal to the weight, which acts through the C. G. in a vertically upward direction, would keep the body in equilibrium. Thus if a body is suspended by a string, the C. G. lies vertically below the point of suspension.

Centre of Mass—If we consider the masses of the particles constituting a body and if a system of parallel forces proportional in magnitude to the masses of the particles be drawn, the point, at which the resultant of all these forces may be supposed to act, is called the **centre of mass** or *mass centre* of the body. Hence, it is the point, at which the whole mass of the body may be supposed to be concentrated. If the forces, proportional to the masses, be all drawn in the vertically downward direction, the centre of mass coincides with the C. G. of the body. The centre of mass is all important in the motion of a body in *free space* where there is no gravitational force.

C. G. of Symmetrical Bodies—The position of the C. G. of a body can be determined from the principles of Statics. But in some simple cases, it may be inferred from the considerations of symmetry, the bodies in such cases being supposed to be of uniform density throughout.

For example, the centre of gravity—

(1) of a *uniform straight rod*, a piece of wire or a beam, etc., is at the middle point of its axis ;

(2) of a *uniform circular lamina* or a *sphere* is at its *geometrical centre* ;

(3) of a *uniform circular ring* is at the centre of the circle ;

(4) of a *uniform parallelogram* or a *regular parallelopiped* is at the intersection of its diagonals ;

(5) of a *uniform cylindrical body*, solid or hollow, is at the middle point of its axis ;

(6) of a *uniform triangular plane* is at its centroid.

66. Determination of the C. G. of a Lamina—When a body is freely suspended from a point by a string, the C. G. of the body lies somewhere on the prolongation of the string by which the body is suspended.

Take a thin sheet of a metal piece and suspend it by a string attached to some point near its edge (Fig. 95). Hang a plumb line from the point of suspension and trace on the sheet a vertical line through the point of suspension. In a similar manner suspend the sheet at two or three different points and find, as before, the vertical line through the point of suspension in each case. It is found that the verticals cut one another at a single point G. The point G is the required *centre of gravity* of the sheet.

The centre of gravity of a lamina may also be found by *balancing* it on a horizontal edge. The plate is placed on a table and moved towards the edge, until it is just on the point of toppling over. A line is then drawn on the undersurface of the lamina along the edge of the table. The C. G. of the plate lies somewhere on this line. By turning the plate and proceeding as before, another line is obtained. The point of intersection of these two lines gives the position of the C. G. of the plate. If the plate is supported at this point, it remains balanced. The centre of gravity of an object of small thickness (*e.g.*, a bicycle) may be determined by the method of suspension or balancing. But, in any case, the steering arm must be fixed rigidly, as any change of position of the front wheel will alter the position of the centre of gravity of the bicycle.

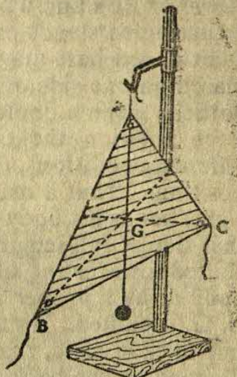


Fig. 95

67. **Equilibrium of Bodies**—If a body remains at rest under the simultaneous action of two or more forces, it is said to be in **equilibrium.** As the weight of body acts vertically downwards through its C. G., the resultant of the reactions at the point of support of the body in equilibrium must act vertically upwards through the C. G. of the body. So in the case of the body, *supported at one point* (for example, when suspended by a string attached at one point or when balanced on a pivot), equilibrium is possible, when the C. G. of the body either coincides with the point or is above or below the point in the same vertical line. Further, the force supporting the body (or the *force of reaction*) at the point of support must be equal and opposite to the weight of the body. If a body is *supported at two points*, the resultant reaction passes through some intermediate point and so for equilibrium the vertical through the C.G. of the body must lie between the two points of support.

In the case of a body *supported at three or more points* (for example, a tripod stand or a glass tumbler placed on a table), the area obtained by joining successively the outermost points of support is called the *base of* the body. Equilibrium is possible, only when the vertical through the C. G. falls within the area of base. For, in that case, the resultant reaction may pass through the C. G. of the body. If it falls outside, the body overturns and takes a new position of equilibrium, in which the vertical falls within the base. If a rectangular block of some heavy material is gradually inclined, it will be found that so long as the vertical through the C. G. of the block falls within the base area, the body will not topple over, whatever its inclination may be.

Evidently then, the wider is the base, the greater is the stability of the body ; for, even if the body is sufficiently inclined, the vertical may still pass through the base. This is clearly shown by the *Leaning Tower* of Pisa and a certain building at the Varanasi Ghat. Again, a quadruped stands or walks more stably than a man. A table with an extended base may not turn over, even when a heavy weight is placed on one side of the top. To make it more stable, a mucilage bottle is given a conical shape with a wider base. In the ordinary erect position the C. G. of a man is nearabout his navel ; under different conditions, the man changes his form to come to a stable position. Thus a man carrying a heavy load in one hand (say, the right) bends his body towards the left and often extends the arm on that side at full length. Here the C. G. of the man with the load is shifted towards the side of the load. But by a suitable bend of the body or by a proper extension of the hand towards the left, the C. G. is shifted. In fact, it is easier to walk with two equal loads, one in each hand, than with the two in one hand. If the man carries the load on the back, he bends his body forward. When a man is seated on a chair with the legs hanging down, he is in an equilibrium position. If he inclines himself backwards, so that the chair rests on the two hind legs, the man has to extend his legs forward as far as possible. In a circus show while walking on a rope, the performer holds in his hand a long pole or an open umbrella, which helps him to keep the C. G. vertically above the rope.

68. Different States of Equilibrium—It has been explained that a body remains in equilibrium, whenever the resultant upward reaction passes through the C. G. of the body. The *static equilibrium* (*i.e.*, the equilibrium of a body at rest) may be of three kinds, namely *stable*, *unstable* and *neutral*.

A body is said to be in a **stable equilibrium** position, if on being slightly displaced, it tends to come back to its original position. Thus a table or a cube resting on its side, a right circular cone (Fig. 96 P), a funnel resting on its base and a pendulum at rest are examples of stable equilibrium.

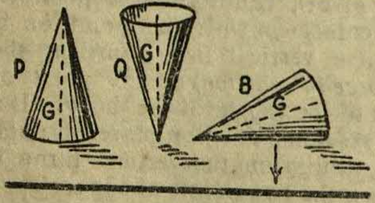


Fig. 96

A body is said to be in **unstable equilibrium**, if on being slightly displaced from that position, it does not come back to its original position but tends to move farther away from it. Thus a cone resting on the apex (Fig. 96 Q), a stick balanced on the finger and an egg standing on its end are examples of unstable equilibrium.

A body is said to be in **neutral equilibrium**, if on being slightly displaced, it neither returns to its original position nor moves farther away from it, but remains in its new position as before. A uniform sphere on a flat surface, a right circular cone (Fig. 96 B) and a cylinder resting on its side are examples of neutral equilibrium.

A body always tends to keep the position of the Centre of Gravity as low as possible and the three states of equilibrium depend on the relative position of the centre of gravity. In all cases of *stable* equilibrium, the C. G. occupies the *lowest* possible position, so that a slight displacement tends to *raise* the position of the C G. Hence, the body returns to its original position as soon as it is free to do so. In all cases of *unstable* equilibrium the C. G. occupies the *highest* possible position, being always higher than the point of support ; so that a slight displacement tends to *lower* the C. G. and thus bring the body to a more stable position. In all cases of *neutral* equilibrium, the C. G. is neither raised nor lowered by a slight displacement of the body and so the body remains in any position.

For a body supported at one point, the point of support is higher than the C. G. in cases of *stable* equilibrium, *lower* than the C. G. in cases of *unstable* equilibrium and *coincides* with the C. G. in cases of *nautral* equilibrium. To keep the C. G. at the lowest possible position a ship is loaded with ballast and a wagon is loaded with heavy materials at the bottom. For the same reason the lower deck of a double-decker omnibus should be loaded more and it is risky for too many passengers to go up on the higher deck.

69. Static Equilibrium and Potential Energy—A body is in equilibrium position when its potential energy is either a *maximum* or a *minimum.* If its potential energy is *maximum,* any disturbance gives rise to a couple tending to rotate the body further. Thus *when the potential energy of the body is maximum, its equilibrium is unstable.* Again, when the potential energy is *minimum,* any disturbance sets up a couple tending to bring back the body to its equilibrium position. Thus *when the potential energy of a body is minimum its equilibrium is stable. In a state of neutral equilibrium,* the potential energy of the body remains *constant* for any small displacement.

Dynamic Equilibrium—A body *in motion* may be in stable equilibrium in a position, in which it is unstable when at rest. The equilibrium in this case depends on the motion of the body and the body is said to be in a state of *dynamic equilibrium.* A rotating top and a cyclist in motion furnish examples of such cases.

70. Simple Harmonic Motion—A body is said to execute a *Simple Harmonic Motion* **(S. H. M.)*** when it shows the following characteristics :—

---

* For a detailed study of Simple Harmonic Motion, the student is referred to Chapter II of the part on Sound of this Book (Part I). In short, if x be the displacement of a particle of mass m, measured at time t from the position of equilibrium and if k be restoring force per unit displacement, the equation of S. H. M. may be written in the form :

$$x=a\sin\left(\sqrt{\frac{k}{m}}\,t+\phi\right). \quad \text{If we put } \frac{k}{m}=\omega^2, \text{ then } x=a\sin(\omega t+\phi)$$

where a=amplitude of oscillation and ϕ is a constant known as epoch.

(i) *The motion is periodic.*—This means that the body in motion passes through the same succession of positions after each definite interval of time.

(ii) *The motion is assumed to take place in a straight line, such that the acceleration of the body is always directed towards a fixed mean position* called the position of equilibrium.

(iii) *The magnitude of the force acting on the body at any point in its path is proportional to its displacement from the fixed mean position.*

71. **Pendulum**—A *simple pendulum* is defined to be a heavy point mass, suspended by a *weightless inextensible* and *perfectly flexible* string from a fixed point, about which the pendulum oscillates *without friction.* Evidently, it is not possible to have such a pendulum in practice, for we cannot isolate a single material particle. Nor can we get a perfectly inelastic and weightless string which can oscillate without friction at any point. The nearest approach to such a pendulum consists of a small heavy metal ball, suspended by a very thin and inelastic thread. A *compound pendulum* consists of a heavy mass supported in such a way that it is capable of oscillating freely in a vertical plane about a fixed point or a fixed line as axis. A practical form of a compound pendulum is that of a clock, which consists of a rigid rod having at its lower end a heavy metallic mass and is capable of a turning about an axis at the upper end.

The heavy spherical mass A (Fig. 97), suspended at the end of the string, is called the *bob* of the pendulum. The distance between the point of suspension O and the centre of gravity of the bob is called the *effective length* of the pendulum. One complete to-and-fro movement of a pendulum is called an *oscillation.* A pendulum is said to execute one **complete oscillation**, when starting from any point in its path, it comes back to the same point, moving in the same direction. Thus when the bob starts from A, moves to B, returns to A, moves towards C and is back to A, it makes one complete oscillation. The maximum displacement (AB or AC) of the bob on either side of its position of equilibrium is called the **amplitude** of vibration. The time to perform one complete oscillation is called the *time period* (or simply, the **period**) of oscillation.

The number of complete oscillations performed in one second by a vibrating body is called the **frequency** of vibration. If the period be the T and the frequency of vibration be n, then

$$nT=1 \quad \text{or,} \quad n=\frac{1}{T}.$$

The phase of a body executing an S. H. M. gives the state of motion at a particular instant and thus determines the instantaneous *position* of the body in the path and the *direction* of motion at that instant.

72. **Motion of a Simple Pendulum.**—Let a spherical bob A of mass m be suspended by a string of length l from the point O (Fig. 97). When at rest at A, the weight of the bob is just balanced by the tension of the string.

Now displace the pendulum through a small angle θ so as to take up the position OB. Resolve the weight mg of the bob acting vertically downwards into $mg \times \cos\theta$ along BD (which is OB produced) and $mg \times \sin\theta$ along BE which is drawn tangential at B to the path AB. The component $mg\cos\theta$ is balanced by the tension T of the string acting along BO. The tangential component $mg\sin\theta$ is the only effective force which tends to move the pendulum towards the position of equilibrium at A.

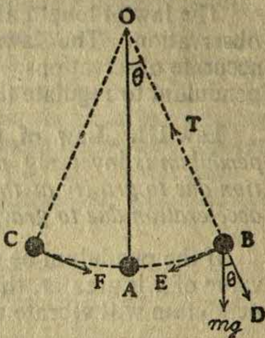


Fig. 97

Now if θ is very small, elementary trigonometry gives $\sin\theta = \theta = \frac{\text{arc}}{\text{radius}}$

$$= \frac{AB}{OB} = \frac{\text{displacement}}{\text{length of pendulum } (l)}$$

$$\therefore \text{ Effective force} = mg\sin\theta = \frac{mg}{l} \times \text{displacement.} \qquad (72,1)$$

Thus the acceleration of the bob at the instant under consideration is given by $\frac{g}{l} \times$ displacement. (72,2)

As g and l are constants, the acceleration along the path of motion is proportional to the displacement from the mean position of rest. Hence *the motion of the bob is simple harmonic in nature.*

It can be proved that $T = 2\pi\sqrt{\frac{l}{g}}$. (72,3)

[For details, *Vide* 'SOUND' Art. 8. See also footnote pp. 104.]

73. Laws of Pendulum—The oscillation of a simple pendulum is governed by the following laws :—

Law I. Law of Isochronism—The *oscillations of a simple pendulum of a given length are isochronous.* This means that oscillations take place in equal intervals of time, provided the amplitude is small (within 4° to 5° of the angle of oscillation on each side). The first law was discovered in 1583 by Galileo (1564-1642) who casually noticed the swings of a lamp hanging by chain from the roof of the Leaning Tower at Pisa. For a measure of time he counted his pulse beats during one oscillation of the lamp. To his surprise he found that in all cases equal number of pulse beats occurred for every oscillation no matter whether the oscillation of the lamp was large or small.

Law II.—Law of Length—*The period of oscillation of a simple pendulum varies* directly *as the square root of its effective length.*

If T be the period of oscillation and l be the effective length, then $T \propto \sqrt{l}$. Thus if the length of the pendulum be increased 4, 9 or 16 times, the period will be 2, 3 or 4 times as large respectively. As the length of a pendulum changes with temperature, the period will be *slightly* affected by changes of temperature.

The law of length also was discovered by Galileo from the same observation. The laws were afterwards verified by him by more accurate observations. In 1658 Huygens (1629-1693) first applied a pendulum to regulate the motion of clocks.

Law III. **Law of Gravity**—*The period of oscillation of a simple pendulum is* inversely *proportional to the square root of the acceleration due to gravity at the place of observation. If 'g' represents the acceleration due to gravity, then* $T \propto 1/\sqrt{g}$.

If the pendulum be taken from one place to another, where the value of g is greater, the period will be smaller. This means that the pendulum will vibrate more rapidly with increasing gravity.

Law IV. **Law of Mass**—*The period of oscillation of a simple pendulum is independent of the mass or the material of the bob, provided the effective length remains the same.*

The period would remain the same, so long as the distance from the point of suspension to the centre of gravity of the bob remains unaltered, even if the bob be of different size or of different material.

If a simple pendulum be made of a thin string and hollow spherical bob, its effective length is the distance between the point of suspension and the centre of gravity of the bob. If now the bob is half-filled with the mercury, the centre of the bob (and hence that of the system) lowers down resulting in a larger effective length. Consequently its time period becomes greater. But if it is completely filled, its C. G. returns to the centre of the bob and there is no change in the period.

From eqn. (72,3) we conclude

(i) that if l and g are constant T is constant ... Laws I & IV)

(ii) that if g is constant, $T \propto \sqrt{l}$ (Law II)

and (iii) that if l is constant $T \propto 1/\sqrt{g}$ ... (Law III)

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* By (72,1) the restoring force per unit displacement $= mg/l$. If the amplitude AB=AC=a (Fig. 97) and if the displacement measured from A at a time t be x the equation of motion of the simple pendulum can be written in the form

$$x = a \sin\left(\sqrt{\frac{mg}{ml}}\,t + \phi\right), \text{ or, } x = a \sin\left(\sqrt{\frac{g}{l}}\,t + \phi\right) \qquad \text{...(72,4)}$$

where ϕ is a constant. If the period of the simple pendulum be T, then after each interval of time T, the displacement x will have the same value.

$$\text{Thus } x = a \sin\left[\sqrt{\frac{g}{l}}(t + T) + \phi\right], \text{ or, } x = a \sin\left[\sqrt{\frac{g}{l}}\,t + \sqrt{\frac{g}{b}}\,T + \phi\right] \qquad \text{...(72,5)}$$

Equations (72,4) and (72,5) are satisfied, if only

$$\sqrt{\frac{g}{l}}\,T = 2\pi, \text{ whence } T = 2\pi\sqrt{\frac{l}{g}}$$

74. Verification of the Laws—To verify the laws, take a small metal ball suspended by a fine thread from a hook attached to a suitable stand. (*Vide* J. Chatterjee's *Intermediate Practical Physics.*)

First Law—Set the pendulum into oscillation in a vertical plane, taking care that the amplitude is very small. By means of a stop-watch, note the total time required to perform a number of complete oscillations, say 20. Then dividing the time by the number of oscillations, calculate the time period. Repeat the experiment with different amplitudes up to the limit of 8° and find the period in each case. It will be found that within the limits of experimental error the time period in all cases is the same.

Second Law—Take a pendulum, the length of which can be altered. Measure the diameter of the bob at several places by a slide callipers and get the mean radius. To this add the length of the string. The sum gives the effective length of the pendulum. Note the time required for a given number (say, 20) of complete oscillations and hence find the time period. Change the length of the string to different values, and in a similar way find the time period in each case. Take a set of readings as shown in the following table. Square the time period T and divide it by the corresponding length *l*.

It will be found $\frac{T^2}{l}$ is a constant, that is, $T \propto \sqrt{l}$.

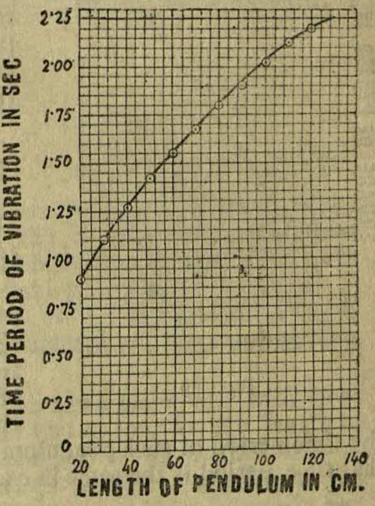

Fig. 98

Effective Length of Pendulum in cm. (l)	Time Period in sec. (T)	Square of Period in sec.^{2} (T^2)	Effective Length of Pendulum in cm. (l)	Time Period in sec. (T)	Square of Period in sec.^{2} (T^2)
20·0	0·89	0·79	80·0	1·78	3·17
30·0	1·09	1·19	90·0	1·90	3·61
40·0	1·26	1·59	100	2·01	4·04
50·0	1·41	1·99	110	2·13	4·58
60·0	1·55	2·41	120	2·19	4·80
70·0	1·67	2·79	130	2·25	5·15

The relation between the length of a pendulum and the time period can be represented by the graph (Fig. 98), in which the length (l) is measured along the abscissa and the corresponding time period (T) along the ordinate. The graph will be found to be parabolic in nature. From the mathematical relation between l and T a parabolic form is evident. But since $T^2 \propto l$, the graphical relation is linear as shown in fig. 99. When a pendulum clock gains or loses, it is regulated by altering the effective length of the pendulum by lowering or raising the bob by a screw nut below it.

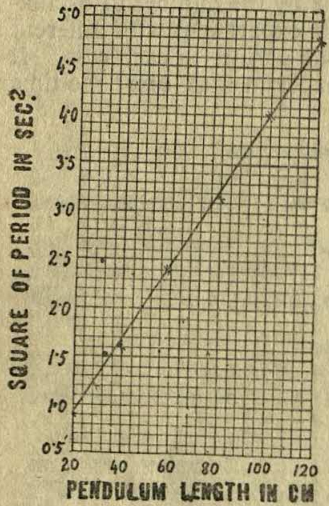

Fig. 99

Third Law—This law can be verified by finding the period of a simple pendulum of a *given* length at different places on the surface of the earth or at different altitudes above the same place. It will be found that for different places the product of the period of oscillation and the square root of the acceleration due to gravity is a constant.

Then, $T \times \sqrt{g} = \text{constant}$, whence $T \propto \dfrac{1}{\sqrt{g}}$.

A relation between T and $1/\sqrt{g}$ is a straight line.

Fourth Law—Take spheres of different materials and of different sizes as bobs. If necessary alter the length of the string, but keep the effective length of the pendulum the same in each case. Determine the time period of such pendulum in the usual way. It will be found that time period is the same in each case.

75. Seconds Pendulum—A simple pendulum, which swings from one extremity to another in one second, is called *seconds pendulum.* Hence, the time period of a seconds pendulm is 2 seconds.

By dcfinition, therefore, we find from eqn. (72,3) that

$$2\pi\sqrt{\frac{l}{g}} = 2, \text{ whence } l = \frac{g}{\pi^2} \qquad \ldots(75,1)$$

As the value of g varies from place to place on the surface of the earth and at different altitudes, the length of seconds pendulum is different at different places. Taking the average value of g as 981 cm. per sec. per sec., the cnrresponding length of the seconds pendulm is given by,

$$l = \frac{981}{\pi^2} = \frac{981}{9{\cdot}87} \text{ cms.} = 99.39 \text{ cms.}$$

Again taking $g = 32{\cdot}2$ ft. per sec², $l = \dfrac{32{\cdot}2}{\pi^2}\text{ft.} = \dfrac{32{\cdot}2}{9.87}\text{ft.}$

There is some misconception about the nature of the pendulum clock. Although every such wall-clock is regulated by the periodic motion of a pendulum, *such a pendulum is neither a simple pendulum not it is generally a seconds pendulum.* In a wall clock the pendulum is *much shorter* than the usual seconds pendulum and as such its beat-period is *much less* than a second. But by the proper adjustment of gear and release wheels, the minute hand of the clock is adjusted to rotate over a complete circle in one hour. However there is one type of *large* clock, popularly known as Grand Father's Clock, which is provided with an actual seconds pendulum. This clock is used in the Laboratory to standardize a seconds pendulum.

76. Determination of the value of 'g' by the Simple Pendulum—It is evident from eqn. (72,3) that a simple pendulum furnishes a very convenient method of determining acceleration due to gravity at a place. For that purpose we note that,

$$T=2\pi\sqrt{\frac{l}{g}}, \quad \text{or} \quad T^2=4\pi^2\frac{l}{g},$$

$$\text{whence } g=4\pi^2\frac{l}{T^2} \qquad \text{...(76,1)}$$

Taking a simple pendulum, its effective length (l) is carefully measured and the mean time period (T) is accurately determined from a number of observations. On substitution in eqn. (76,1) the value of g is calculated. Instead of taking the mean value of T, the values of l and T^2 may be plotted on a graph as in Fig. 99 from which the mean value of l/T^2 is obtained. Thus on substituting in eqn. (76,1) the value of g is found.

If the experiment is performed at the same place but with bobs of *different* substances, it will be found that so long as the length of the pendulum remains the same, the time period will be the same at the same place. This shows that acceleration due to gravity at the same place is the same for all bodies.

Variation in the value of T at different places—As the value of g varies from place to place, the value of the time period T of a simple pendulum of a given length also changes. Thus if g decreases, T increases, and *vise versa.* It has been explained in Art. 52 that the value of g increases from the equator to the poles. Hence if a pendulum clock be taken from the equator to the poles, T decreases and so the clock *gains* ; this means that it goes faster. Again, if at the same place, the clock is taken from the surface of the earth to the top of a mountain or to the bottom of a mine, the value of g decreases and so T increases : hence the clock *loses,* meaning that it goes slower. Within an orbiting satellite moving in a circular orbit any object is weightless. If then g becomes zero, the pendulum will not move at all.

Examples :

1. Find the length of a simple pendulum beating $\frac{3}{4}$ths of a second at a place where the value of g is 980 cm. per sec.^{2}.

Ans. Let the length of the pendulum be l cm.
The time period T of this pendulum=$(2\times\frac{3}{4})$ secs.$=\frac{3}{2}$ secs.

Then we have

$2\pi\sqrt{\frac{l}{980}}=\frac{3}{2}$, or, $l=\frac{9}{4}\times\frac{980}{4\times(3{\cdot}1416)^2}$, whence the required length (l) $=55{\cdot}85$ cms.

2. A faulty seconds pendulum loses 20 seconds per day. Find the required alteration in its length so that it may keep correct time ; given $g=32$ ft. sec.^{2}.
[Pat. U.—1953]

Ans. Let the length of the faulty seconds pendulum be l ft, and the alteration in its length in order to make it correct be x ft. Since it is losing time, its effective length must have increased and so its correct length should be $l-x$.

A correct seconds pendulum beats seconds and hence by the standard formula we get

$$\pi\sqrt{\frac{l-x}{g}}=1 \text{ or, } \pi\sqrt{\frac{l-x}{32}}=1 \qquad \ldots \quad \ldots \quad (1)$$

The faulty pendulum loses 20 seconds a day which consists of 86400 seconds ; and so its number of beats in one day $=86400-20=86380$. Its time for a single beat is,

therefore. $=\frac{86400}{86380}$

$$\text{Hence } \pi\sqrt{\frac{l}{32}}=\frac{86400}{86380} \qquad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$\text{or, } \frac{\pi^2}{32}l=\left(\frac{86400}{86380}\right)^2, \text{ or, } l=\left(\frac{86400}{86380}\right)^2\times\frac{32}{\pi^2} \qquad \ldots \quad \ldots \quad \ldots \quad (3)$$

$$\text{Now by eqn. (1), } \pi^2\frac{l-x}{32}=1, \text{ or, } l-x=\frac{32}{\pi^2} \text{ or, } l=x+\frac{32}{\pi^2} \qquad \ldots \quad (4)$$

From eqn. (3) and (4), we deduce

$$x+\frac{32}{\pi^2}=\left(\frac{86400}{86380}\right)^2\times\frac{32}{\pi^2},$$

$$\text{or, } x=\frac{32}{\pi^2}\left[\left(\frac{86400}{86380}\right)^2-1\right]=\frac{32}{(3{\cdot}1416)^2}\left[\left(1+\frac{20}{86380}\right)^2-1\right]$$

$$=\frac{32}{9{\cdot}8696}\left[1+\frac{40}{86380}-1\right], \text{ nearly}=\frac{32}{9{\cdot}8696}\times\frac{40}{86380}=0{\cdot}0015 \text{ ft. (nearly).}$$

This gives the desired alteration in length in feet.

3. A pendulum, which keeps correct time at the foot of a mountain, loses 16 seconds a day when taken to the top. Find the height of the mountain. (Neglect the attraction due to the mountain and take the radius of the earth as 21×10^6 feet.).

Ans. Let the height of the mountain be h ft. Denote the acceleration due to gravity at the foot and the top of the mountain by g and g' respectively. Then

$$g:g'=\frac{1}{(21\times10^6)^2}:\frac{1}{(21\times10^6+h)^2}, \text{ or, } \frac{g}{g'}=\left(\frac{21\times10^6+h}{21\times10^6}\right)^2 \qquad \ldots \quad (1)$$

The pendulum makes 86400 and 86384 oscillations respectively at the bottom and top of the mountain, in a day of 86400 seconds, Then

$$\pi\sqrt{\frac{l}{g}}=1 \text{ and } \pi\sqrt{\frac{l}{g'}}=\frac{86400}{86384}, \text{ whenc } \frac{g}{g'}=\left(\frac{86400}{86384}\right)^2 \qquad \ldots \quad (2)$$

Combining [1] and [2], $\left(\frac{21\times10^6+h}{21\times10^6}\right)^2=\left(\frac{86400}{86384}\right)^2$, or, $1+\frac{2h}{21\times10^6}$ $=1+\frac{16\times2}{86384}$, whence the required height $h=3891$ ft. (approximately).

EXERCISES ON CHAPTER IV

Reference

1. What is meant by the phrase "Constant of gravitation is $6·6 \times 10^{-8}$ C. G. S. unit" ? (Raj. U.—1954) — Art. 50

2. State Newton's law of gravitation. Obtain an expression for the acceleration due to gravity in terms of the mass of the earth, radius of the earth and the gravitational constant. (Raj. U.—1955) — Arts. 50 & 52

3. What is meant by acceleration due to gravity ? How does it vary from place to place ? How does the rotation of the earth affect this quantity ? (*Cf.* Raj. U.—1955 ; And. U.—1961) — Art. 52

4. Distinguish between mass and weight. How are the mass and weight of a body affected by variations of latitude ? Is weight an essential property of matter ? (C. U.—1956) — Arts. 53 & 54

5. A stone is dropped from a rising balloon at a height of 200 ft. above the ground and it reaches the ground in 6 secs. What was the velocity of the balloon just at the moment when the stone was dropped ? (C. U.—1952) — Art. 56

Ans. 62·6 ft. per sec. (upwards).

6. A body of mass 50 gm. is allowed to fall freely under the action of gravity. What is the force acting upon it ? Calculate the momentum and the kinetic energy it possesses after five seconds. [$g = 980$ cm. per sec.^{2}.] (Del. H. S.—1951 ; C. U.—1947) — Art 56

Ans. 49×10^2 dynes ; 245×10^2 gm. cm./sec. ; $60·025 \times 10^7$ ergs.

7. Describe an Atwood's machine and explain how you would use it to determine the value of *g* in the laboratory. (Pat. U.—1955) — Art. 57

8. Describe an experiment to show that acceleration of a body falling freely under gravity is uniform. (Utkal U.—1954) — Art. 57

9. A stone is let fall from the top of a tower 180 ft. in height, and at the same time another is projected vertically upwards with a velocity of 60 ft. per sec. Find where and when the two will meet.

Ans. 36 ft. above the ground ; 3 seconds later.

10. A ball is thrown horizontally from the top of a tower with a velocity of 80 ft. per sec. Show thal the curve traced by the ball is a parabola. (Del. U.—1942) — Art. 58

11. A body weighing 200 lbs. is tied to a rope and lowered with an acceleration of 5 ft./sec.^{2}. Find the tension in the rope. What would be the tension in the rope, if the body is pulled up with an acceleration of 10 ft./sec.^{2} ? [$g = 32$ ft./sec.^{2}] (Pat. U.—1954) — Art. 59

Ans. 2400 poundals, 1400 poundals.

12. A man weighing 12 stones on earth is standing on a lift. Find his weight as recorded by a weighing machine (or spring balance) when (*a*) the lift moves upwards with a uniform velocity of 10 ft./sec. ; (*b*) the lift moves downwards with a uniform acceleration of 4 ft./sec^2, and (*c*) the lift moves upwards with a uniform acceleration of 4 ft./sec.^{2}. (U. P. B.—1942) — Art. 59

Ans. 12 st. ; 10·5 st. ; 13·5 st.

13. Two masses of 80 gm. and 100 gm. are connected by a string passing over a smooth pulley. Find the tension of the string when they are in motion. Find also the space described in 4 seconds, *g* being 981 C. G. S. (Mad. U.—1951) — Art. 59

Ans. 87200 dynes ; 872 cm

Reference

Art. 64

14. A common balance has equal arms, each 20 cm. long. An unknown weight W, placed on the left pan, with 1 gm. rider mounted 5 cm. from the end of the left-side is counterpoised by 15 gm. weight on the right pan. Calculate the value of W. (C U.—1958)

Ans. (Take moments of all the forces about the mid-point of the beam). $W=14{\cdot}25$ gm.

Art. 63

15. Explain why a balance, which is sensitive, cannot by very stable. (*Cf.* Bom. U.–1955 ; *Cf.* Guj. U.—1962 ; P. U.—1952)

Art. 63

16. Obtain an expression for the sensitivity on a balance. State the requirements of a good balance, giving reasons. (Guj. U.—1952 ; *Cf.* Bom. U.–1964 ; Poo. U.–1953)

Art. 63

17. What are the requirements of a good balance ? Explain them. (Guj. U.—1951, '54 ; Raj. U.—1952, *Cf.* '54 ; Pat. U—1952. '54 ; Utkal U.—1954 ; U. P. B.—1952, '55 ; C. U.—1960 ; Dac. U.–1934 ; Del. H. S.–1950 ; P. U.–1952.)

Art. 64

18. A balance has arms of equal length but pans of unequal weights. A body is found to weigh W_1 gm. in one pan and W_2 gm in the other. Show that the excess of the weight of one pan over that of the other is $(W_1-W_2)/2$. (Raj. U.–1954)

Art. 64

19. A dealer has correct weight ; but one arm of his balance is $\frac{1}{20}$th part shorter than the other. If he sells two quantities, each apparently weighing 9·5 lbs, at Rs. 40 per pound. weighing one in one scale and the other in the other what will be gain or loss ? (Nag. U.–1952)

Ans. Re. 1 (loss).

Arts. 71 & 72

20 What is a Simple Pendulum ? Prove that its motion is simple harmonic. Derive an expression for its period and justify its use as a time-keeper. (Poo, U.–1952)

Art. 72

21. A pendulum of length 96 cm. loses 20 seconds per day. Find by how much must it be shortened to keep correct time. (Vis. U.—1955)

Ans. $\frac{4}{9}$ mm.

Art. 72

22. A seconds pendulum, which keeps correct time on the surface of the earth, is taken to the surface of the moon. Find the time period there, taking the mass of the earth to be 81 times that of the moon and the radius of the earth to be 4 times that of the moon.

Ans. 4·5 seconds.

Art. 72

23. A pendulum, which beats seconds at a place, where $g=32{\cdot}02$, is taken to a place, where $g=32$. How many seconds does it gain or lose per day ?

Ans. It loses 27 seconds per day ?

Art. 72

24. A pendulum clock keeps correct time in London ; but it loses 4 minutes per day when taken to the equator. What is the factor that causes the loss ? In what ratio does this factor change in the present case ?

Ans. For a compensated pendulum the effect is due to change in the acceleration due to gravity which is less at equator.

Art. 72

25. A Seconds Pendulum has a hollow spherical bob attached to its thread. Will the period alter, if the hollow bob is half-fille[illegible] mercury ? (C.U.–1960)

Ans[illegible] it will. 99·29 cm.

CHAPTER V

WORK AND ENERGY

77. **Work**—The term *work* gives us an idea of *something being done.* Work is said to be done by a force on a material particle when that force is able to displace that particle through some distance. If the particle is absolutely free to move and a force acts upon it, the result is a motion of the particle according to Newton's Second Law of motion. Then we say that work is continuously done *by the force on the particle* producing its accelerated motion. The work then appears as a moving or kinetic energy (Art. 80). Very often material bodies are made to move under an external force against some opposing forces. In scuh cases also the external force does work on the body ; and some work is done by the body against the opposing force. Thus a man in raising a load does work on the load ; but the load does work against the force of gravity. A horse in drawing a carriage does work on the carriage ; but the carriage in moving against the frictional forces due to road, etc., does work against the frictional force (vide Ex. 1pp. 113).

Work done by a Force—*The magnitude of the work done by a force is given by the product of the force and the distance through which the point of application of the force moves,* provided that the direction of the force and that of the displacement coincide (Fig. 100). Thus if a force P acting on a body displaces it through a distance s, the *work* done is given by

$$W = P \times s \tag{77,1}$$

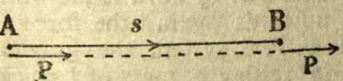

Fig. 100

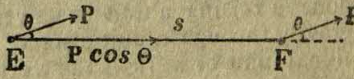

Fig. 101

If the direction of the force and that of the displacement are *not* the same, but make an angle θ with each other, the component of the force in the direction of displacement is effective in doing work. (Fig. 101) In this case the work done by the force is given by

$$W = P\cos\theta \times s \tag{77.2}$$

It is evident from (77.1) that *if there is no displacement, no work is done.* If a material particle is made to move through a distance s right against a constant opposing force P, the work done by the material particle is $P \times s$. Or, if it goes through the same distance making an angle θ with the opposing force, the work done against the force is $P \times s \cos\theta$. Again, as a force has no component in a direction at right angles to its own line of action, no work is done by or against a force in a direction perpendicular to its line of action.

Work done by a Couple—A couple has tendency to rotate a material body and when that body *actually rotates* under the couple, work is done by the couple on the body. Here also from the same sense we measure the work done. Let the material body under the action of couple PP having as the *arm* A_1B_1 move through an angle θ (Fig. 102), The points of application of the forces move through an equal arc A_1A_2 or B_1B_2 and these forces at every point of the arc are tangential. Hence, the work done by the two equal forces is $2P \times \text{arc } A_1A_2$ (or arc B_1B_2), $=2P \times OA_1 \times \theta$

$\therefore$ $W = P \times A_1B_1 \times \theta =$ moment of the couple × angular rotation.

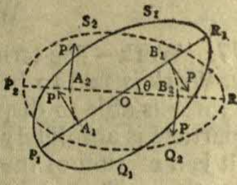

Fig. 102

78. Units of Work—The unit of work is the work done when a unit force acts through a unit distance in its own direction. Evidently, the unit of work depends on the units of force and distance. The **absolute unit of work** is the work done by the *obsolute unit of force* acting through unit distance in its own direction.

In the F. P. S. system, the *absolute unit of work* is the work done by a force of 1 *poundal* in moving its point of application through a distance of 1 *foot* along the line of action of the force. This unit is termed a **foot-poundal.** In the C. G. S. system, the *absolute unit of work* is the work done by a force of 1 *dyne* in moving its point of application through a distance of 1 *cm.* in its own direction. This unit is termed an **erg.** As this unit is extremely small, the unit chosen for practical purposes is the **joule,** 1 joule being 10^7 ergs.

Gravitational Unit of Work—In practice, work is often expressed in the gravitational unit. The *gravitational unit of work* is the work done in raising a unit mass vertically upwards against the force of gravity through a unit distance. In the F. P. S. system, the gravitational unit of work is the work done in raising a mass of 1 *pound* through a vertical distance of 1 *foot.* The unit is termed 1 **foot-pound** (ft.-lb). In the C. G. S. system, this unit is termed 1 **gramme-centimetre** and is equal to the work done in raising a mass of 1 *gram* through a vertical distance of 1 *centimetre.* As a gramme-centimetre is very small, the unit chosen by the the engineers is the *kilogramme-metre* which is the work done in a mass of 1 *kgm.* through a vertical distance of 1 *metre.* 1 kgm. metre$=10^5$ gm.cm.

Relation between the two Units of Work—It has been explained already that the gravitational unit of force is g times the absolute unit of force. Hence,

gravitational unit of work$=g$ times absolute unit of work,

$\therefore$ 1ft-lb.$=31{\cdot}2$ ft. poundals and 1 gramme-centimetre$=981$ ergs.

Now as 1 foot-poundal$=13825$ dynes and 1 foot$=30{\cdot}48$ cms., we get

1 ft-poundal$=(30{\cdot}48 \times 13825)$ ergs$=(4{\cdot}214 \times 10^5)$ ergs.

Also 1 foot-pound$=32{\cdot}2$ foot-poundals$=(32{\cdot}2 — 4\text{-}214 \times 10^5)$ ergs

$=(1{\cdot}357 \times 10^7)$ ergs.

Hence **1 foot-pound$=$1·357 Joules.**

Work is of the dimension of the product of a force and corresponding displacement. Now the dimension of force is MLT^{-2} and that of displacement is L. Hence, the work is of dimension ML^2T^{-2}.

79. **Power**—*The power* (or *activity*) of an agent is the *rate of doing work* and is measured by the quantity of work done by the agent in unit time. The absolute unit of power in the F. P. S. system is the rate of doing 1 foot-poundal of work per second, and in the C. G. S. system it is the rate of doing **1 erg of work per second.** The practical unit of power, as used by the engineers, is termed a *horse-power.*

The term was first used by James Watt (1736-1819), the inventor of steam-engine. In order to establish a relation between the working capacity of a horse and of his newly invented engine, he performed an experiment in which, on an average a weight of 150 lbs. of coal was raised by a horse from a coal pit through a height of 220 ft. in 1 minute. The work done by the horse in 1 minute was thus equal to (150 × 220) or 33,000 ft.-lbs. This was, therefore, considered to be the average rate of working of a horse. Although the estimate is found to be too high, the term has still been retained.

Hence a **horse power (H. P.)** is the power of an agent doing 33,000 ft.-lbs. of work per minute or 550 ft.-lbs. per second.

Their is another unit of power called the **watt** so named after James Watt. This is defined as 1 Joule per sec. $=10^7$ ergs per second.

Now 1 H. P. = 550 ft. -lbs per second
$=(550\times1{\cdot}357\times10^7)$ ergs per second
$=746{\cdot}3$ Joules per second $=746$ watts (app.)
∴ 1 kilowatt $=1000$ watts $=(1000\div746)$ H.P. $=1{\cdot}34$ H.P.
$=(1{\cdot}34\times550)=737$ ft.-lbs. per second

∴ 1 kilowatt hour $=(1{\cdot}34\times550\times60\times60)=2653{,}200$ ft.-lbs. of work or energy.

The power being work per unit time, its dimension is clearly $ML^2T^{-2}\div T=ML^2T^{-3}$.

Examples :

1. A constant force of 384 poundals acts vertically upwards on a mass of 10 lbs. at rest on the earth's surface. Calculate the amount of work done by the force as well as the work done by the body against gravity while it is passing through an altitude of 100 ft., given $g=32$ ft,/sec². Find also the velocity of the body at that altitude.

Ans. The work done by the force = force × distance = (384 × 100) ft. poundals = 38400 ft.-poundals.

Also the work done by the body against gravity = weight × distance
$=(10\times32\times100)$ ft.-poundals $=32{,}000$ ft.-poundals.

So the work done by the impressed force is here greater than the work done against the opposing gravitational force by 6400 ft.-poundals. This excess work shows itself in form of motion of the body at that altitude. The resultant force in the body is $(384-320)=64$ poundals. Hence, the acceleration upwards $=64\div10=6{\cdot}4$ ft./sec.². If the required velocity be v ft./sec. we have $v^2=2\times6{\cdot}4\times100$, whence the required velocity is $35{\cdot}75$ ft./sec.

2. A man weighing 140 lbs. takes his seat on a lift which weighs 2 tons. He is taken on the third floor, which is at a height of 75 ft. from the ground floor in 2 minutes. Calculate the work done and the power required in the process.

Ans. The total weight of the man and the lift$=(140+2\times 2240)$ lbs.$=$ 4620 lbs.

Now the work done in raising the mass through 75 ft.$=(4620\times 75)$ ft. lbs.
$=3,46,500$ ft.-lbs.

As this work is done in 2 minutes, the power required for is

$$=\frac{346500}{2\times 60}\text{ ft.-lbs. per sec.}=\frac{346500}{2\times 60\times 550}\text{ H.P.}=5{\cdot}25\text{ H.P.}$$

3. An engine of mass 5 tons drags a train of mass 23 tons up a slope of inclination 1 in 28 at 15 miles per hour. If the frictional resistance be equivalent to a force of 0·4 ton weight parallel to the rails, find the horse-power of the engine.

Ans. The speed of the train$=\left(\frac{15\times 1760\times 3}{60\times 60}\right)$ ft./sec.$=22$ ft./sec.

Total mass of the engine and train$=28$ tons$=(28\times 2240)$ lbs.$=62720$ lbs.

∴ the effective force against motion along the slope
$=(62720\times\frac{1}{28}+0{\cdot}4\times 2240)$ lbs.-wt.$=(2240+896)$ lbs.-wt.$=3136$ lbs.-wt.

∴ the work done by the engine per second$=3136\times 22$ ft.-lbs.

$$\text{Hence, power of the engine}=\frac{3136\times 22}{550}\text{ H. P.}=\frac{3136}{25}\text{ H.P.}=125{\cdot}44\text{ H.P.}$$

4. Show that if a gas expands against a constant external pressure, the work done by the gas is equal to the product of the pressure and the increase in volume.

Ans. Suppose that the gas of volume v is contained in a cylinder with an air-tight piston of cross-sectional area A. Let p be the pressure exerted on the gas.

Suppose now that the gas expands to a volume v_2 and that in consequence the piston is displaced through a small distance x. As the pressure is supposed to remain the same during the change, the force on the piston$=Ap$. Hence, the external work done by the gas during expansion$=Ap\times x=p(v_2-v_1)$. The result holds goods irrespective of the shape of the container.

80. Energy—The *energy* of a body is defined as its ability for doing work and is measured by the total quantity of work that a body can do. We shall consider here only the *mechanical energy.* It is of two kinds, *viz.*, Kinetic and Potential.

Kinetic Energy—The *kinetic energy* (K.E.) of a body (or a system of bodies) is the energy it possesses by virtue of its motion. It is measured by the amount of work that the body (or the system) can do against an external impressed force before coming to rest.

In nature we find mainly two types of mechanical K. E. A load (*e.g., a pile driver*) falling from a height acquires sufficient momentum to do some work. A torpedo in motion can do work by breaking through the side of a ship at which it is aimed. Here the energy of the body is due to its motion along a straight line and the energy so developed is called *translational kinetic energy.* The other type is the energy of a fly-wheel in revolution. In this case the fly-wheel has a rotational motion and in coming to a stop it can do some such work as the raising of a bucket of water from a well. The energy acquired by a revolving material body is called the *rotational kinetic energy.*

Sometimes there may be a combination of translational and rotational kinetic energies. A ball rolling on a surface possesses the linear as well as the rotational kinetic energy. Illustrations of this combinational kinetic energy are the motions of a billiard ball, a football, a rifle bullet or the earth.

Potential Energy—The *potential energy* (P. E.) of a body (or a system of bodies) is the energy it possesses by virtue of its position or configuration. It is measured by the amount of work the body can do in coming from its present position (or configuration) to some standard position (usually called its *zero position*) or configuration. In nature we find mechanical potential energies of various types. In the gravitational P. E. the surface of the earth is usually taken to be the standard position, so that when, a body is on the surface of the earth, its potential energy is zero. But if raised above the ground, the body owing to its elevated position possesses potential energy.

In the case of a pile driver a large piece of iron, called the *ram*, is connected to a rope passing over a pulley. The mass, raised up to a certain height by a number of men pulling at the other end of the rope, possesses potential energy against the gravitational force. When released, the mass falls down upon the pile and drives it to some depth into the ground. The stretched spring, a cross-bow when fully stretched, the compressed spring in an air-gun, the spring of a clock or watch when wound, all possess different types of potential energy by virtue of their altered configuration.

81. Measurement of Energy—The energy of a body is always measured by the maximum amount of work that can be done by a body.

Translational Kinetic Energy—Let a body of mass m be moving with a velocity u at any instant. We are to find the amount of work it can do before coming to rest. Let a constant force P resist the motion of the body and thereby produce a retardation f, so that $P=mf$. Let the body thus retarded traverse a distance s before it comes to rest.

Then (Art. 20), $u^2-2fs=0$, whence $u^2=2fs$. (1)

Now the K. E. of the body=work done against the force before coming to rest=force×distance$=P.s=mfs$.

$\therefore$ by (1), the K. E.$=mfs=\frac{1}{2}mu^2$ (81,1)

Thus the kinetic energy of a moving body at *any* instant is given by half the product of its mass and the square of its velocity at that instant. The K.E. is measured in the same unit as work.

If a force P acting upon a mass m increases its velocity from u to v, while moving through a distance s along straight path, we have

$$v^2-u^2=2fs, \quad \text{or,} \quad s=\frac{v^2-u^2}{2f}.$$

$$\therefore \text{ work done by the force}=P.s=mf.\frac{v^2-u^2}{2f}=\tfrac{1}{2}mv^2-\tfrac{1}{2}mu^2.$$

∴ work done = final K. E. − initial K. E.
= increase of K. E. of the body.

Thus *the work done by a force on a freely moving body is equal to the increase of kinetic energy of the body provided the question of potential energy does not arise.*

Gravitational Potential Energy—We know that a body of mass m is attracted by the earth vertically downwards with a force mg which is the weight of the body, g representing the acceleration due to gravity. Hence, if the body be raised through a vertical distance h, the work done *against* the force of gravity is mgh. The energy spent for this purpose is stored up in the body and constitutes the potential energy *gained* by the body by virtue of its position. Again if a body of mass m falls through a vertical height h, the work done by the force of gravity is mgh, which is the *loss* of potential energy of the body.

Hence P. E. $= mgh$. ... (81,2)

∴ in F. P. S. system P. E. $= mgh$ ft.-poundals $= mh$ ft. lbs.

Also in the C. G. S. system, P. E. $= mgh$ ergs $= mh$ gm.-cm.

Rotational Energy—Let a body rotate uniformly with an angular velocity ω about an axis passing through a *fixed* point O perpendicular to the plane of the paper (Fig. 65). We may consider the body of mass M to be composed of a large number n of elementary particles of masses $m_1, m_2, m_3, \ldots m_n$, so that the summation of all these tiny masses gives the entire mass of the body. Let the distances of $m_1, m_2, m_3 \ldots, m_n$ from the axis of rotation be respectively $r_1, r_2, r_3, \ldots r_n$. Now from Art. 23 we know that the tangential velocity is given by the product of the angular velocity and the radial distance. Let $v_1, v_2, v_3, \ldots, v_n$ be the tangential velocities of the particles. Then $v_1 = r_1\omega$, $v_2 = r_2\omega$, $v_3 = r_3\omega$, ..., $v_n = r_n\omega$.

Now, at any instantaneous position of the rotating body, we may imagine that these particles have instantaneous linear velocities v_1, v_2, v_3 etc., and so they have at that instant a total kinetic energy which is equal to the sum of individual kinetic energies. Hence :

$$\begin{aligned}
\text{Total K. E.} &= \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + \tfrac{1}{2}m_3v_3^2 + \ldots + \tfrac{1}{2}m_nv_n^2 \\
&= \tfrac{1}{2}m_1r_1^2\omega^2 + \tfrac{1}{2}m_2r_2^2\omega^2 + \tfrac{1}{2}m_3r_3^2\omega^2 + \ldots + \tfrac{1}{2}m_nr_n^2\omega^2 \\
&= \tfrac{1}{2}(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots + m_nr_n^2)\omega^2 \\
&= \tfrac{1}{2}I\omega^2 \qquad \ldots \quad (81,3)
\end{aligned}$$

Here 'I' is called *moment of inertia* of the body about the axis of rotation (Art. 43). Now comparing the two equations (81, 1) and (81,3), we find that their forms are alike. In the case of linear motion the K. E. is given by half the product of the mass and square of the linear velocity and for a given mass, this energy is proportional to the square of the linear velocity. But in the case of angular motion the K. E. is given by half the product of the moment of inertia of the body about the axis of rotation and the square of the angular velocity. Thus, for a given mass the rotational energy depends on the position of the axis of rotation as well as on the square of the angular velocity.

There are instances of P. E. acquired by a rotational body. For example, the tiny fly-wheel of a watch controlled by the hair-spring moves a little while in one direction coiling the hair-spring more. The spring, on being coiled more, produces an opposite couple on the wheel to stop its motion and at this stage the wheel possesses a rotational P. E. It can be proved* that if c be the torque on the wheel for a rotation of one radian and if θ be the actual rotation of the wheel, the P. E. acquired by the wheel is $\frac{1}{2}c\theta^2$.

Examples :

1. An engine is used to pump 22,000 gallons of water per hour through a vertical height of 100 ft. Find the H. P. required. (1 gallon of water=10 lbs.)

Ans. Since 1 gallon of water weighs 10 lbs., the weight of 22,000 gallons is 22×10^4 lbs.

$\therefore$ in one hour the work done$=(22\times10^4\times100)$ ft.-lbs$=22\times10^6$ ft./lbs.

$\therefore$ work done per second$=\frac{22\times10^6}{60\times60}$ ft.-lbs.

Since 550 ft.-lbs. per sec.=1 H. P.; the required H. P.$=\frac{22\times10^6}{60\times60\times550}=11{\cdot}1$.

2. Find the energy stored in a train weighing 250 tons and travelling at a rate of 60 miles per hour. How much energy must be added to the train to increase its speed to 65 miles per hour ? [C. U.—1925]

Ans. The mass of the train$=(250\times20\times4\times28)$ lbs.$=560000$ lbs.

Also the speed of the train=60 miles per hour$=\frac{60\times1760\times3}{60\times60}=88$ ft./sec.

$\therefore$ the K. E. of the train$=(\frac{1}{2}\times560000\times88^2)$ ft.-poundals
$=216832\times10^4$ ft,-poundals.

The increased speed=65 miles per hour$=\frac{65\times1760\times3}{60\times60}$ ft./sec.$=\frac{286}{3}$ ft./sec.

$\therefore$ K. E. at the time$=\frac{1}{2}\times560000\times\left(\frac{286}{3}\right)^2$ ft.-poundals.
$=254476{\cdot}4\times10^4$ ft./poundals.

$\therefore$ the additional energy$=(254476{\cdot}4-216832)\times10^4$ ft.-poundals.
$=37644{\cdot}4\times10^4$ ft./poundals.

3. An engine increases the speed of a train of mass 200 tons from 30 miles per hour to 60 miles per hour, while it moves through a distance of 3 miles. If the resisting force be 15 lbs.-wt. per ton. find the work done by the engine and the power at which it is working. (given $g=32{\cdot}0$ F.P.S).

Ans. The total resisting force$=(15\times200)$ lbs.-wt $=3000$ lbs.-wt.
This force acts over 3 miles,$=(3\times1760\times3)$ ft.$=15840$ ft.
$\therefore$ work done against the force$=(3000\times15840)$ ft.-lbs.$=47,520,000$ ft.-lbs.
Now the mass of the train=200 tons=448,000 lbs.
Since 30 miles per hour=44 ft. per sec., it follows that the increase of energy$=\frac{1}{2}\times448,000\times(88^2-44^2)$ ft.-poundals$=40,656,000$ ft. lbs.

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* Let $\mathcal{L}$ be the rotation of the wheel at any instant. Then the restoring torque on the wheel in that position is $c\times\mathcal{L}$. If now the wheel be made to rotate through an infinitely small angle $d\mathcal{L}$, the work done by the wheel is torque×angular rotation$=c\mathcal{L}, d\mathcal{L}$. Then the total work done on the wheel when it is made to rotate through an angle θ is obtained by integrating the expression between the limits 0 and θ.

That is the P. E.$=\int_0^\theta c\mathcal{L}.\, d\mathcal{L}=\left[\frac{1}{2}c\mathcal{L}^2\right]_0^\theta=\frac{1}{2}c\theta^2$.

Hence, the total work done by the engine $= (47520 \times 10^3 + 40656 \times 10^3)$ ft.-lbs.
$= 88176 \times 10^3$ ft.-lbs.

Again, the average speed of the train $= \frac{1}{2}(30+60) = 45$ miles per hour, so that the time to cover 3 miles is $(3 \div 45)$ hour $= 240$ seconds.

Hence, the required power of the engine $= \dfrac{88176 \times 10^3}{240 \times 550}$ H. P. $= 668$ H.P.

4. A bicycle wheel, 28 inches in diameter, has a mass of 2 lbs. and the radius of gyration is 13 inches. The bicycle is travelling at 12 miles per hour. Find the total kinetic energy. ($g = 32{\cdot}3$ ft./sec.^{2}).

Ans. In moving linearly the wheel has a translational K. E. given by $\frac{1}{2}mv^2$. Now a linear velocity of 12 miles per hour is equivalent to

$$\frac{12 \times 1760 \times 3}{60 \times 00} \text{ ft./sec.} = \frac{88}{5} \text{ ft./sec,} = 17{\cdot}6 \text{ ft /sec.}$$

$\therefore$ the linear K. E. $= (\frac{1}{2} \times 2 \times 17{\cdot}6^2)$ ft.-poundals. $= 309{\cdot}76$ ft.-poundals.

Also, the angular velocity $\omega = \dfrac{17{\cdot}6 \times 12}{14} = 15{\cdot}85$ rad /sec.

$\therefore$ the rotational K. E. $= \frac{1}{2}I\omega^2 = \frac{1}{2}MK^2\omega^2 = \frac{1}{2} \times 2 \times \left(\frac{13}{12}\right)^2 \times \left(\frac{17{\cdot}6 \times 12}{14}\right)^2$
ft.-poundals $= 293{\cdot}92$ ft.-poundals.

Thus the total K. E, $= (309{\cdot}76 + 293{\cdot}92)$ ft.-poundals $= 603{\cdot}68$ ft.-poundals.

82. Different forms of Energy—The energy possessed by a body manifests itself in a variety of *forms*. The different forms in which energy may appear are :

(1) Mechanical energy, (2) Heat, (3) Light, (4) Sound, (5) Electrical and Magnetic energies, (6) Chemical energy.

Transformation of Energy—The two kinds of energy, kinetic and potential, are mutually interchangeable. Again, energy does not only appear in various forms but it can *pass from one form into another*. This is known as the **transformation of energy**. Thus when a body is raised to a height, the energy possessed by it is wholly potential, and when it falls, the P. E. is gradually changed into K. E. When the body is just on the point of striking the ground, the energy is wholly kinetic and when it finally strikes the ground, the K. E. is transformed into sound, heat and mechanical energies.

When we wind a clock, the energy, is stored up in the spring as potential energy. As the spring unwinds and keeps the clock going, the P. E. is continuously transformed into K. E. The P. E. of the compressed spring in an air gun is transformed into K. E. when the shot is released.

A moving pendulum furnishes another example. When the bob in its course of oscillation is at the uppermost position, its energy is wholly potential. As the bob comes down, the P. E. gradually decreases and K. E. is gradually gained. When the bob is at the lowest position the energy is wholly kinetic. Again when it moves up from the lowest position towards the other side, it gradually loses kinetic energy and gains potential energy till at the highest position at the other side, the energy is again wholly potential. This process goes on repeatedly.

In a *hydro-electric plant*, the water at the higher level possesses P. E. The water in moving down acts against the turbine pits and rotates the wheel when the P. E. is transformed into K. E. The motion of the turbine is utilised by suitable arrangement to revolve the armature in a dynamo. The mechanical work thus done is converted into electrical energy. This electrical energy may be converted into heat in an electric oven, heat and light in an electric lamp, mechanical energy in driving a tram car or a fan and into magnetic energy in an electromagnet. The Tilaiya dam across the river Barakar near Hazaribag has provided an opportunity of erecting a hydro-electric power station having an installed capacity of 4000 kilowatts of power for the present, with a provision for future expansion of an additional 2000 kilowatts. The steam turbine station at Bokaro situated at the Damodar Valley zone is already producing 150,000 kilowatts of power. The Bandel Power Station in West Bengal and various other Power Stations including Atomic Power Stations in Tarapur in Rajashan are generating millions of kilowatts of Electric energy for peaceful social purposes.

When coal burns, the carbon of the coal combines with the oxygen of the air and the chemical energy is converted into heat and light energies. The heat of the sun evaporates vast quantities of water from the seas and the oceans. The heat gives sufficient energy to the molecules of water so as to enable them to evaporate. The vapour so formed condeneses into clouds which melt down into rain ; rain in its turns fills the running streams that flow down to the seas again. The energy of the running stream may, therefore, be ultimately traced to the heat energy of the sun. It is to be noted that the earth is receiving energy from the sun at the rate of about 232×10^{12} H. P.

In the steam-engine, the chemical energy of combustion is changed into heat energy ; the heat energy is utilised in producing steam atnd is thus converted into mechanical energy in the motion of the piston and of the wheels. When the engine drives a dynamo, the mechanical energy is changed into electrical energy.

When two insulated conductors with opposite kinds of electricity are placed at a distance from each other, the system consisting of the two charged bodies and the space between them possesses some amount of electrical energy. If the two bodies are brought near each other, a spark passes between them, and then the electrical energy is converted into heat, light and sound energies.

Again when a body changes from a solid state to liquid or vaporous state even at the same temperature, some amount of heat is absorbed. This heat energy is transformed into molecular potential energy and is utilised in bringing about the change of state. On the other hand, when water freezes, heat energy is given out. Thus when the water in a lake freezes, sufficient heat is given out. This heat keeps temperature of the vicinity greater than that of inland localities. All these cases show transformation of energy from one kind into another.

83. Illustrations of Transformation of Energy—The following are further examples of transformation of energy :

(1) **Mechanical**—(*a*) *Kinetic energy into Potential energy*—the motion of a pendulum bob from its position of rest to the extremity

of the swing. (*b*) *Potential energy into Kinetic energy*—a body falling from a height : the bob of a pendulum moving down from the highest position on either side to its mean position. (*c*) *Kinetic energy into Heat energy*—the wheels of a moving engine stopped by putting brakes ; a piece of stone being struck against another and giving out sparks. (*d*) *Kinetic energy into Sound energy*—sound by vibration of bodies. (*e*) *Kinetic energy into Electrical energy*—action of a dynamo.

(2) **Heat**—(*a*) *Heat energy into Mechanical*—action of heat engines. (*b*) *Heat energy into Light energy*—a white-hot body, *e.g.* —singing flame. (*d*) *Heat energy into Electrical energy*—thermo-electric phenomena. (*e*) *Heat energy into Chemical energy*—formation of chemical compounds by the application of heat (*e.g.*, oxygen and hydrogen combine with each other when heated). (*f*) *Heat energy into Molecular energy*—change of state of a substance.

(3) **Light**—(*a*) *Light energy into Electrical energy*—the working of a photo-voltaic cell in a photophone. (*b*) *Light energy into Chemical energy*—the action of light on photographic plates ; combination of hydrogen and chlorine in light.

(4) **Sound**—(*a*) *Sound energy into Mechanical energy*—forced vibration and resonance. (*b*) *Sound into Electrical energy*—telephone.

(5) **Magnetic**—(*a*) *Magnetic energy into Heat energy*—rapid magnetisation and de-magnetisation generate heat in steel bars. (*b*) *Magnetic energy into Electrical energy*—electro-magnetic induction.

(6) **Electrical**—(*a*) *Electrical energy into Mechanical energy*—electric motors and tram cars, electric fans, lifts, etc. (*b*) *Electrical energy into Heat energy*—heating of conductors by the passing of electric current, as in electric furnace, electric iron, etc. (*c*) *Electrical energy into Light energy*—electrical lamps. (*d*) *Electrical energy into Sound energy*—electric bells, telephones, microphones (sound is produced when discharge takes place in electric machines or induction coils). (*e*) *Electrical energy into Magnetic energy*—magnetic field produced by electric current ; electromagnet. (*f*) *Electrical cnergy into Chemical energy*—electrolysis.

(7) **Chemical**—(*a*) *Chemical energy into Heat energy*—chemical combination (as in the burning of coal or when lime is slaked) ; various other chemical rections. (*b*) *Chemical energy into Light energy*—phosphorus combines with oxygen producing a dull glow light ; magnesium in combining with oxygen produces bright light. (*c*) *Chemical energy into Mechanical energy*—the chemical energy of a loaded cartridge is changed into the kinetic energy of the flying bullet. (*d*) *Chemical energy into Electrical energy*—Voltaic cells. (*e*) *Chemical into Heat, Sound Mechanical energies*—explosion of gun-powder and other explosives.

84. **Conservation of Energy**—It is thus found that in all cases of transformation, energy disappears in one form but reappears in another. But in no case is the energy destroyed in any way ; nor does any form of energy appear of itself without loss of some other.

It has been found also that the energy lost in any form by one system is equal to the gain of the same amount by some other system, so that the total energy of combined system remains constant. This is known as the **Principle of Conservation of Energy** first expounded by Michael Faraday and then by Mayer in the middle of Nineteenth Century. There is no general proof of the principle ; but it has been verified in various cases. So it is taken to be true. This very important principle forms the foundation of the whole Physical Science. It may be stated thus.

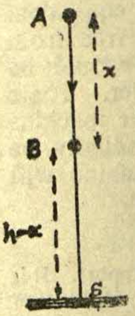


Fig. 103

The total amount of energy possessed by a material system can neither *be created* nor *destroyed by any action between the parts of the system, though it may be transformed from one form into another.*

The principle may be illustrated by the following simple example of gravitational energy. Suppose a body of mass m is raised to a vertical height h at A from earth's surface S (Fig. 103). The gravitational energy possessed by the body is wholly potential and is equal to mgh.

Drop the body freely from A through a distance x and let the velocity just at the end of this fall be v. Then $v^2=2gx$. Thus the mechanical K. E. of the body at that instant at B is$=\frac{1}{2}mv^2=mgx$. Also the gravitational P.E. of the body at the instant at B is$=mg(h-x)$;

$\therefore$ the total energy of the body at the time$=mgx+mg(h-x)$ $=mgh$, which is the energy it had initally at A.

Thus we find that as the body drops down more and more, the P. E. decreases and the K. E. increases. But the sum of the potential and kinetic energies at any instant remains constant. When the body strikes the ground and is at rest, both its kinetic and potential energies disappear. But, even then the principle is not violated, for the energy reappears as heat, sound and mechanical energies.

The oscillations of the bob of a simple pendulum present another example of the conservation of gravitational energy. When the bob of the pendulum hangs vertically as in the position A (Fig. 97), it possesses an initial gravitational potential energy mgH, where H denotes the height of the bob of mass m above the ground level. If now, the bob be displaced to a position B, when the string makes an angle θ with the vertical, the bob is virtually raised vertically up through a distance $l-l\cos\theta$. Thus the *additional* potential energy taken by the pendulum is $mg\,(l-l\cos\theta)=mgl\,(1-\cos\theta)$.

When the pendulum is released from B, it moves down the arc BA. This means that it virtually falls down under gravitational force through a distance $l-l\cos\theta$. Ultimately when it reaches A, this *additional* potential energy is converted into kinetic energy of the bob. If v be the linear velocity of the bob, as it moves past A, we derive from the principle of conservation of energy that

$$\tfrac{1}{2}mv^2=mgl(1-\cos\theta),\text{ whence } v=\sqrt{2gl(1-\cos\theta)}.$$

At any intermediate position where the string makes an angle ϕ with vertical, the bob still possesses some excess P. E. It possesses some K. E. as well. If v_1 be the velocity of the bob at this position, the K. E. here is $\frac{1}{2} mv_1^2$. Further the P. E. at this position is $mgl(1-\cos\phi)$.

Therefore the total excess energy at $F=\frac{1}{2}mv_1^2+mgl(1-\cos\phi)$. From the principle of conservation of energy this quantity=additional potential energy $mgl(1-\cos\theta)$=the maximum K. E.$=\frac{1}{2}mv^2$.

The above principle denies the possibility of the existence of a *perpetual motion machine*, that is, a machine from which work can be continually obtained without the expenditure of an equivalent amount of energy. Even if the machine be perfectly free from friction, the amount of work taken out of the machine would be *equal* to the amount put in, but it would never be greater. There are, however, always some frictional losses. So, even if the machine does no useful work, the energy initially supplied will gradually be used up to overcome frictional resistances and the machine will ultimately come to a stop.

Example :

The bob of a pendulum has its centre 1 metre below the support. It is pulled aside till the string makes an angle of 30° with the vertical. If it is now released, calculate its speed when it moves past the rest position.

[Mys. U.—1951]

Ans. When the bob of the pendulum is suspended with a string to a support and is *hanging vertically*, it must have been above the ground level. So it has some initial P. E.=V say. When the string is pulled aside at an angle of 30° to the vertical, the centre of gravity of the bob is raised up from original level through a vertical distance of 100 (1 − cos 30°) cms.=13·4 cms.

Taking the mass of the bob to be *m* and acceleration due to gravity to be *g* we note that the P. E. of the bob increases from V by the amount 13·4 *mg. ergs*. If the string is now released, the bob moves down over the arc in which the excess P. E. is gradually converted into K. E. At its lowest position, all its *excess* P. E. is converted into K. E. provided there is no other transformation of energy.

Thus $\frac{1}{2}mv^2=13{\cdot}4\times mg$. or $v^2=26{\cdot}8$ g.

Taking $g=980$ *cms.*/sec.^{2}. the actual speed $v=162$ cms./sec.

85. Dissipation of Energy—The principle of conservation of energy enunciates that the disappearance of energy in one form is always accompanied by the appearance of an equivalent amount in some other forms. But in every case of transformation some amount of energy may appear in a form in which we cannot utilise it further for doing useful work. So far as practical purpose is concerned, this amount may be said to be lost to us. This is known as the *dissipation of energy*. When a stone falls to the ground from a considerable height, its K. E. disappears completely. But the stone as well as the ground, where it strikes become a little warmer as already mentioned. The energy is wholly converted into the heat, sound and mechanical energies. But we cannot get back any of these into practical work and so to us the energy of the stone is lost.

In a railway engine, a considerable portion of the mechanical energy, derived out of heat energy, is used up in overcoming frictional resistances, which again appear as heat in the rails and the axle bearings of carriages. Similarly, in all machines a part of the energy supplied is used up in overcoming friction and is thereby changed into heat. This heat cannot be used to do work by any process hitherto known to us. Similarly, the heat radiated by a source in space cannot be received back and may therefore be taken to be lost. The total amount of energy in the universe being supposed constant, the quantity avilable for effective work is apparently diminishing.

EXERCISES ON CHAPTER V

Reference
Arts.
77, 79, 80

1. Define work, energy and power. Give some examples of transformation of energy. (Dac. U.—1958 ; C.U —1956, '62, Gau. U.—1952 ; B. U.—1953, '55 ; Del. U.—1951 ; P. U.—1951, '52 ; Pat. U.—1947, '54, '56, 58 ; Nag. U.—1952 ; *Cf.* Guj. U.—1951, '62, '63).

Art. 79

2. A company sells electrical energy for lighting purposes at $4\frac{1}{2}d.$ per unit. What is the weekly bill for a factory in which ten 5 horse-power motors run for forty hours per week and fifty 100 watt lamps are alight for forty-four hours per week? (1 H. P. $=\frac{3}{4}$ kilowatt,)
Ans. £ 13. 10s.

Art. 79

3. An engine, by applying a steady pull to a railway train at rest, is able to impart to it a velocity of 30 miles per hour in 10 minutes. If the mass of the train including the engine is 500 tons and the resistance to motion is 20 lbs. per ton, find the horse-power of the engine.
Ans. 582·67 H. P.

Art. 81

4. A man weighing 130 lbs. lifts a weight of 80 lbs. to the top of a building 33 ft. high in one minute. Find the work done by him and horse-power. (Del. H. S.—1951)

Ans. 6300 ft. lbs ; 19 H. P.

Art. 81

5. Water is pumped from a well through a height of 30 ft. by means of a 5 H. P. motor. If the efficiency of the pump be 85%, find in gallons the quantity of water pumped up per minute (1 gallon of water weighs 10 lbs.) (C. U.—1954)
Ans. 467·5 gals.

Art. 81

6. What is the work done when a weight of 500 kilograms falls through a height of 50 metres and is then suddenly stopped. Assume the normal value of gravity. (Dac. U.—1953)
Ans. 24525×10^{8} ergs.

Art. 81

7. A bullet of mass 25 gms. travelling with a velocity of 500 metres per second strikes a target which it penetrates, the velocity on emergence being 100 metres per second. What energy was expended in passing through the object?
Ans. 3000 joules.

Art. 81

8. A man climbs a tower 500 ft. high in 10 minutes. If the man weighs 165 lbs., find the rate at which he works. (P. U.—1952)

Ans. 137·5 ft. lbs./sec.

Art. 81

9. An elevator is designed to lift a load of 1,000 lbs. through 5 floors of a building averaging 11 ft. per floor in 5 seconds. Calculate the power of the elevator. (Anna. U.—1951)
Ans. 20 H. P.

Art. 80

10. A solid mass of 100 grams is allowed to drop from a height of 10 metres. Calculate the amount of kinetic energy gained by the body, g being 980 cms. per sec.^{2} (Dac. U.—1940)
Ans. $9{\cdot}8\times10^{7}$ ergs.

Reference

11. Find the expression for the kinetic energy of a body of mass m moving with a velocity v. (P. U.—1950 : Mys. U.—1951 ; Pat. U.—1952, '58)

Art. 81

12. The hydro-electric station at Konar dam project is to generate a power of 40,000 kilowatts, and the reserves of coal in the Damodar valley coal fields are estimated to be 25,950 million tons. Supposing that the coal is deposited at a mean depth of 300 ft. below the surface level and the power of the station is incessantly utilised in raising coal, calculate the time required to empty all the deposits of coal of that area. ($g = 32$ F.P.S.)

Ans. 54495×10^4 hours.

Art. 81

13. A simple pendulum is formed by suspending a metal bob from a long string. If it swings through a small angle of 60 on either side of its mean position so that the vertical descent of the centre of the bob from the extreme to the lowest position is 5 mm. Find the velocity and acceleration of the centre of the bob at the ends of the swing and when crossing the mean position, given, $g = 980$ cm./sec^2. (Raj. U.—1951)

Ans. 0 and 102·4 cm./sec^2 ; 31·3 cm./sec. and 0.

Art. 81

14. State the law of conservation of energy and prove it in the case of a body falling freely under gravity. (C. U.—1959 ; Pat. U.—1952 ; Mys. U.—1952 ; Anna. U.—1951 ; *Cf.* P. U.—1951)

Art. 81

15. Trace the transformations of energy which occur when a pendulum bob is displaced from its position of rest and is let free to oscillate till it comes to rest again. (C. U.—1953)

Arts. 82 & 84

16. A body falls under gravity and strikes the ground. Explain how the phenomenon supplies an illustration of the transformation of energy. Does it also illustrate the principle of conservation of energy? (*Cf.* C. U.—1957 ; P. U.—1947 ; Pat. U.—1952)

Art. 84

17. State and explain the principle of conservation of energy with suitable illustration. (Pat. U.—1952 ; Dac. U.—1943 ; *Cf.* B. U.—1955, '57 ; C. U.—1958)

Art. 84

18. Bring out the similarity between the moment of inertia and the mass of a body having translatory motion. Derive expressions for the Kinetic Energy and the angular momentum of a rotating body. (Guj. U.—1962)

Art. 81

CHAPTER VI

SIMPLE MACHINES

86. Machine—A machine is a contrivance by which 'work done upon it' is converted into 'work done by it' in a desired manner. To do work on the machine some amount of force is to be applied which is called the **effort or power,** and the resisting force overcome by the machine is called the **resistance or load.** The effort is usually denoted by P and the resistance by W.

The **purposes** served by a machine may be roughly grouped into three parts :

(*i*) to apply the force at a more convenient point or in a more convenient manner (*e.g.*, a poker) ;

(*ii*) to overcome greater resistance and to raise heavier load ; for example a crow-bar ;

(*iii*) to convert a slow motion at one point in into a more rapid motion at some other point : (*eg*. a bicycle, a sewing machine).

Principle of Work—The principle of conservation of energy propounds that the work that may be derived from a machine can never be more than that done upon it. It is possible for a small force exerted through a machine to lift a heavy weight ; but if there is no loss of energy in the from of friction within the machine, the work done by the effort on the machine is always equal to the work done by the machine against external resistance. In simple language the 'input' energy is equal to the 'output' energy. This is known as the *principle of Work.*

87. Efficieney of a Machine—A part of the energy supplied to a machine is used in overcoming internal resistance in the form of friction and this part is always dissipated in the form of heat energy. So the useful work done by a machine is always less than the emergy supplied.

The ratio, which indicates what fraction of the total energy supplied to a machine is utilised by it in doing useful work, is termed the efficiency or *modulus* of the machine. Hence,

$$\textit{Efficiency } (E) = \frac{\textit{energy utilised or work usefully performed}}{\textit{total energy supplied}} \quad \ldots(87\text{-}1)$$

The efficiency of any machine is therefore, always less than unity. On being multiplied by 100, it is often expressed as a percentage. For example, suppose a machine takes in 400 ft. pounds of work in a certain interval and delivers 320 ft. pounds of work during the same interval. Then the efficiency of the machine is 320÷400=·8 In terms of percentage, it is 80%.

A perfect machine is that which would utilise the entire energy supplied and so the *efficiency* **of perfect machine is unity.** Thus, if F denotes the effort applied and W the resistance overcome and the distance through which the effort and the resistance move be d_1 and d_2 respectively, then in a *perfect machine*—

$$P \times d_1 = W \times d_2.$$

If P is much smaller than W, d_2 is much smaller than d_1; so *what is gained in power is lost in speed.*

Velocity Ratio—The ratio of the distance d_1, through which the effort works to the distance d_2, through which the resistance is overcome, is called the *velocity ratio* of the machine.

In other words, the velocity ratio is given by

$$r=\frac{\text{distance through which applied force moves}}{\text{distance through which resistance is overcome}}=\frac{d_1}{d_2}\cdots(87,2)$$

Mechanical Advantage—The *mechanical advantage* (or the *force ratio*) of machine is the number whieh expresses the ratio of the resistance overcome to the effort applied to the machine to produce equilibrium. Or, in simple terms,

$$\text{Mechanical advantage}=\frac{\text{resistance overcome}}{\text{effort applied}}=\frac{W}{P}\quad\cdots\quad(87,3)$$

Machines are usually so constructed that the mechanical advantage is greater than unity.

Again it is obvious from the above, *Efficiency* $(E)=\frac{Wd_2}{Pd_1}=\frac{W/P}{d_1/d_2}$

Clearly therefore, *efficiency* $(E)=\frac{\text{mechanical advantage}}{\text{velocity ratio}}\cdots(87,4)$

In a *perfect machine* however, $E=I$, so that for such a machine the mechanical advantage=the velocity ratio.

88. Lever—It is simply a rigid bar, straight or bent, capable of turning about a fixed point of support, which is called the fulcrum. The effort and resistance are supplied at any two points either on the same side or on opposite sides of the fulcrum. The perpendicular distance between the fulcrum and the line of action of effort is called the *power arm*, and that between the fulcrum and the line of action of the resistance is called the load arm. When the lever is straight both the effort and the resistance act perpendicularly to the lever. Since levers are used to overcome resistance, soft and flexible substances should not be used as the materlal of the levers, as otherwise they might get bent by the forces and a part of the energy supplied may be spent in bending levers.

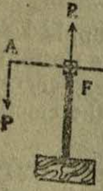


Fig. 104

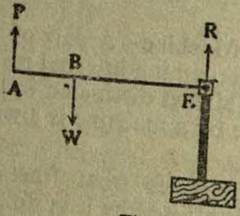


Fig. 105

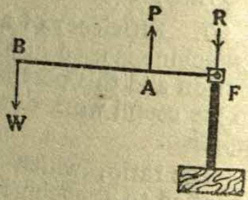


Fig. 106

Different Classes of Lever—According to the different positions of the fulcrum relative to the point of application of the effort and the resistance, levers are usually divided into *three classes*.

Class I—Here the fulcrum lies between the points of application of the effort and resistance. Thus, in Fig. 104, P represents the effort acting at A, W is the load acting at B and F is the fulcrum which is placed between A and B. The effort P, applied to overcome the load W, tends to rotate the lever about F in the counter-clockwise direction, while W tends to rotate in a clockwise direction. Hence for equilibrium.—

The moment of P about F=the moment of W about F,

i.e., $$P \times AF = W \times BF \quad \ldots \quad (88,1)$$

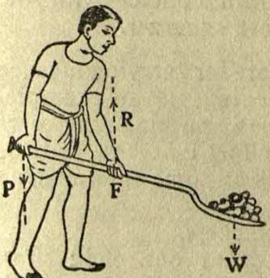


Fig. 107

Mechanical advantage $= \frac{W}{P} = \frac{AF}{BF} = \frac{a}{b}$, where a and b are lengths of power arm and the load arm.

The reaction R at F is always equal and opposite to the resultant of P and W. As P and W are parallel and both act downwards, the *resultant reaction* at F is given by R=P+W, which acts upwards. Hence, with this lever, the mechanical advantage may be greater than, equal to, or less than unity, according as the power arm a is greater than, equal to or less than the load arm b.

Examples of Single lever of Class I—A crow-bar used to raise a weight (Fig. 107) a poker used to raise coal in a grate; a claw hammer (Fig. 108); a spade in digging the earth; the handle of a common pump; the beam of a common balance; a boat's rudder, a see-saw. To use a poker, for example, the hand gripping at the middle F of the handle acts as the fulcrum (Fig. 107). The other hand pressing down at one end with a force P acts as the power and raises the load W at the other end.

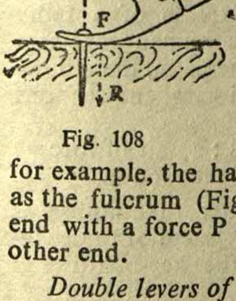


Fig. 108

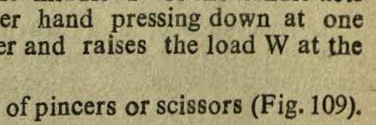


Fig. 109

Double levers of this class—a pair of pincers or scissors (Fig. 109).

Class II (Fig. 105)—Here the fulcrum F lies at one end. The effort P and load W act at A and B respectively on the same side of F but in opposite directions, the fosmer acting at a greater distance from the fulcrum than from the latter.

Now the *mechanical advantage* $= \frac{W}{P} = \frac{AF}{BF}$.

Since AF is greater than BF, *the mechanical advantage is greater* *than unity.* As P and W act in opposite directions and W is greater than P the resultant reaction at F acts upwards and is equal to W—P.

Example of Single Lever of Class II—A wheel barrow with the fulcrum at the axle and the power applied at the handle ; a punching machine ; an oar (when used in rowing ; the end of the oar in this case acts as the fulcrum, which is supposed to be fixed) ; a crow-bar with one end in contact with the ground : a cork squeezer.

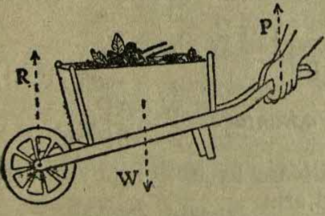


Fig. 110

A push cart for carrying luggage or road sweepings is an example. Here the man pushing the cart applies the effort P at the ends of the handles and the weight W of the load acts at some intermediate point (Fig. 110).

Double lever of this class—a pair of nut-crackers.

Class III (Fig. 107)—Here the fulcrum F is at one end ; effort P and the resistance W, are at A and B respectively on the same side of F but in opposite directions, the former acting nearer to the fulcrum than the latter. As before.

$$\textit{Mechanical Advantage}=\frac{W}{P}=\frac{AF}{BF}.$$

Since AF is less than BF, *the mechanical advantage is always less than unity.* Hence to overcome a small resistance a large effort is to be applied ; in other words, the lever has *mechanical disadvantage.* This class is generally used to convert a slow motion into a more rapid one. As P and W act in opposite directions and P is greater than W, the resultant reaction at F acts downwards and is P—W.

Examples of Single Lever of Class III—The tredle of a lathe or a sewing machine ; the pedals of a piano ; the human fore-arm (when used to raise or support a load placed on the palm, the elbow-joint acting as the fulcrum and the tension of muscles and tendons as the effort).

Double lever of this class—a pair of coal tongs ; a pair of forceps in a weight-box ; the upper and the lower jaws of the mouth.

Lever system of a Typewriter—In a typewriter the motion is conveyed from the key-board to a type through a system of three interconnected levers. K represents one stud of the key-board and

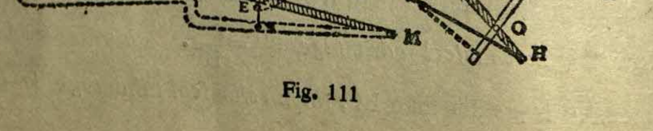


Fig. 111

KDM is one of the levers made of thin steel bar with fulcrum at M (Fig. 111). There is a light spring below the bend D of this lever which keeps it ordinarily pressed upwards at the position shown. With finger pressure on K, it is depressed down to the dotted lines. At some intermediate point of this lever, another lever FNG is connected with a short thick wire and has got its fulcrum at N. The second lever is connected with the third lever HOT with a light rod. The third lever has its fulcrum at O and carries a type at the end T. In the normal condition, this lever rests in an inclined position with the back against a rubber band B. But when the key is depressed, all the levers take up the positions as shown by the associated dotted lines and the type impinges against the roller R with a ticking sound. Here the lever KDM belongs to the second system, and the other two to the first system.

89. Pulley—A pulley consists of a small circular disc or wheel of wood or metal L (called the *sheave*), the circumference of which is cut into a groove (Fig. 112). The disc can revolve freely about an axle passing through its centre at right angles to the plane of the disc, the axle being supported in a framework B, called the *block*. The pulley can be fixed to *any* stout support by means of its hook R.

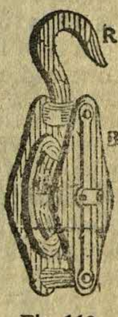


Fig. 112

If the block B be fixed, as in (Fig. 113), the pulley is said to be *fixed*, while if the block can ascend or descend as shown in (Fig. 114) the pulley is *movable*, In a single **movable pulley** the weight W is attached to one end of the string and the effort P is applied by pulling at the other end. If the pulley is smooth and the string is considered weightless and perfectly flexible, the tension of the string is the same throughout. Also the distance through which the load ascends, is equal to the distance through which the power descends. Hence from the principle of work it follows that W=P.

∴ the *Mechanical Advantage*

$$=\frac{W}{P}=1 \quad \dots \quad (89,1)$$

In practice, however, due to friction at the pulley, W is always less than P and so the mechanical advantage is slightly less than I. The pulley is used only in *changing the direction of the force*, a downward pull in this case being changed into an upward one. It is used for raising weights, drawing curtains, etc.

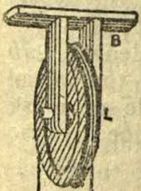
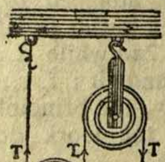


Fig. 113



Fig, 114

In a single **movable pulley** (Fig. 114) the pulley is supported by a string, one end of which is attached to a fixed support, while the effort P is applied at the other end. The weight W to be raised is attached to the pulley block. As the pulley is smooth, the tension T in any part of the string is P. The weight of the movable pulley being neglected, the weight W is supported half by the tension of the fixed portion of the string and half by the tension of the moving portion. Hence W=2P.

$$\therefore \text{ the } \textit{Mechanical Advantage} = \frac{W}{P} = 2 \qquad \ldots(89{,}2)$$

Hence a given effort can raise a weight twice as much. In practice, as the pulley is not weightless, the mechanical advantage is a little less than 2. (For different systems of pulleys and their mechanical advantages *vide* Appendix IB in Supplementary Volume).

90. Wheel and Axle—It consists of two cylinders arranged so as to turn on the same horizontal axis ; one of these is a grooved wheel AB (Fig. 115) of larger radius called the **wheel**, while the other is a coaxial cylinder of much smaller radius called the **axle** which can turn freely on fixed support. A rope is wound around the axle, one end of which is fixed to the axle, while the free end supports the weight W. Another rope is coiled round the wheel in the *opposite* direction, one end of which is attached to the wheel, while the effort P is applied downwards at its free end. When the effort P is applied downwards, the rope round the wheel unwinds and that round the axle coils up. Thereby W moves upwards.

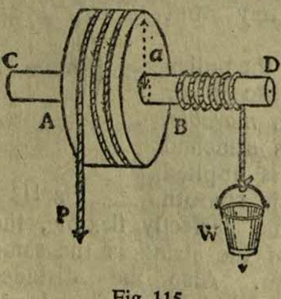


Fig. 115

Let the radii of the wheel and the axle be a and b respectively. When they make one complete revolution, a length $2\pi a$ of the rope attached to the wheel unwinds, *i.e*,. the initial point of application of P moves down through the length $2\pi a$ while a length $2\pi b$ of the rope attached to the axle coils round it ; *i. e.*, the initial position of W moves up through $2\pi b$. Now, the principle of work in an ideally frictionless machine states that the work done by the effort is equal to that done on the resistance.

Hence, $P \times 2\pi a = W \times 2\pi b$,

or, $P \times a = W \times b$.

$$\therefore \text{ The } \textit{Mechanical Advantage} = \frac{W}{P} = \frac{a}{b} = \frac{\text{radius of the wheel}}{\text{radius of the axle}} \qquad \ldots(90{,}1)$$

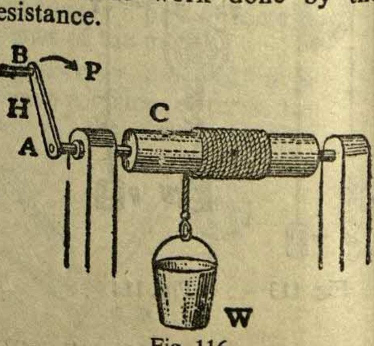


Fig. 116

By making a large and b small within practical limits, the mechanical advantage can be increased. A modification of the wheel and axle is the windlass which is a device used to draw water from wells. It consists of a horizontal roller with its handle attached to one end (Fig. 116). Another modification is the *capstan* which is used on board a ship to raise anchors by coiling the rope round a vertical axis.

91. Toothed Wheels—In machinery, the speed of revolution of different parts is often required to be adjusted to suitable needs, although the main shaft possesses a constant revolution. When the main shaft is at a considerable distance, it is usual to fix a fly-wheel with a broad rim at each shaft and to connect these two wheels with a belt of leather or rope running over the rims. The driven shaft can be geared to its fly-wheel, whenever necessary. Such a belting arrangement is shown in the diagram of the steam engine in Chapter X of the part on Heat. When the load on the driven shaft circuit is high or the friction between the rim of the wheel and belting is not sufficient, there may be a slip between the two and the desired rotation may not be obtained. Consequently, sufficient precaution must be taken to secure a good grip between the belt and the fly-wheel. Nevertheless, belting has one advantage, *viz.*, that it can be run with a high speed.

To secure the best possible grip on the fly-wheel without any slip, the rim of each wheel is cut into a number of identical teeth, and a grooved chain fitting into such teeth may be made to pass over the two. Such an arrangement is called *chain gearing or chain driving*. The pedal of a bicycle is connected to the axle of the rear wheel by means of chain drive (Fig. 117). In such an arrangement the disadvantage at increasing speed is the considerable friction between the chain and the axle.

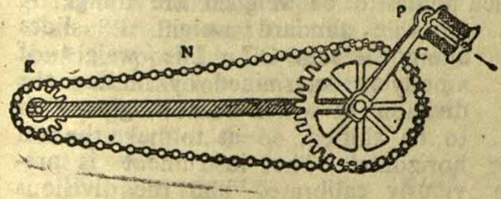


Fig. 117

When the shafts are quite close together, the rims of the fly-wheels are provided with teeth which engage with each other as shown in Fig. 118. The teeth of both the wheels are made exactly identical. The radius of a wheel is taken to be the distance from the axis of rotation to a point at half the height of any tooth. The effective rims of the wheels are represented by dotted circles. The distance along this dotted line between the mid-points of any two consecutive teeth is called the *pitch* of the tooth. Since the pitch of the tooth is equal on both the wheels, the number of teeth multiplied by the pitch is equal to the circumference of a wheel. Thus if n_1 and n_2 be the numbers of teeth on the wheels, p the pitch, and r_1 and r_2 their respective radii, we note that,—

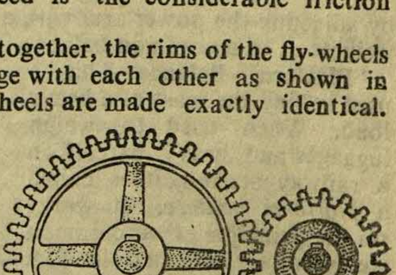


Fig. 118

$n_1 p = 2\pi r_1$ and $n_2 p = 2\pi r_2$, so that $\frac{n_1}{n_2} = \frac{r_1}{r_2}$ ... (91,1)

If N_1 be the number of revolutions of the first wheel per second, then across any point, $N_1 n_1$ teeth would pass by during the period. A similar number of teeth of the second wheel would pass by during one second across that point. Let this number be $N_2 n_2$, where N_2 represents the number of revolutions per second of the second wheel.

Thus $N_1 n_1 = N_2 n_2$,

i.e., $\frac{N_2}{N_1} = \frac{n_1}{n_2} = \frac{r_1}{r_2}$, by (91, 1) ... (91,2)

Therefore, the number of revolutions is *inversely* proportional to the radius of the fly-wheel. To make the speed of the shaft double that of the driver, the secondary toothed wheel would have a radius half that of the primary wheel.

92. Common (or Roman) Steelyard—The common steelyard is a machine for quickly weighing heavy bodies. It belongs to a lever of Class I. It consists of a long uniform rod CD (Fig. 119), turning about a fixed fulcrum O, which is situated a little away from the centre of gravity of the rod. A hook is attached to one end D of a graduated rod from which bodies to be weighed are hung. A movable standard weight P slides along the arm OC. The weight of a body is determined by finding the division at which the weight P is to be shifted, so as to make the rod horizontal. The instrument is previously calibrated and the divisions are engraved on the rod. Thus we see that this steelyard is a class of balance in which weighing is done by adjusting the power arm with a fixed load but not by changing weights as in a common balance.

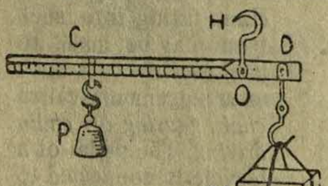


Fig. 119

Platform Balance—This is used to find the mass of a heavy load. When used to weigh luggages and heavy parcels in a railway station, it is called a *platform balance*. It works on the principle of a common steelyard as indicated in Fig. 120. It consists of three levers fF_3g, F_1de and cbF_2, having their fulcrums respectively at F_3, F_1 and F_2. The platform P, upon which the load is to be placed, rests on two knife-edges *a* and *b*, fixed on the levers *ad* and *cb* close to the respective fulcrums. Any pressure exerted on the platform

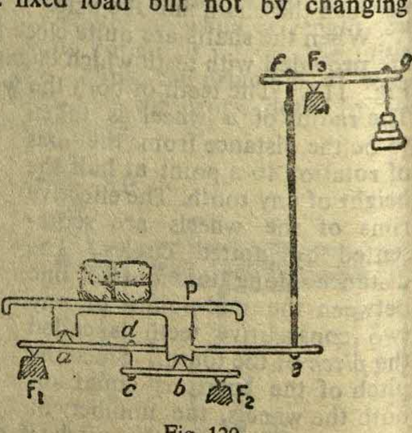


Fig. 120

may be regarded as being distributed at the points *a* and *b*. The pressure is communicated to the end *e* of the lever *ae* and a very small force is required to balance it, as the arm F_1e is much longer than F_1a. Then *e* is again attached by a vertical rod to the point *f* of the lever *fg* which has its fulcrum F_3 very near to *f*. Thus, a small effort acting at the end *g* of this can balance the force at *f*. Standard weights suspended from *g* constitute the effort. Very small fractions of weights are measured by sliding a small weight along F_3g, which has been previously graduated for the purpose.

It is evident that a considerable mechanical advantage is gained here, inasmuch as quite a small weight balances a very large one. The *weigh bridge* used for weighing trucks of coal, etc., or any loaded cart with its contents also consists of a platform resting on a system of levers, in which a system of small weights balances the load, so that there is also a considerable mechanical advantage.

93. **Inclined Plane**—Sometimes a heavy load may be conveniently lifted upwards with the help of an inclined plane. Suppose AC to be a rigid plane inclined at an angle θ with the horizontal line AB (Fig. 121). A load W is placed on the plane. Suppose further that there is little or no friction betweent hem.

The weight of the load acts vertically downwards. The component of this force parallel to the plane is W′ which is W sin θ. Hence, if an effort P equal to this component be applied in the opposite direction, the load may be kept stationary at *any* position on the plane. Thus, for equilibrium, $P = W \sin\theta$.

$$\text{or,} \quad \frac{W}{P} = \frac{1}{\sin\theta} = \frac{l}{h} \qquad \ldots(93,1)$$

where *l* and *h* are respectively AC and CB. Therefore, the mechanical advantage of an inclined plane is the ratio of its length to its vertical height.

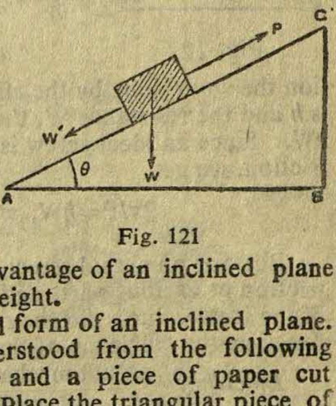


Fig. 121

94. **Screw**—The screw is a modified form of an inclined plane. The formation of a screw thread is understood from the following illustration : Imagine a vertical cylinder and a piece of paper cut in the form of a right-angled triangle. Place the triangular piece of paper with its base at right angles to the axis of the cylinder L and gradually wrap it round the cylinder (Fig. 122) The hypotenuse will trace a spiral path over the cylinder, which is called the *thread* of the screw. A screw is generally provided with an arm by which it can be rotated. Sometimes a groove is cut at the head of the screw so as to facilitate its rotation by means of a screw driver. When a screw is fixed up in a piece of wood or in a nut, then by the application of a torque, it turns round and moves bodily forward in the direction of the length. For each complete turn, the screw moves forward through a distance which is equal to the difference between consecutive threads measured parallel to the axis of the screw, *e.g.*, a length PQ. This distance is called the *pitch* of the screw. There are various forms in which the thread of screw may

be cut. Figure 123 represents screw with a square thread. Figure 124 shows a screw jack which is used to raise a heavy load (such as a motor car) through a small height. The jack is placed on a firm base in such a way that its head B very nearly touches the underside of the load W to be raised. The screw S is then rotated by the handle H, where by it rises through a stout nut fixed within the cylinder C acting as its base.

Suppose that a screw is rotated with a force P applied at right angle to the end of the arm of length l. Then, for one complete rotation the work done by the effort is $2\pi l P$. If the pitch of the screw is h and the resistance W, the work done against the resistance is hW. Since an ideal screw is supposed to have no loss of energy in friction, we get

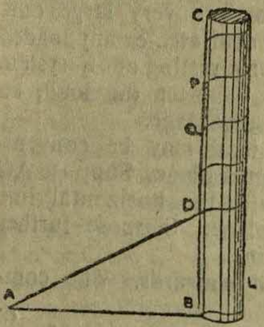


Fig. 122

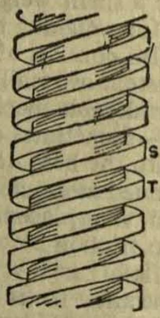


Fig. 123

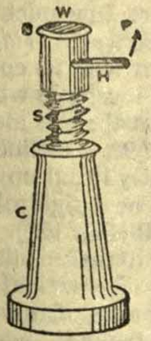


Fig. 124

$$2\pi l P = hW, \text{ whence } \frac{W}{P} = \frac{2\pi l}{h} \qquad (94,1)$$

But, for an actual screw, there is some loss of energy due to friction at various parts and so the force of resistance W falls short of the ideal value. So the ratio W/P becomes less than $2\pi l/h$. In practice, the efficiency of the screw is given by :

$$\textit{Efficiency} = \frac{\text{actual mechanical advantage}}{\text{velocity ratio}} = \frac{W/P}{2\pi l/h} = \frac{Wh}{2\pi l P} \quad \cdots \quad (94,2)$$

Examples :

1. 10 men raise a marble block weighing 1 ton with the help of two triple pulley blocks, each weighing 100 lbs. If half the energy is wasted in friction, find (*i*) the average pull per man, and (*ii*) the average horse-power per man, if the block rises 10 ft. in every minute.

Ans. Total weight to be raised = weight of marble block + weight of lower pulley block = 1 ton + 100 lbs. = 2340 lbs.

As half the work is used in overcoming friction, total resistance to be overcome is given by W = 4680 lbs. wt.

As the number of segments of strings on the lower block is 6 : W = 6P.

∴ the total efforts to be applied by 10 men = (4680 ÷ 6) = 780 lbs.-wt.

Hence, average effort put in per man = (780 ÷ 10) = 78 lbs.-wt.

∴ the total work done in one *minute* = (4680 × 10) = 46800 ft.-lbs.

∴ the work done per man per minute = (46800 ÷ 10) = 4680 ft.-lbs.

Hance the average horse-power per man = (4680 ÷ 33000) = 0·142 (approx)

2. A uniform bar AB, 2 ft. long and weighing 3 lbs., is used as steel yard being supported from a point 4″ from A. Find the greatest weight that can be weighed with a movable weight of 2 lbs. and find also the point from which the graduations are measured.

Ans. When the movable weight is at the extreme end B, the weight, that can be weighed, is the greatest. Let this weight be W which acts at A. The distance of the fulcrum from A is 4″.

The weight of the bar passes through the middle point of AB and the distance (from the fulcrum) of the point of application of the weight is thus equal to 1 ft.−4 in.=8″,

Hence taking moments about the fulcrum, we find

$W \times 4 = 3 \times 8 + 2 \times 20$, whence the required weight $W = 16$ lbs.

As the weight of the steelyard acts through a point on the right of the fulcrum, the movable weight must act at some point on the left of the fulcrum to counterpoise the weight of the steelyard. Let the distance of this point from the fulcrum be x inches.

Then $2 \times x = 3 \times 8$, whence $x = 12$ inches

Hence, the point does not lie in the bar, but is $(12-4)=8$ inches to the left of A.

3. A screw jack with a pitch of 0·25 inch has a handle 30 inches long. A force of 20 lb. must be applied when a load of 6600 lb. is being lifted. Calculate the velocity ratio, the actual mechanical advantage and the efficiency. [E. P. U.—1953]

Ans. The *actual mechanical advantage* $= \frac{W}{P} = \frac{6600}{20} = 330$.

Also the velocity ratio $= \frac{2 \times 3 \cdot 14 \times 30}{\cdot 25} = 753 \cdot 6$.

$\therefore$ the *efficiency* $= \frac{\text{actual mechanical advantage}}{\text{velocity ratio}} = \frac{330}{753 \cdot 6}\% = 43 \cdot 8\%$.

EXERCISES ON CHAPTER VI

	Reference
1. Define the terms 'velocity ratio', 'mechanical advantage' and 'efficiency' as applied to machines. [Mys. U.—1952]	Art. 87
2. Describe a lever of Class III Calculate its mechanical advantage and show how the principle of work is satisfied there. [Utk. U.—1949]	Art. 88
3. Give a very brief description of the second system of pulleys and deduce the mechanical advantage. [E. P. U.—1952]	Art. 89
4. Describe a simple pulley and explain its action. A pulley system consists of a fixed block and a moving block each containing three pulleys. Explain what the mechanical advantage is. [Del. H. S.—1950]	Art. 90
5. Deduce the mechanical advantage of a wheel and the axle from the general principle of the conservation of energy. [P. U.—1959, '61]	Art. 90
6. Explain the construction and action of a railway platform balance used for weighing heavy parcels on luggages. [U. P. B.—1962]	Art. 92
7. What are the two ways in which effort can be applied in raising a body up an inclined plane ? Which is more advantageous and why ? [Mys. U.—1952]	Arts. 92 & 93
8. Derive an expression for the mechanical advantage of a screw. [P. U.—1959]	Art. 94

Reference

Art. 94 9. The arm of a screw jack is 15 inches long and the screw has 10 threads per inch. Find the force (in seers) that will be required to lift one side of a lorry weighing 60 mds. [P.U.—1962]

Ans. 2·55 seers.

Art. 94 10. Find the mechanical advantage and efficiency of a screw jack if an effort of 1 kgm. weight is applied at the end of the arm 21 cm. long to lift a mass of 250 kgm., the pitch of the screw being 5 mm. [E. P. U.—1960]

Ans. 250; 94%.

Art. 94 11. Describe a jack-screw and state one of its practical applications with which you are familiar. Neglecting friction and weight of the machine, find out an expression for its mechanical advantage.

A jack-screw having a pitch of ·25 inch is turned with a force of 50 lbs.-wt., applied at the end of a handle 3 ft. from the axis of rotation of the screw. Calculate the load which the jack will be able to raise. [C. U.—1956]

Ans. 45257 lbs.-wt.

CHAPTER VII

PROPERTIES OF MATTER

95. Physical States of Matter—We already know that any thing that can be felt by senses is called matter. Matter can exist in three different states, namely *solid. liquid* and *gas*. A **solid** preserves a *definite shape* and *volume* and opposes any force which tends to change its shape. A given mass of **liquid** has a *definite volume* but no *fixed shape*, and when at rest it takes the shape of the containing vessel which it fills, maintains a free horizontal surface and offers no resistance to any external force tending to change its shape. A gas has *neither* a definite shape *nor* a definite *volume*; it entirely fills up any space in which it is enclosed and always tends to expand. Liquid and gas are both included under the common term **fluid**, for if unopposed, they can flow from one place to another.

A few solids (like pitch and sealing-wax) do not, however, maintain a definite shape, while liquids like treacle and tar do not readily assume a free horizontal surface. Such substances are said to be *viscous*. On being heated, a solid may be converted into a liquid and a liquid into a gas, while on cooling the process may be reversed. Thus ice, water and steam are the three different states of a substance of the same chemical composition.

Constitution of Matter :—Atoms and Molecules—The problem of 'how a matter is composed' struck the minds of people from ages past. They found that any material body is capable of being broken to pieces. They argued that if any substance be divided, subdivided and further subdivided into smaller and smaller pieces, *can an ultimate limit be reached beyond which further fragmentation is not* *possible ?* The natural argument also leads to the conclusion that subdivision of a matter cannot lead to 'nothingness' since something cannot be made out of nothing.

The earliest record regarding the constitution of matter is obtained from the writing of a Greek Philosopher Democretus (460-370 B. C.) who said that the universe is composed of atoms and void ; and such atoms are indivisible and everlasting. An Indian sage Kanad also propounded the same theory but he moved a step farther and said that void (vacuum) and atoms might be interchangeable : *atoms can be formed of void* and viceversa. Kanad's speculation has been theoretically supported by Einstein (1879-1955) in his Theory of Relativity by assuming 'void' as 'energy' and experimental evidences have come recently from unclear Physicists.

Any way we assume that the Atomic Theory of ancient Greece and India was more or less on *speculation not based upon experimental evidence.* The first logical and experimental support came from an English Chemist John Dalton (1766-1844) in 1805, along with a team of workers Antoine Laurent Lavoisier, a French Chemist (1743-1794), Joseph Louis Proust, another French Chemist (1754-1826) etc. But even then the difference between the atoms and molecules could not be established. It was not until in 1811 that Amedeo Avogadro (1776-1856) along with Joseph Gay Lussac (1778-1850) established beyond any doubt the distinction between the atoms and molecules and the part played by such particles in the formation of a matter. We now know that *in all chemical changes and combination the atoms of elements take part. A molecules is the smallest particle of any kind of matter which can exist in free state and can show the properties of matter.*

Intermolecular Spaces—When a body is compressed or when it is cooled, its volume contracts. On the other hand, its volume increases, when it is stretched out or heated. Again, if sugar is dissolved in water, the volume of the solution is less than the total volume of sugar and water taken separately. It is now known that in most *solid substances,* the molecules may be taken touch each other like a number of balls in a basket. As there is some space in between the balls, there is also a space of this kind in between the molecules of a solid body. This is known as **intermolecular space.** Any alteration in these spaces by any physical process produces a change in volume.

Intermolecular Forces—The forces of attraction, which hold together the molecules of a body, are called *intermolecular forces.* Such a force is exceedingly great when the distance between them is small compared to their dimensions ; but this force becomes exceedingly small, when the distance exceeds a certain critical value. The molecules are supposed to be in a state of *rapid motion* due to which they tend to separate from one another ; but due to mutual attraction they cannot leave each other permanently. Thus the molecules are under simultaneous action of two opposing tendencies and a change in the relative proportion of such forces that gives rise to different physical states of matter.

In *solid*, the intermolecular attraction is far stronger than the molecular tendency to separate, so that the molecules are held together with a strong force, first investigated by Van der Waal (1837-1923). In crystalline solid, the molecules retain their relative positions by a regular arrangement. It is for this reason that a solid has a definite *shape* and *volume* and offers a great resistance to any change in its shape or volume. Fig. 125 is a very highly magnified picture of the element 'iodine' in a solid crystalline state. Note that each molecule is composed of *two atoms in contact* and how such molecules have arranged themselves in regular rows, in a horizontal or vertical direction. Each ball, representing an atom, has a diameter of the *order of one-hundred millionth part of a centimetre*. In a *liquid state* the molecular attraction is rather feeble, so that any portion can move relatively to the other. A liquid has, therefore, no definite shape and yields to any external force tending to change the shape. Fig. 126 represents iodine in liquid state. Note that the regularity of structure of molecules is lost. In a *gas* the molecules are widely separated from one another, so that the molecular attraction is practically absent and hence molecules can move about independent or each other. Fig. 128 is

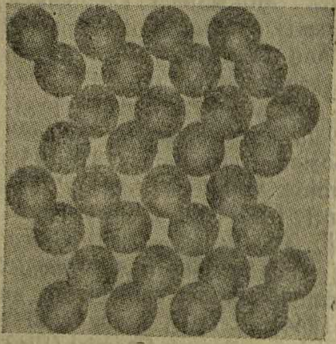


Fig. 125—Solid State

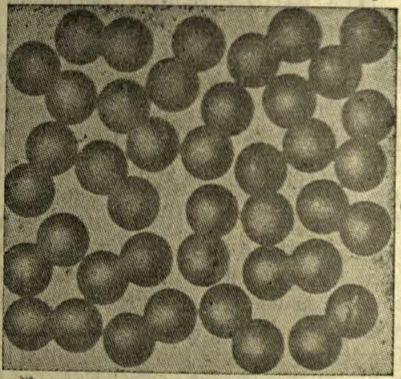


Fig. 126.—Liquid State

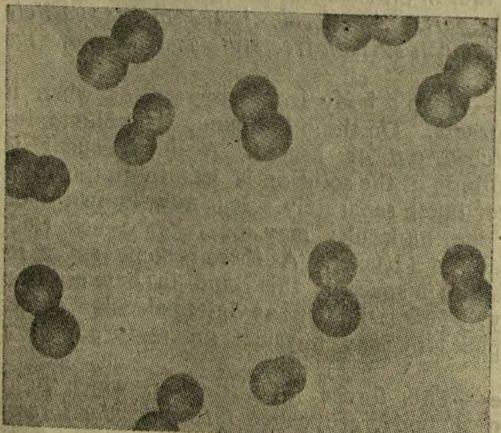


Fig. 127—Gaseous State

a picture of iodine in gaseous state. A gas, therefore, has no definite shape or volume but completely fills the containing vessel and offers much less resistance to any change of volume.

Physical and Chemical Changes—Water when sufficiently cooled, changes into ice, and when heated, passes into vapour. But in each state the constituent molecules retain the same composition. The changes, in which the constituent molecules (and consequently the substance as a whole) retain the same composition, are called *physical changes.* If now electric current is passed through pure or acidulated water, two kinds of gases are produced. The properties of these gases are quite different from each other and also different from those of water. Again, if electric sparks are passed through a definite proportion of hydrogen and oxygen, the gases explosively combine to form water. Such changes, in which the constituent parts lose their identities and give rise to a substance having different properties, are called *chemical changes.*

96. **Properties of Matter.**—Properties possessed by bodies in all their physical states—solid, liquid or gaseous—are termed *general properties*, while those properties, that are found in a particular state (or states) but *not* in all the three states are called *special properties.*

Density—Every substance, whether a solid, a liquid or a gas, contains some quantity of matter and so possesses some inertia. If the body be homogeneous, the mass contained in a unit volume of the body is called the *density* of the substance of which the body is made. Hence, density is a general property.

Elasticity—It is the property, by virtue of which a body offers resistance to external forces tending to change its volume or shape or both. Whenever deformed up to a certain limit, it would tend to regain the original volume or shape.

A body is said to be *perfectly rigid*, when any external force, however large, cannot produce any relative displacement of its constituent elements. No body, however, is perfectly rigid ; but substances such as glass and steel possess rigidity to a very high degree. Again, a body is said to be *perfectly elastic* when it completely recovers its original volume or shape after the deforming force has ceased to act.

Elastic Limit—No body is perfectly elastic. It is, however, found that a boby would behave as a perfectly elastic body (*i.e.*, it would completely regain its original condition), when the deforming force does not exceed a certain limit depending on the nature of the body. This is called the elastic limit of the substance, of which the body is composed. If the force is increased still further, the body acquires a permanent deformation and is said to be overstrained.

Elastic Fatigue—If a comparatively large force even within the elastic limit acts upon a bony for a time, the body does not *readily* return to the original state, even when the deforming force has ceased to act. The body is then said to be in a state of *elastic fatigue.* If the force is taken away and the body is allowed to rest for some time, it gradually recovers and comes back to its former state.

Hardness—It is the property, due to which one solid offers resistance to being scratched by another. There is no absolute value for the hardness of a substance ; but it is a relative property and is determined by referring the particular substanee to a scale of hardness (usually *Mohr's Scale of hardness*), in which substances are arranged in such a way that each of them can be scratched by any that follows in the list but *not* by any that precedes it. Steel is harder than many substances, and diamond is the hardest of substances known.

Cohesion—It has already been mentioned that the molecules of a body attract each other with a force which tends to bind the molecules to one another. The binding force is called the *cohesive force* and the phenomenon is known as *cohesion* or Van der Waal force. In the case of cohesion, the forces of attraction bind molecules of the *same* kind. Cohesion is very high in solids, so that the molecules are bound to one another with a great force and this gives a *definite shape* to a solid. It is, howevcr, different in different solids. Cohesion is also exhibited to a slight extent in liquids and is practically absent in gases. The formation of drops by liquids when in small masses (*e.g.*, dew or rain drops) is due to cohesion.

Adhesion—The force of attraction, that binds molecules of *different* kinds, is called *adhesion.* Due to adhesion water adheres to glass. If a glass plate is made to float on water, great force will be necessary to take off the plate. Sticking wood by glue, joining bricks by mortar and the process of soldering or nickel plating are instances of adhesion. Properties such as surface tension, viscosity, diffusion, etc., which are characteristics of fluids, are dealt with in Supplementary Volume of the part of General Physics.

97. **Stresses and Strains**—When a force (or a system of forces acting upon a body) causes a relative displacement of its various parts, a change of length, volume or shape takes place. The body is then said to be *strained* by the force and the change in length, volume or shape of the body relative to the original quantity is called a *strain*. The strain is, therefore, a pure *ratio* and has got no unit.

When a body is put to strain by external forces, internal forces of reaction are called into play due to relative displacement of its parts, which tends to restore the original shape of the body. The reactions, set up within the body, are equal and opposite to the forces applied, so long as there is no permanent distortion. The restoring force generated per unit area of the body is called a *stress*. The internal forces of reaction called into play during a state of deformation of the body are, by Newton's Third Law of Motion, equal and opposite to the externally applied forces. Hence, we measure a stress by examining external forces per unit area causing the deformation of the body.

(i) **Volume Stress and Strain**—When the body is subjected to uniform pressure acting perpendicularly at *any* point on the surface, there is a *change in volume only* without any change in shape. Suppose that a body of the shape of a cube represented by ABCDKLMN (Fig. 128) is acted upon by a uniform force P perpendicular to its faces. This system of forces will tend to squeeze the body uniformly from all directions and consequently the body would contract in volume, its shape remaining unaltered. The amount of contraction is proportional to the magnitude of the pressure applied at each face, provided that a certain limit, called the *elastic limit* dependent on the material of the body, is not surpassed. Figure 129 shows the nature of contraction, in which the face ABCD contracts to EFGH. Correspondingly other faces contract by the same amount. If V be the original volume of the body and v the change produced in this volume, then : $Volume\ Strain = v/V$. ...(97,1)

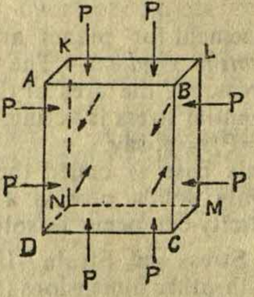


Fig. 128

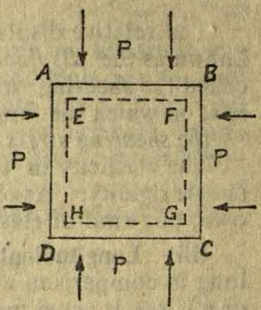


Fig. 129

Bodies in all the three states may suffer **volume strain.** In the case of *solids* and *liquids*, the strain produced is very small even for a large force, while in the cases of gases, the strain is appreciable.

When the body is squeezed in volume by the application of external forces perpendicular to the surface of the body, is tends to regain the original volume due to elasticity. Hence, the stress developed within the body is equal to the externally applied force per unit area of pressure. If P be the force applied on any face of the cube and if the face area be α the *normal stress* is P/α.

(ii) Shearing Stress and Strain—When a solid is strain ed in such a way that it undergoes a change only in shape or form but no change in volume it is said to suffer a *shearing strain.*

Let ABCDEFGH represent a rectangular block (Fig. 130) whose face DCGH is kept fixed to a horizontal bed. Let a system of force act uniformly and tangentially over the faces DG and BE, so that the plane BE is displaced through a small distance relatively to the face DG and the block assumes a form CDLKGHIJ of a rhombus. The material of the block between the faces DG and LJ undergoes a change in form only but no change in volume. The block, therefore, suffers a shearing strain, which is measured by the angle ADL (Fig. 131) called the *angle of deformation* **or angle of shear.**

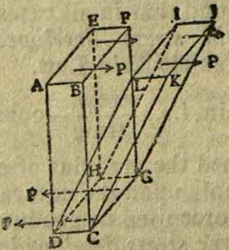


Fig. 130

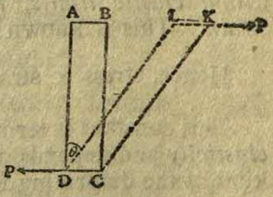


Fig. 131

Let $\angle ADL = \theta$, $AD = b$ and $AL = x$.

As θ is small, $\theta = \tan\theta$.

∴ the *shearing strain* $=\theta=\tan\theta=\frac{x}{b}$ ...(97,2)

The relative displacement for planes at unit distance apart is known as the *displacement gradient.* The corresponding stress is called the **shearing stress.** If the area of the upper or lower face is s, on which the tangential stress is acting, then :

the shearing stress $=\mathrm{P}/s=p$, say.

The elasticity in this case is called the elasticity of shape or **simple rigidity.** As only solids possess a definite shape of their own, this kind of elasticity can occur in solids only.

(iii) **Longitudinal Stress and Strain**—If a body, which is very long in comparison with other dimensions (for example, a thin rod or a wire), is acted upon by stretching forces P, P at both ends in the direction of its length, the deformation takes place lengthwise, *i.e.*, along the length only (Fig. 132) ; the strain in this case is termed the **longitudinal strain** or **tensile strain.**

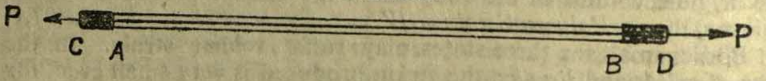


Fig. 132

If L be the original length of the body and l the change produced in this length,

the longitudinal strain $=l/\mathrm{L}$. ...(97,3)

The corresponding stress is sometimes called the **tensile stress** and in such a case the elasticity is termed *longitudinal elasticity.* If P be the force acting at each end of the specimen over an area of cross-section α, the tensile stress is P/α.

98. Hooke's Law ; Modulus of Elasticity—In all cases of strains, there should be some relation between the strain (or deformation) produced and the deforming force. Robert Hooke (1635-1703) experimentally found that in all cases of stresses and strains *within elastic limits, the stress developed is proportional to the strain.* This is known as **Hooke's Law.**

Hence, stress $\propto$ strain, *i.e.*, $\frac{\text{stress}}{\text{strain}}=\text{constant}$ ...(98,1)

This constant is termed the modulus of elasticity or *coefficient of elasticity* and depends only upon the nature of the material and the kind of the deforming force used to produce the strain. The modulus may also be defined as the *stress developed to produce unit strain.*

Different Moduli of Elasticity—The modulous of elasticity becomes different according to the nature of the strain produced. As strain is a pure ratio, each modulus is expressed in the same unit as the corresponding stress.

(i) **Bulk Modulus**—Let a force of p dynes per unit area act normally all over the surface of a body of volume V and thereby produce a diminution in volume v. Then ;

stress = force (in dynes) per unit area $=p$ and strain v/V.

∴ the modulus of volume elasticity (or **bulk modulus**) k is given by $k=\frac{p}{v/V}$ dynes per sq. cm. ... (98,2)

If the force is expressed in poundals and area in square inches, the unit of bulk modulus is then expressed in poundals per sq. inch. As a body possesses some volume in every state matter, the bulk modulus is a general property. A liquid possesses very high volume elasticity. The reciprocal of bulk modulus is called *compressibility.*

(ii) **Modulus of Rigidity**—Let a body undergo a change only of shape or form, as in Fig. 130, without any change in volume, due to a tangential force p per unit area. Then :
shearing stress$=p$ and shearing strain$=\theta=x/b$.

∴ the *Modulus of Rigidity* $n=p\left/\frac{x}{b}\right.=\frac{pb}{x}$ ... (98,3)

The unit of rigidity modulus is of the same nature as that of bulk modulus.

The shape of a solid may be changed in various ways, *viz.,* by *flexure* or bending, by *torsion* or twisting and by *tension* or stretching. The *elasticity of flexure* is the property, by virtue of which a straight rod or plank bent by the application of a force would recover itself when the force is taken away. Elasticity of flexure is found in clock and watch-springs, bows, carriage springs, etc.

The *elasticity of torsion* is brought into being, when one end of a wire is kept fixed, while the other end is twisted. This is shown by suspending a ball from a long wire fixed at the upper end. When the ball is rotated about a vertical axis, every longitudinal fibre of the rod would be twisted and elastic forces due to shear would act within the rod.

(iii) **Young's Modulus** (Modulus of Longitudinal Elasticity)—Suppose a wire of length L cm. and of cross-section α sq. cm. is stretched by a force P dynes along its length and that thereby it suffers an increase of l cm. in the length. (For a detailed exposition *vide* J. Chatterjee's *Intermediate Practical Physics.*)

Then the tensile stress$=\mathrm{P}/\alpha$ and the tensile strain$=l/\mathrm{L}$.

∴ Young's Modulus $\mathrm{Y}=\frac{\mathrm{P}/\alpha}{l/\mathrm{L}}=\frac{\mathrm{PL}}{\alpha l}$ dynes/sq.cm. ...(98,4)

The modulus is named after an English scientist Thomas Young (1773-1829).

(iv) **Poisson's Ratio**—When a rod is subjected to a longitudinal extension by a tensile stress, it is accompained by a lateral contraction. The ratio of the lateral contraction and longitudinal strain is called *Poisson's* (1801-1872) *Ratio.* If b be the amount of contraction of a rod of width B, the lateral contraction is b/B. The longitudinal strain is l/L. Thus Poisson's Ratio, usually denoted by σ is given by :

$$\sigma=\frac{b/\mathrm{B}}{l/\mathrm{L}}=\frac{b\mathrm{L}}{l\mathrm{B}} \qquad \text{... (98,5)}$$

It has got no dimension. For a long and thin wire, this ratio is negligibly small and may be neglected.

Work done in a strain—When any type of strain is produced on a system by some external force, the strain gives rise to a corresponding type of stress. So long as the external force is not applied, no strain (and hence no stress) can be generated. If now, the external force, applied to the system, be continuously increased it is clear that so long as the elastic limit is not exceeded, the deformation of the system increases proportionally. At every stage, the stress developed within the body is equal and opposite to the external force applied.

When the external force is gradually increased, there would be a gradual deformation of the system. Consequently, the external force, originally applied at *some region of the system*, would shift its point of application along with the displacement of that region due to strain. Hence in moving through some distance the external force *does some work* on the system. The greater is the strain, the larger is this amount of work done. Suppose that an external force in rising from a zero value to F makes a total displacement S of its point of application. We may simplify the idea by assuming that the average value of the external force is ½F. Hence the total work done in bringing about the deformation is ½FS.

For a *unit cube* of a medium, the force F applied at any face is the *stress*, since the face area is unity. The displacement of any face area represents the strain, since the distance between opposite faces is unity. Hence the work done *per unit volume* of a medium due to elastic deformation is ½ × (stress) × (strain). This work goes to increase the heat energy of the medium. When the external force is withdrawn, this amount of work is given out by the system.

99. Experimental Determination of Young's Modulus—Two exactly similar wires A and B (Fig. 133) of the material under investigation are hung up close to each other from the same fixed support. The wire A has a fixed load W attached to its lower end so as to keep it taut and a short scale S, graduated in mm. is fixed to this wire towards the lower end. The wire B is the experimental wire, to which is attached a weight hanger at the lower end. On this hanger weights are placed to stretch the wire. The wire carries a vernier V which slides along the main scale S.

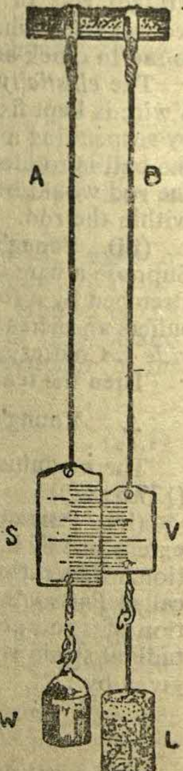


Fig. 133

At the start, the experimental wire is stretched by a load of 1 or 2 kg. to make it free from kinks. This is usually called the *dead* load. The length of the wire B, from the point of suspension to the point where the vernier is attached, is measured with an ordinary metre scale. The diameter of the wire is measured at several places by means of a screw gauge, and at each place two readings are taken at right angles to each other. The mean of all such readings is the average diameter of the wire half of which gives the average radius.

If the fixed load W is not already with the main scale, it is to be put. The reading of the vernier is taken. Then loads on the experimental string are increased by equal instalments of 1 kg. and reading of the vernier is taken at each step. The difference of readings gives the corresponding elongation in each case. The total load must not however stretch the wire beyond the limit of elasticity which is about half the breaking load. For finding the breaking load see next page. The weights are then removed one by one in equal instalments as before and the readings of the vernier are again noted. The two sets of readings should agree fairly well and from these sets the mean elongation for a given load is calculated.

Let the length of the wire be L cm., the mean radius of the wire be r cm. and the mean elongation of the wire be l cm. corresponding to the load of W gms.

$$\therefore \text{ Young's Modulus } Y = \frac{Wg}{\pi r^2} \Big/ \frac{l}{L} = \frac{WgL}{\pi r^2 l} \ \frac{\text{dynes}}{\text{cm.}^2} \qquad \ldots \quad (99, 1)$$

Very often a graph (Fig. 134) is drawn with the *load* as *abscissa* and the corresponding *elongation* as *ordinate*. Within the limits of elasticity this graph is a *straight line*, showing that Hooke's Law is true. This is because we can write equ.(99,1) in the form $l = \frac{Lg}{Y\pi r^2} \times W = K \times W$ where K is an instant. Hence relation between l and W is linear. Any suitable elongation and corresponding load may be taken from the graph and the calculation for Y may be made as before.

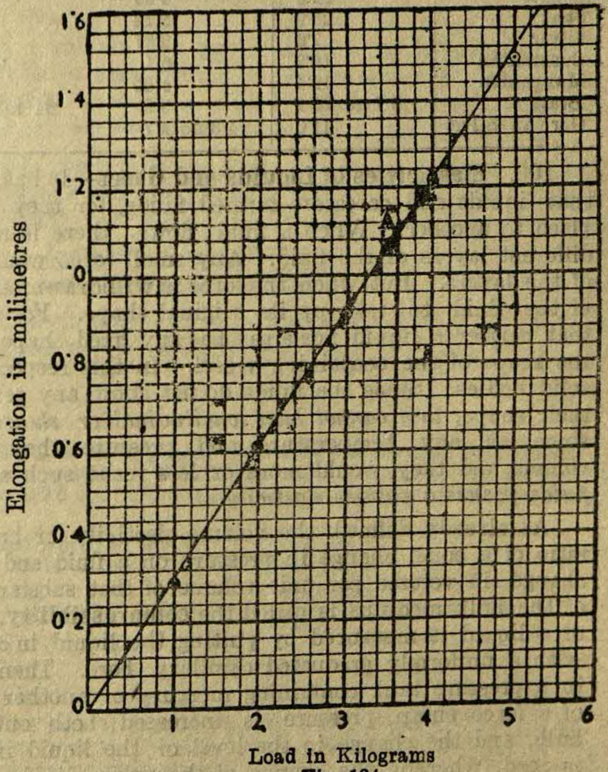


Fig. 134

The two wires are fixed to the same support, since any yielding of the support due to a load on either wire would equally affect the depression of both the wires. They are also of the same material, so that any change of temperature may affect both of them equally without altering the reading.

When the extra load over the dead load is zero, the excess strain also is zero, so that the graph passes through the origin. If the load is increased too much, the stress goes beyond the *elastic limit* and the wire gets a permanent elongation until for a certain limiting load the wire breaks down. This load is called the breaking weight for the wire. The breaking weight for a wire of unit cross-section is called the **breaking stress** for the material of the wire. To find the breaking weight for a certain wire, we are to multiply the corresponding breaking stress by the area of cross-section of the wire.

The following Table supplies the various types of elastic constants for common substances:

Substance	Young's Modulus (dynes/cm.^{2} $\times 10^{11}$)	Rigidity (dynes/cm.^{2} $\times 10^{11}$)	Poisson's Ratio	Volume Elasticity (dynes/cm. $\times 10^{11}$)
Aluminium	7·0	1·67	·34	7·46
Copper	12·3	4·55	·337	13·1
Steel	20·9	8·12	·29	16·4
Silver	7·6	2·87	·34	10·0
Constantan	16·2	6·1	·325	15·5
Manganian	12·4	4·65	·33	12·1
Brass	9·7 to 10·2	3·5	·34 to ·4	10·65
German Silver	11·6	4·3 to 4·7	·37	...

100. Elasticities of Liquids and Gases—It has already been stated that liquids and gases are termed fluids, for they can flow from one place to another. When a fluid flows, there is a relative motion of different layers of the fluid. Any small force would start the motion of the layers. But when the force is withdrawn, there is no tendency of the fluid to take up its original shape. For this reason we say that neither a liquid nor a gas has any fixed shape; but both take up the shape of the containing vessel. In this respect they differ from solid bodies. Since the fluids do not show any tendency to maintain their shape, *they cannot have longitudinal or shearing elasticity.* But since at any temperature and pressure they have a measurable volume and they would more or less resist such a change in volume, *fluids possess a volume elasticity.*

As already defined, the volume elasticity (or bulk modulus) is the ratio of a small change in pressure on a fluid and the corresponding change in volume per unit volume of that substance. The reciprocal of the bulk modulus is called the **compressibility.** The bulk modulus of a liquid is measured by putting the liquid in a glass or metal bulb with a uniformly graduated capillary bore. Then the bulb is placed in a pressure tank containing mercury or another liquid. By means of a force pump pressure is increased both outside and inside the bulb, and the change in the level of the liquid in the capillary bore is read, whereby the change of the volume is known. Knowing the increase of pressure by an apparatus called a *manometer* (*vide* Art. 168), the bulk modulus can be found. The chamber, in which the bulb is placed and is subjected to a uniform pressure both inside and outside, is called a **piezometer.** The bulk modulus of liquids is high, since liquids are almost incompressible. For water at 25°C, the value is $2{\cdot}12 \times 10^{10}$ dynes per sq. cm.

It would be seen subsequently that at a uniform temperature all gases very closely obey Boyle's Law. (*Vide* Art. 142). If P be the pressure on the gas having a volume V at some temperature and if by increasing the pressure by a small amount p, the volume of the gas is decreased by v, temperature remaining constant, then by Boyle's Law, we obtain the following relation—

$PV = (P+p)(V-v) = PV - Pv + pV - pv$. Now pv is negligibly small as compared to other quantities, and so it can be omitted from the equation

$$Pv = pV, \text{ whence } P = \frac{n}{v/V} = \frac{\text{increase of pressure}}{\text{proportional change of volume}}.$$

Hence if a gas obeying Boyle's Law be compressed, the volume elasticity is equal to its pressure. This is also called **isothermal elasticity** of a gas.

If, however, a volume of a gas be compressed so suddenly that the heat produced due to compression cannot escape but goes entirely to heat up the gas, the process is called *adiabatic transformation*. In such a case the pressure-volume relation of a gas is given by

$PV^{\gamma} = \text{constant}$, were γ is the ratio of the specific heat of the gas at constant pressure and the specific heat of the gas at constant volume (*Vide* HEAT, Art. 49). In adiabatic compression of a gas,—

$$\gamma P = \frac{p}{v/V} = \frac{\text{increase of pressure}}{\text{proportional change of volume}} = \text{Bulk Modulus.}$$

Thus the adiabatic elasticity of a gas is equal to γ times is pressure. (For detailed study, *vide* SOUND, Art. 17).

Examples :

1. A mass of 20 kgm. is suspended from a vertical straight wire 600 cm. long and 1 sq. mm. in cross-section. When the load is removed, the length of the wire is found to be $599{\cdot}5$ cm. Find the Young's Modulus for the material of the wire.

Ans. The original length of the wire $L = 599{\cdot}5$ cm.
and the elongation in the wire $l = (600 - 599{\cdot}5)$ cm. $= 0{\cdot}5$ cm.

$\therefore$ the deforming force per unit area $= \dfrac{20 \times 10^3 \times 981}{0{\cdot}01}$ dynes/sq. cm.

$$\text{Hence, } Y = \frac{20 \times 10^3 \times 981}{0{\cdot}01} \times \frac{599{\cdot}5}{0{\cdot}5} \text{ dynes/cm.}^2$$
$$= 2{\cdot}35 \times 10^{12} \text{ dynes/cm.}^2$$

2. A uniform steel wire of density $7{\cdot}8$ gm. per c.c. weighs 16 gm. and is 250 cm. long. It lengthens by $1{\cdot}2$ mm. when stretched by a force of 8 kgm. weight. Calculate the value of Young's Modulus of steel. [Gau. U.—1958]

Ans. Let the area of cross-section of the wire be α sq./cm.
Then the volume of the wire $= 250\alpha$ c.c.
Since mass $=$ volume $\times$ density, $250\alpha \times 7{\cdot}8 = 16$ gm.

whence $\alpha = \dfrac{16}{250 \times 7{\cdot}8}$ sq. cm.

Now $\text{stress} = \dfrac{\text{force}}{\text{cross-section}} = \dfrac{8 \times 1000 \times 981 \times 250 \times 7{\cdot}8}{16}$ dynes/sq. cm.

and $\text{strain} = \text{elongation per unit length} = \dfrac{{\cdot}12}{250}$.

$\therefore \quad Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{8 \times 1000 \times 981 \times 250 \times 7{\cdot}8 \times 250}{16 \times {\cdot}12}$ dynes/cm.^{2}

By use of a log-table for calculations we find

log 8 = 0·9031
log 1000 = 3·0000
log 981 = 2·9917
log 250 = 2·3979
log 7·8 = 0·8921
log 250 = 2·3979
12·5827

log 16 = 1·2041
log ·12 = 1·0792
·2833

12·5827
0·2833
12·2994

antilog 12·2994 = $1{\cdot}993 \times 10^{12}$

$\therefore$ Young's Modulus for steel $= 1{\cdot}993 \times 10^{12}$ dynes/sq. cm.

3. A stress of 1 kgm. per square millimetre is applied to a wire, of length 100 cm. of which the Young's Modulus is 10^{12} dynes per sq. cm. Find (*a*) the increase in length and (*b*) the work done per unit volume of the wire during the stretching. [Raj. U.—1951]

Ans. The word 'stress' in the problem is rather loosely used to mean the external force.

(*a*) Thus the external force being 1 kgm. per sq. mm., the stress on it is $\dfrac{\text{force}}{\text{area}} = \dfrac{1000 \times 981}{{\cdot}01}$ dynes/cm.$^2 = 981 \times 10^5$ dynes/cm.^{2}.

Now $Y = \dfrac{\text{force per unit area}}{\text{elongation per unit length}}$,

whence the elongation per unit length $= \dfrac{981 \times 10^5}{10^{12}}$.

$\therefore$ the increase in length $= \dfrac{100 \times 981 \times 10^5}{10^{12}} = {\cdot}00981$ cm.

(*b*) Again, stress $= 981 \times 10^5$ dynes/cm.^{2} and strain $= 981 \times 10^{-7}$

$\therefore$ work done per unit volume $= \frac{1}{2}(\text{stress}) \times (\text{strain})$

$= \frac{1}{2} \times 981 \times 10^5 \times 981 \times 10^{-7}$ ergs $= 4811{\cdot}8$ ergs (approximately).

4. Find the change in volume of a solid iron sphere of radius 5 cm. subjected to a uniform hydrostatic pressure of 0·1 ton weight per square inch, the bulk modulus of iron being 14×10^{11} dynes per sq. cm.

Ans. The volume of V of the sphere $= \frac{4}{3}\pi r^3 = \dfrac{4 \times 3{\cdot}14 \times 5^3}{3}$ c.c.

Normal stress $= {\cdot}1$ ton/sq. inch $= \dfrac{{\cdot}1 \times 20 \times 4 \times 28 \times 32{\cdot}2 \times 13825}{2{\cdot}54 \times 2{\cdot}54}$ dynes/cm.^{2}

Since $k = \dfrac{pV}{v}$, it follows :

$$v = \frac{pV}{k} = \frac{4 \times 3{\cdot}14 \times 5^3 \times 8 \times 28 \times 32{\cdot}2 \times 13825}{3 \times 2{\cdot}54 \times 2{\cdot}54 \times 14 \times 10^{11}} \text{ c.c.} = 5{\cdot}786 \times 10^{-3} \text{ c.c.}$$

5. Show that the energy spent in stretching a wire of length L cm. and radius *r* cm. by *l* cm. is given by $\dfrac{Y\pi r^2 l^2}{2L}$, where Y is its Young's Modulus of elasticity. [Raj. U.—1954]

Ans. We know $Y = \dfrac{\text{stres}}{\text{strain}} = \dfrac{\text{force}/\pi r^2}{l/L}$.

$\therefore$ the force required to stretch the wire by l cm. $= \frac{Y\pi r^2 l}{L}$ dynes. So long as the wire is not stretched, the pulling force is zero. Thus we can say the average force on the wire is $\frac{1}{2} \cdot \frac{Y\pi r^2 l}{L}$ dynes with which it has been stretched by l cm.

$\therefore$ work done = energy spent = force × displacement $= \frac{Y\pi r^2 l^2}{2L}$ ergs.

101. Friction—The surface of a solid is never perfectly smooth. Consequently, whenever one body slides over another, there is a sort of resistance to its motion. Hence if two bodies be in contact with each other and if we try to drag one of them over the other, a force is set up at the surface of contact and this force tends to resist the motion. This is called the force of **friction** between the surfaces in contact. The friction is due to the *roughness* of the material surfaces in contact. So if the surfaces be perfectly smooth, there is no force of friction to oppose the motion. The force of friction always acts parallel to the surface in contact and opposite to the direction of motion. Friction is a special property of solids. When a liquid or a gaseous mass flows, there is something like frictional resistance between various moving layers. This peculiar type of friction within fluid media is called its *viscosity*.

Suppose a rectangular solid body G (Fig.135) is at rest on a plane horizontal surface. The forces acting on G are (*i*) its weight *mg* acting vertically downwards, and (*ii*) the reaction B, acting vertically upwards. In a state of rest the upward reaction B balances the weight *mg* and no friction is brought into play. If now a small force P be applied to G parallel to the surface, a resistance (say, F) is offered against the motion. If this body is still at rest, it is in equilibrium under the action of the four forces B, F, P and *mg*. As B is equal and opposite to *mg*, the force F must be equal and opposite to P.

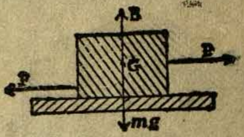


Fig. 135

As P is increased, F also increases. It is found that as long as P does not exceed a certain limit, there is no motion, F being so long always equal to P. The resistance F, which is thus brought into play by the external force P in a direction opposite to that of the latter is a *self-adjusting force* and so long as the body is at rest, that force is equal to the pulling force. The force F is called the **frictional force** between the two bodies in contact.

102. Limiting Friction—Although friction is a self-adjusting force, it does not however increase indefinitely with the external force. Thus, if the external force P is gradually increased, the force of friction reaches a maximum (or limiting) value which depends on the nature of the surfaces in contact and the magnitude of the pressure (*i e.*, the normal force) between them. The body is now on the point of sliding and the friction then exerted is called **limiting friction** between the two surfaces under the impressed normal force. If the external force be increased further, equilibrium is lost and the body begins to move.

The friction is said to be *sliding* or *rolling*, according as one body slides or rolls over another. The dragging of a heavy load over the ground, the rubbing of hands, etc., furnish examples of sliding friction, while the rolling of a cart-wheel on the road offers an example of rolling friction.

When motion has once set in, the force to be applied so as to maintain a steady mention is found to be slightly less than that necessary for starting the motion. *Sliding friction is* thus *slightly less than limiting friction.* If, when the equilibrium is limiting, the normal reaction and the frictional force be compounded into a resultant single force. The angle, which this resultant makes with the normal to the surface, is called the **angle of friction** and the single force is called the *resulant reaction.*

Friction plays an important part in our everyday life. Without friction on the ground, we would not have been able to walk; without friction we cannot fix nails, tie knots, climb trees, etc. Again, it becomes sometimes necessary to *increase* the friction, for example, when sand is thrown on uphill railway lines after rains or when violin bow is rubbed with resin. When a brake is applied, it comes in contact with the moving wheel and owing to the friction produced the car is stopped.

Also in many cases friction is to be minimised. Thus, in the moving parts of different machines, friction is reduced as much as possible by lubricating oil, graphite, vaseline, etc. Figure 136 shows the cross-section of the *collar bearing* in which the axle S of the revolving part is loosely fitted so as to be able to rotate. The space between the two is well lubricated. Roads are covered with tar macadam so as to present an even surface, while the rims of wheels are provided with rubber tyres and motor cars with pneumatic tyres. To diminish friction to the greatest possible extent, agate knife-edges are used in balances and jewel centres in watches. Again, since a rolling friction is less than sliding friction, heavy pieces of furniture are provided with wheels at the base, which can rotate in different vertical planes. In bicycles, etc., the sliding friction is replaced by rolling friction with *ball-bearing* arrangement, in which a number of hard steel balls are placed loosely in a metal case round the axle. Figure 137 shows a ball-bearing arrangement, in which the axle S is very free to move.

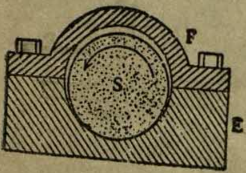


Fig. 136

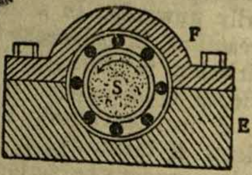


Fig. 137

103. Laws of Friction—When one body is at rest in contact with another, the friction between them is said to be **static**. When it is just on the point of sliding over the other, the friction is said to be *limiting*; and when one body is actually sliding over another, the friction it termed *kinetic* or *dynamic*.

Laws of Limiting Friction :

(1) The *direction* of the limiting friction is opposite to the direction in which the point of contact tends to move.

(2) The *magnitude* of the limiting friction always bears a constant ratio to the normal reaction.

(3) The *constant ratio* depends only on the material and the nature of the surfaces in contact but not on their extent or shape so long as the normal reaction remains the same.

Coefficient of Friction—The constant ratio of the limiting friction to the normal reaction acting across two surfaces in contact is called the **coefficient of friction** and is usually denoted by μ. Thus if F be the limiting friction and R the normal reaction, then

$$\mu = \frac{F}{R}, \text{ whence } F = \mu R \quad \ldots \quad \ldots \quad (103, 1)$$

The value of the coefficient of friction is different for different pairs of substances ; but it is always less than unity for any pair. When one body is sliding over another, the force that is required to maintain the motion, is slightly less than that at start. Let the force for the maintenance of motion be F_1. Then the coefficient of dynamic friction μ_1 is denoted by $\mu_1 = \frac{F_1}{R}$, whence $F_1 = \mu_1 R$....(103, 2)

Again, if the resultant reaction between the two surfaces is denoted by Q and the angle of friction by α, then

$$F = Q \sin \alpha \quad \text{and} \quad R = Q \cos \alpha,$$

$$\text{whence } \frac{F}{R} = \tan \alpha, \text{ or } \mu = \tan \alpha. \quad \ldots \quad (103, 3)$$

In other words, *the coefficient of friction is equal to the tangent of the angle of friction.*

Angle of Repose—Let a body of weight W be placed somewhere on a rough inclined plane AB (Fig. 138). At that region the weight W acts vertically downwards and the reaction R acts normally to the plane AB. Under the action of these two forces the body would tend to move down the inclined plane. But the motion is opposed by the frictional force between the plane and the body, such force acting opposite to the direction of motion. If the inclination of the plane to the horizontal is small, the force of friction may be equal to the resultant force tending to drag the body down the inclined plane and the body would remain at rest. If the angle of inclination is gradually increased, the sliding force increases ; so also does the force of friction. But ultimately the friction reaches its limiting value F, say. The angle, which the plane now makes with the horizontal, is called the **angle of repose.**

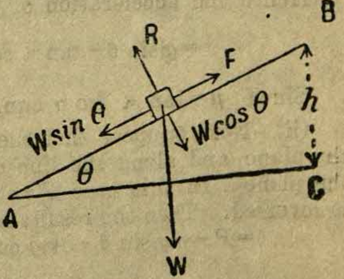


Fig. 138

From the centre of gravity of the body drop a normal to the horizontal line AC.

Let the force of friction be $=F$ and the angle of repose be $=\theta$.

Then the component of W parallel to the plane $= W \sin\theta$

and the component of the W normal to the plane $= W \cos\theta$.

$\therefore$ for equilibrium, $W \sin\theta = F$ and $W \cos\theta = R$;

or $\tan\theta = \dfrac{F}{R} = \mu = \tan\alpha$, whence $\theta = \alpha$.

Also $$\mu = \tan\theta = \frac{BC}{AC} = \frac{\text{height of the plane}}{\text{length of the base}} \qquad \ldots \quad (103, 4)$$

Hence the angle of friction is equal to angle of repose, which is, therefore, also called the *limiting angle* of friction. The coefficient of friction μ is obtained by dividing the height by the base of the inclined plane at the limiting angle.

Example :

A solid cube is placed on an inclined plane. If the angle of friction is 30°, calculate the coefficient of friction. [P. U.—1951]

Ans. Since the coefficient of friction is equal to the tangent of the angle of friction,

$$\mu = \tan 30^\circ = \frac{1}{\sqrt{3}} = 0{\cdot}5774.$$

104. Motion of a body on a Rough Inclined Plane—Let a body of mass m be placed upon a rough inclined plane of inclination θ to the horizontal when the equilibrium is limiting. Let μ be the coefficient of friction between the body and the plane. Let the normal reaction be R.

(i) Let the body be on the point of moving *down* the plane, so that the frictional force F acts *up* the plane. According to the preceding Article, we get for the reaction R and frictional force F,

$$R = mg\cos\theta \quad \text{and} \quad F = \mu R = \mu mg\cos\theta.$$

$\therefore$ the resultant force on the body down the plane

$$= mg\sin\theta - \mu mg\cos\theta = mg(\sin\theta - \mu\cos\theta). \qquad \ldots (104, 1)$$

Hence the acceleration of the body $= g(\sin\theta - \mu\cos\theta)$

$$= g(\sin\theta - \tan\alpha\cos\theta) = \frac{g\sin(\theta-\alpha)}{\cos\alpha}. \qquad \ldots (104, 2)$$

Since $\mu = \tan\alpha$ from eqn. (103, 3)

(ii) Next suppose that the force P, acting on the body parallel to the plane and along the line of the greatest slope, drags the body *up* the plane. In this case F acts down the plane and its algebraic sign is reversed. Then the resultant force acting on the body *up* the plane

$$= P - mg\sin\theta - \mu mg\cos\theta = P - mg(\sin\theta + \tan\alpha\cos\theta)$$

$$= P - mg\frac{\sin(\theta+\alpha)}{\cos\alpha}. \qquad \ldots (104, 3)$$

Hence the acceleration $= \dfrac{P}{m} - g\dfrac{\sin(\theta+\alpha)}{\cos\alpha}$. When, however, the body is just in equilibrium, there is no acceleration, so that

$$\frac{P}{m} - g.\frac{\sin(\theta+\alpha)}{\cos\alpha} = 0, \text{ or, } P - mg\sin\theta - \mu mg\cos\theta = 0,$$

whence $P = mg(\sin\theta + \mu\cos\theta)$. ...(104,4)

105. Determination of the Coefficient of Friction—The coefficient of friction between two soild surfaces can be determined by the following method :

(1) A horizontal table is made of one material. Place a block of another material (Fig. 139) on the table. One end of a light string is attached to a hook fixed to the block, while the other end passes over a smooth pulley fixed at the end of the table and carries a scale pan. A known weight is now placed on the block. Weights are then placed on the scale pan, till the block is just on the point of motion. This condition can be ascertained by tapping the plane gently. Let the total weight of the block and the load be equal to W and the total weight on the scale pan be $= w$. Then the force of limiting friction $= w$ and the normal reaction $R = W$.

Hence the Coefficient of Friction is given by $\mu = \frac{F}{R} = \frac{w}{W}$.

Repeat the experiment with different weights on the slab and take the mean value of μ.

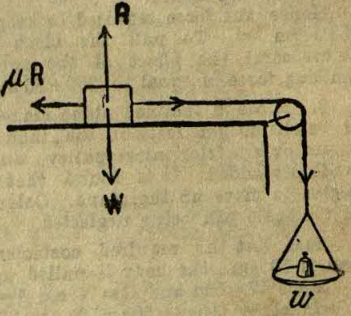


Fig. 139

(2) An inclined plane is made of one material and a rectangular slab of another material is placed on it. The angle of inclination of the plane is gradually increased, till the slab is just on the point of sliding down the plane.

Since the slab is in limiting equilibrium, the tangent of the angle of inclination of the plane to the horizontal is equal to the coefficient of friction. Keeping the plane fixed at the position, the height and length of the base are measured. The tangent of the angle of inclination is thus found, whence the coefficient of friction is obtained.

(*Vide* J. Chatterjee's *Intermediate Practical Physics*)

The following Table supplies the Coefficients of Limiting Frictions between common substances :

Coefficients of Limiting Friction

Substances	Coefficients	Substances	Coefficients
Wood on wood	0·3-0·5	Stone on stone	0·7
Metal on wood	0·1-0·2	Metal on leather	0·5
Metal on metal	0·2-0·5	Metal on stone	0·55-0·65

Examples :

1. A metal block of mass 2000 gm. slides from rest down a rough inclined plane, the slope being 1 in 5. Find the resultant force down the plane, the coefficient of friction being 0·15 ($g = 980$ cm. per sec. per sec.)

Ans, The resultant force down the plane $=mg \sin \sphericalangle - F = mg \sin \sphericalangle - \mu R$
$= mg \sin \sphericalangle - \mu mg \cos \sphericalangle = mg (\sin \sphericalangle - \mu \cos \sphericalangle)$.

Here $\sin \sphericalangle = \frac{1}{5}$, $\left(\text{so that } \cos \sphericalangle = \frac{2\sqrt{6}}{5}\right)$, $\mu = 0{\cdot}15$ and $m = 2000$ gm.

$\therefore$ the required resultant force $= 2000 \times 980 \times \left(\frac{1}{5} - 0{\cdot}15 \times \frac{2\sqrt{6}}{5}\right)$ dynes
$= 103880$ dynes (approx.).

2. A metal block is placed on a metal plane inclined at 30° to the horizontal. the mass of the block be 500 gm. and the coefficient of friction be 0 2, what rce must be supplied (*a*) to keep the body from sliding down the plane, (*b*) to ove it up plane ? [Utkal U.—1954]

Ans. The component of the weight of the block moving it down the plane $mg \sin \theta = (500 \times g \times {\cdot}5)$ dynes $= 250g$ dynes.

Also the normal reaction of the plane $= mg \cos \theta = 500 \times g \times \frac{\sqrt{3}}{2}$ dynes
$= 250\sqrt{3}g$ dynes.

The frictional force is thus equal to $\mu mg \cos \theta = {\cdot}2 \times 250\sqrt{3}g$ dynes $= 50\sqrt{3}g$ nes $= 86{\cdot}5 \times g$ dynes.

Hence, the force required to keep the block in position $= (250 - 86{\cdot}5)g$ dynes $=$ 3·5 gm. wt. To pull the block up the inclined plane, the force required is overcome the effect of the two forces $mg \sin \theta$ and $\mu mg \cos \theta$. Thus the lting force is equal to $(250 + 86{\cdot}5)g$ dynes $= 336{\cdot}5$ gm. wt.

3. A rough inclined plane makes an angle of 30° with the horizontal. A load weight 50 gm. rests on the inclined plane and is attached to a string which sses over a frictionless pulley, and supports a scale-pan in which weights are dually added. It is found that when the weight is 50 gm., the weight just gins to move up the plane. Calculate the coefficient of friction, the weight the scale pan being neglected. (Dac. U.—1958)

Ans. Let the required coefficient of friction be μ. As the weight in the n is 50 gm., the body is pulled up along the plane with a force $50g$ dynes. Since $W = 50g$ and $R = W \cos \theta = 50g \cos 30°$ and $F = \mu R = 50\mu g \cos 30°$. Hence by (104,4) $50g = 50g \sin 30° + 50\mu g \cos 30°$,

or, $\frac{1}{2} + \mu \frac{\sqrt{3}}{2} = 1$, whence the required coefficient of friction $= \frac{1}{\sqrt{3}}$.

EXERCISES ON CHAPTER VII

	Reference
1. What is Hooke's Law ? Explain the terms 'stress' and ain'. In what units are they expressed ? (Del. U.—1962 ; P. U.—1952 ; U. P. B.—1953 ; Raj. U.—1964)	Art. 98
2. A wire 0·4 cm. in diameter is loaded with 25 kgm. weight. ength of 100 cm. is found to be extended to 102 cm. Calculate Young's Modulus of the material of the wire. (C. U.—1953) *Ans.* $\cdot 9 \times 10^{10}$ dynes/cm.^{2}.	Art. 98
3. Define Young's Modulus of Elasticity. How do you ermine Young's Modulus for a steel wire ? (C. U.—1953 ; l. U.—1951 ; U. P. B.—1942 ; Raj. U.—1951 ; Dac. U.—1942 ; Del. H. S.—1960 ; Del. U.—1949, '62 ; Vis. U.—1964)	Arts. 99 & 100
4. A copper wire, 2 metres in length and ·5 mm. in diameter, tretched by a weight of 3 kgm. and the elongation of the wire ·38 mm. Find Young's Modulus of the wire. *Ans.* 2×10^{12} dynes/cm.^{2}.	Art. 99
5. A steel wire of length 200 cm. and of diameter 0·025 cm. uspended from an unyielding support and loaded with 500 gm. elongation of 0·231 cm. is observed. Calculate Young's Modulus steel. ($g = 981$) (Vis. U.—1954) *Ans.* 2×10^{12} dynes/cm.^{2}.	Art. 99

Reference
Art. 99 6. Calculate the work done is stretching a wire assuming Hooke's Law. A copper wire, 2 metres long and 3 sq. mm. cross-sectional area, is suspended vertically and a load of 5 kilograms is attached to the lower end. Calculate the work done is stretching the wire, if Young's Modulus for copper is $1\cdot2\times10^{12}$ dynes/cm.^{2} and $g=981$ cm./sec.^{2}. (Poona U.–1960)

Ans. $6\cdot7\times10^4$ ergs.

7. Calculate the shearing stress and strain produced in a rod of diameter 1 inch subject to a shearing force of 2 tons weight; $n=$ 2100 tons weight per sq. in.

Ans. 2·55 tons weight; $1\cdot2\times10^{-3}$ radian.

Art. 100 8. Define modulus of elasticity. Obtain an expression for the isothermal elasticity of a gas obeying Boyle's Law.

What is the isothermal elasticity of air when 1 gm. of air at 27°C occupies 100 c. c.? One litre of air at N. T. P. weighs 1·293 gm. (Nag. U.—1964)

Ans. $8\cdot6+10^6$ dynes/cm.^{2}.

Art. 103 9. Show that once a body is just ready to slide down an inclined plane, the tangent of the angle of the inclination of the plane is equal to the coefficient of friction.
(Del. H. S.—1962; *Cf.* Nag. U.—1952; Dac. U.–1963)

Art. 104 10. A body of mass 2 lbs. rests in limiting equilibrium on a rough plane whose slope is 45°. If the plane is raised to a slope of 60°, what force along the plane will just support the body? ($g=32$ ft./sec.^{2}) (Poona U.—1954)

Ans. ·723 lbs. wt.

Art. 104 11. What is the acceleration of a block sliding down a 30° slope when the coefficient of friction is 0·25? (Poona U.—1954)

Ans. ·2835 g.

Art. 104 12. A block of stone of mass 4 kgm. is just moving up a slope 1 in 20 by a force of 2×10^6 dynes applied parallel to the inclined plane. What is the value of the coefficient of friction between the surface in contact?

Ans. 0·46.

Art. 104 13. A body of mass 5 kilograms is being pulled up along a rough inclined plane with an acceleration of 5 cm./sec.^{2} by means of a weightless string running parallel to the plane. If the inclination of the plane be 45° to the horizontal and coefficient of friction between the body and the plane be 0·3, find the pull in the string. (Pat. U.—1962)

Ans. $45\cdot285\times10^5$ dynes.

Art. 104 14. A body of mass 4 lbs. rests in limiting equilibrium on a rough plane whose slope is 30°. The plane being raised to a slope of 60°, find the forces required along the plane when the body (*i*) is on the point of sliding down and (*ii*) is on the point of moving up the plane. (Nag. U.—1959)

Ans. (*i*) 2·31 lbs.; (*ii*) 4·62 lbs. wt.

Art. 103 15. Explain fully the terms 'Friction' and 'Limiting Friction'. Justify the statement that friction as a necessity and an evil. Give some familiar methods for reducing and increasing friction. (P. U.—1952)

Art. 103 16. A ladder 30 ft. long rests with one end against a smooth vertical wall and with the other on the ground, which is rough, the coefficient of friction being 0·5. Find how high a man, whose weight is four times that of the ladder, can ascend before it begins to slip, the foot of the ladder being 6 ft. from the wall.

Ans. 88 ft. Hence the man may ascend the full length without a chance of slip of the ladder.

HYDROSTATICS

CHAPTER VIII

PRESSURE IN LIQUIDS

106. Hydrostatics—It is that branch of Physics which deals with the properties of fluids which are at static equilibrium under the action of external forces and pressures exerted within their own masses or on the sides on the containing vessel.

Fluid—A fluid is a substance which has no shape of its own and which takes up the shape of the containing vessel. Liquids and gases fall under the category of fluids. A given mass of liquid has got a definite volume at a given temperature under ordinary pressures but a gas possesses a volume dependent on temperature and pressure. Maintenance of a definite shape, as is the characteristics with solids, is due to rigidity as illustrated in Art 95. Every solid possesses elasticity of shear. But from the fact that a liquid or a gas cannot maintain a definite shape we understand that it possesses no shearing elasticity, although the *compressibility* of a liquid is extremely small, while that of a gas is considerable.

Fluid Pressure—If a number of holes be made at the side and the bottom of a vessel containing a liquid, the liquid is found to come out through all the openings. If any body wants to stop the outflow of the liquid by covering the holes with suitable plates or with fingers, he will feel that some force is to be applied from outside on each plate to keep it in position. This shows that the liquid inside exerts some force on that part of the containing vessel with which it is in contact. It is also found that this force always acts at right angles to the wall of the vessel at that point. The force thus exerted by a liquid on a surface is called the *thrust* of the liquid (Fig. 140). It is due to this thrust (shown by arrow-heads) that the liquid forces its way through an opening of the containing vessel. If the wall of the containing vessel is strong enough to withstand this thrust from point to point, the liquid remains steady within it. Hence, the wall exerts a reaction to fluid pressure.

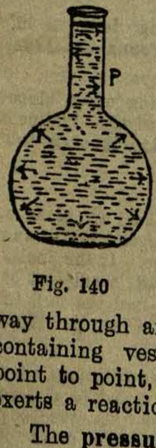


Fig. 140

The **pressure of** a fluid at rest on any surface is the normal force or thrust exerted by the fluid on a unit area of that surface. The pressure is said to be *uniform*, when the thrusts on *any* two equal areas of the surface are equal. If P be the total thrust exerted by the fluid uniformly on a surface of area A, the pressure p on it is given by

$$p=\frac{P}{A} \qquad \dots \quad (106,1)$$

The pressure exerted by a liquid on the wall of a vessel may be uniform or may vary from point to point. If the pressure is not uniform, the expression P/A gives the *average pressure* over the given surface.

Pressure at a Point—The *pressure at a point* due to a fluid is defined to be the normal force exerted by the fluid *on an unit area containing that point.* In the C.G.S. system the pressure is expressed in dynes or in grammes-weight per square centimetre, while in the F.P.S. system it is expressed in poundals per square foot or pounds-weight per square inch. Suppose, for instance, that a hole of 6 sq. cm. in area is made at the side of a vessel containing a liquid. A plate just fitting the hole is pressed against to stop the outflow of water. If a force of 48 gm. wt. is *just* required to check the outflow, the average pressure is 8 gm. wt. per sq. cm. or 8g dnyes per sq. cm.

107. Pressure in a Liquid—That a liquid exerts pressure on the side of the containing vessel may be shown by making a hole anywhere in its wall when the liquid would be forced out. There is also some pressure at any point *within the mass of liquid.* This can be shown by the following experiment :

Take a thistle funnel F and stretch a thin sheet of rubber over its mouth. Connect the funnel to a glass tube G (Fig. 141) of a narrow bore, by means of a length of rubber tubing T. A drop (I) of a coloured liquid, introduced into the glass tube, acts as an index. The glass tube is supported horizontally and a scale S is placed along its side. If the rubber membrane is now pressed with finger, the index moves forward and the larger is the pressure, the more does the index move away. When the pressure is withdrawn, the drop comes back to the initial position.

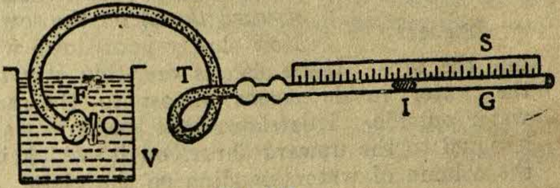


Fig. 141

Now take a deep vessel V containing water and introduce the funnel into the liquid ; the index is seen to move outward. Move the funnel to greater and greater depths and note that the index moves out more and more from its initial position. This proves that pressure continuously increases with the depth of the liquid. Now keep the funnel at a certain depth (say, at O) and mark the position of the index on the scale. Turn the mouth of the funnel slowly to different directions about this point. Note that the index does not move with the rotation of the funnel. Therefore the amount of pressure around a certain point within the liquid is equal. Now displace the funnel to some other point in the same horizontal plane and turn it to different directions as before. Note that the position of the index remains unchanged. Liquid pressure at the same horizontal level is, therefore, the same. This experiment was successfully carried out by Blaise Pascal (1623—1662).

Thus about a mass of liquid at rest the experiment gives the following facts—

(*i*) *A liquid exerts pressure at every point within its mass and in all directions.*

(*ii*) *The pressure at any point in the liquid or at different points on the same horizontal plane is the same in all directions.*

(*iii*) *The pressure at a point within a liquid is directly proportional to its depth.*

108. **Vertical Upward Pressure**—If an empty test tube is pushed down into water with the closed end downwards, the tube on being released springs up due to the upward force exerted by the liquid at its bottom. The following experiment further demonstrates the pressure of vertical upward pressure.

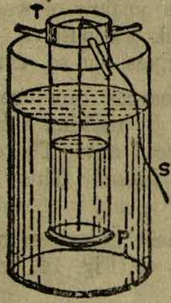


Fig. 142

Take a wide glass tube T with one end evenly grinded. Hold a thin plate P of glass (*or metal*) against the grinded end with a string attached to it (Fig. 142). Immerse the whole thing vertically to some depth in a vessel of water and release the string. The plate is found to remain in position as if pushed from below. This is due to the *upward thrust* on the lower surface of the plate. Now slowly pour down water (preferably coloured) into the tube. If the plate is very thin, it will drop when the level on the water inside comes almost to the same height as that of the water outside. This shows that the downward thrust on the plate is equal to the upward thrust on it, which is equal to the weight of the column of water standing on the plate.

Lateral Pressure—A cylindrical vessel of thin tin sheet fitted with a stop-cock T on one side near the bottom is filled with water. On being placed on a suitable piece of cork it is arranged to float on water (Fig. 143). When the stop-cock is opened water flows out and the cylinder is seen to float away slowly in a direction opposite to that of the issuing water.

The explanation lies in the fact that when the stop-cock remains closed, the lateral pressure exerted by water on the stop-cock is equal and opposite to the reactive pressure exerted by the stop-cock on water and consequently the vessel shows the tendency of motion, When the tap is opened, the pressure exerted by the stop-cock is released due to which water flows in a stream. The back push of the stream makes the vessel move in that direction.

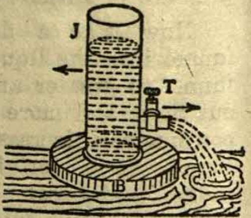


Fig. 143

The existence of lateral pressure can be demonstrated by the *hydraulic tourniquet* or Barkers's Mill apparatus. It consists of a vessel of water free to rotate about a vertical axis, having (near the bottom) horizontal discharging tubes, all being bent at the ends in the same way. As water rushes out of the spouts of the bent tubes, the lateral reaction of the stream at the bends of the tubes opposite the openings causes the whole apparatus to rotate in a direction opposite to that of the issuing water.

109. Magnitude of the Pressure within a Liquid—Let a surface of area A (Fig. 144) be immersed in a liquid at a depth h below the free surface of the liquid and kept there in a horizontal position. Imagine vertical lines to be drawn from every point of the boundary line of this surface till they reach the surface of the liquid. Such vertical lines envelope a column which is filled with the liquid. This column of liquid is clearly standing on the surface of area A which serves as its base. The weight of this liquid gives the thrust on the base area. If the density of the liquid be ρ and the acceleration due to gravity at the place be g, the volume V of the water column on the area is Ah and its weight is equal to $Ah\rho g$. Therefore, the total downward thrust T upon the surface is given by

$$T = Ah\rho g. \quad \ldots \quad \ldots \quad \ldots \quad \ldots (109,1)$$

The total pressure on a surface immersed *horizontally* in a liquid is equal to the weight of the column of the liquid, whose base is the given surface and whose height is equal to the depth of the surface below the free suface of the liquid.

If P denotes the downward pressure at *any* point on the surface, then remembering that pressure is the force or thrust per units area,

we get $$P = \frac{Ah\rho g}{A} = h\rho g. \quad \ldots \quad \ldots \quad \ldots (109,2)$$

This is also the magnitude of the pressure in the upward as well as in any other direction at the point. The above relation shows that the pressure at any point within the liquid is proportional to (*i*) the depth of the point, (*ii*) the density of the liquid and (*iii*) the acceleration due to gravity at the place of observation. The air above the free surface of the liquid also exerts the force of its weight on the liquid surface. The weight of air acting per unit area is called the *atmospheric pressure*, and if this pressure per unit area be π, the total pressure at a point at a depth h below the free surface of the liquid is equal to $\pi + g\rho h$.

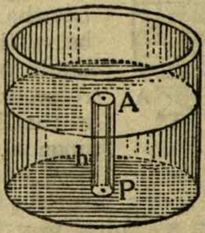


Fig. 144

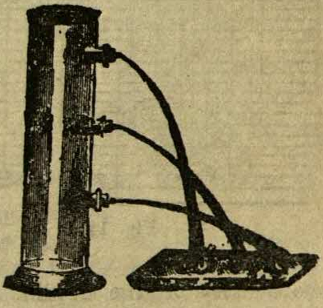


Fig. 145

A tall jar with a number of openings on the side is taken (Fig. 145). When the jar is filled with water, the water is seen to come out in streams from different holes ; but the flow of a stream increases with the depth of a hole from the free surface of water in the jar. This indicates that within the liquid the pressure increases with the depth. Since pressure is force per unit area, the dimension of pressure is force ÷ area or $MLT^{-2} \div L^{2} = ML^{-1}T^{-2}$.

***110. Whole Pressure or Resultant Thrust**—It has already been stated (Art. 106) that the *effective* pressure exerted by a fluid on the walls of the containing vessel is always at right angles to the surface of contact. This is equally true, if there be any par ition wall within the liquid. In the experiment to demonstrate the existence of pressure in all directions around a point within a liquid (Art. 107), a thistle funnel, covered with a piece of rubber membrane on the side, was taken down within the liquid to a certain depth and was slowly rotated in different directions. The position in the index was found to be fixed; this shows that the force exerted by water on the rubber membrane was the same at a given depth, being always *at right angles to the surface* of the membrane. This force is called *the whole pressure* on the membrane, which may defined thus :

If, for every small element of area of a material surface immersed in fluid, the force perpendicular to the surface of this small element be found, the sum of all such forces taken over the whole area of the surface is called the **whole pressure** *or* resultant thrust *of the fluid upon the given surface.*

Consider any plane surface of area A immersed in a liquid in any position, horizontal or inclined [Fig. 146(*a*)]. Suppose that the surface is composed of small elementary areas snch as a_1, a_2, a_3,...a_n. Then the vertical columns of the liquid, as shown in Figure 146(b), are standing upon the elementary areas, each contributing to the whole pressure on the surface. Let the depths of these surface below the free surface of the liquid be h_1, h_2, h_3, ... , h_n respectively. Then the thrust of the liquid column standing upon a_1 at a depth h_1 perpendicular to the element of the surface is $g\rho a_1 h_1$, where ρ is the density of the liquid. In a similar way, it may be shown that the thrusts on other elements are $g\rho a_2 h_2$, $g\rho a_3 h_3$, etc. Hence, the resultant thrust is the sum of the individual thrusts. Thus :

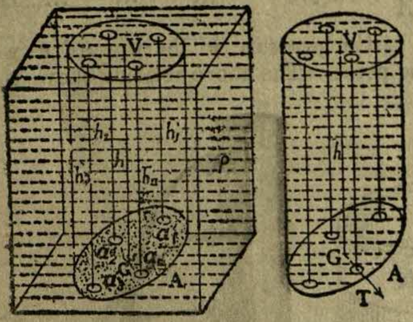


Fig. 146

(*a*) (*b*)

Whole pressure $= g\rho a_1 h_1 + g\rho a_2 h_2 + \ldots\ldots + g\rho a_n h_n$
$= g\rho(a_1 h_1 + a_2 h_2 + \ldots\ldots + a_n h_n)$.

If h denotes the depth of the centre of gravity G of the surface A below the free surface of the liquid, it can be shown from the principles of Statics that

$Ah = a_1 h_1 + a_2 h_2 + \ldots\ldots a_n h_n$.

Hence, the **whole pressure** $T = Ahg\rho$. ...(110,1)

In the preceding article it has been shown that the thrust on a horizontal surface of area A at a depth h below the surface of a liquid equals $Ahg\rho$. Hence the magnitude of the whole pressure is independent of the inclination of the surface, provided that the depth of the centre of gravity of the surface below the free surface of the liquid remains same.

Centre of Pressure—If a surface be immersed in a liquid, the point, at which the resultant thrust on the surface acts, is termed the *centre of pressure* of the surface. When the surface is kept in a horizontal position within a liquid, the point of the surface, where a vertical line dropped from the centre of gravity of the liquid column meets the surface, is the centre of pressure.

111. Hydrostatic Paradox—Since the total pressure on an area A immersed horizontally at a depth h in a liquid of density ρ is given by $Ahg\rho$, the thrust at the bottom of a vessel due to the liquid within it depends only on the area of the base and the height of the liquid column but *neither* on the total quantity of the liquid contained *nor* on the shape of the vessel. This can be demonstrated by **Masson's experiment**, as described below.

A, B, C and D are glass vessels, known as *Pascal's vases* each being open at both ends. They are of different shapes and sizes ; but the base of each is of equal sectional area, so that each one can be screwed to the same socket of the platform F (Fig. 147). A disc R, attached to one arm of the lever L, can press water-tight against the bottom of each vessel which can be screwed to the platform. At the other end of the lever is attached a scale pan S, on which weights can be placed. A pointer P slides along a vertical stand attached to the platform and can be fixed at *any* height by a clamp.

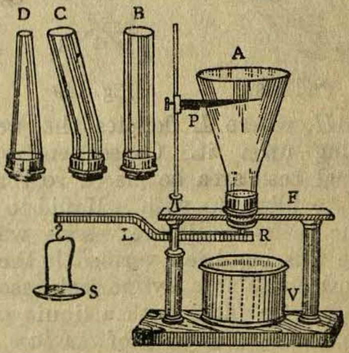


Fig. 147—Thrust on the Base

Screw one of the vessels (say, A) to the platform and put suitable weights on the scale plan. Now slowly pour water into the vessel till the disc just gets loose and allows some water to fall down. Mark the height of the water by the pointer P. Remove the vessel A and repeat the experiment with another vessel. It will be found that the water in it begins to trickle down when the height of water in this vessel is the same as in the former one. The experiment shows that *the thrust on the base depends on its area and on the height of the liquid column* but *not* upon the shape of the vessel. At first sight this appears inconsistent. That is the reason why the experiment is known as the **hydrostatic paradox**.

Pascal (1623-1662) demonstrated hydrostatic paradox by a very simple experiment. A long narrow tube about 30 ft. in length, was fixed vertically to the top of a hollow stout cask. Water was gradually poured into the tube till the pressure became so high as to burst the cask. Although the quantity of water added was very small, the thrust at the bottom of the cask became equal to the weight of a column of water whose base was that of the cask and whose height was equal to that of the water column from the bottom of the cask to the level of water in the tube.

***112. Thrust at the Base**—The resultant thrust on any surface immersed within a liquid depends upon the surface area, the depth of the centre of gravity of the surface below the free surface of the liquid, the density of the liquid and the acceleration due to gravity at the place of observation. Hence, if the position, shape and area of the base of the vessel containing a liquid are known, the whole pressure at its base may be found from an observation of the height of the liquid level within the vessel. For a *horizontal base*, the whole pressure upon it is $g\rho AH$, where H denotes the verticsl height of the liquid surface standing upon it. Consequently, if vessels of different shapes but of equal base area be placed so that their bases are horizontal and if all be filled up with a liquid to an *equal level*, the resultant thrust at the bases of all the vessels would be equal. In the preceding article this has been verified in the experiment with Pascal's vases.

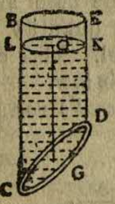


Fig. 148

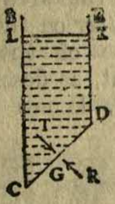


Fig. 149

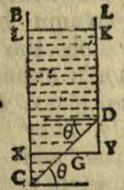


Fig. 150

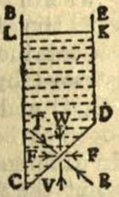


Fig. 151

Now consider a cylindrical base (Fig. 148). Up to a level LK let the vessel be filled with a liquid of density ρ. Let G be the position of the *centre of gravity of the base.* Denote by H the height of the free surface of the liquid above G. If the area of the base be A, the whole pressure (or thrust) T of the liquid column is $g\rho AH$ acting perpendicularly to the surface area CD, as shown in the sectional diagram (Fig. 149). If the liquid within the vessel be steady, the base offers a reaction R which is equal and opposite to T.

Imagine a horizontal plane surface XY (Fig. 150) to pass through the point G of the base of the vessel extending up to the walls. Then the volume of the liquid within the vessel is equal to the volume KLXY, since the volume XGC = the volume YGD. Now the surface XY = Area of the base CD × cos $\theta = A \cos\theta$. Therefore the volume of the liquid = $HA \cos\theta$ and hence its weight W acting vertically downwards is $g\rho AH \cos\theta$.

Again, the base reaction R may be resolved into a vertical component R cos θ and a horizontal component R sin θ (Fig. 151). But since R is equal to $g\rho AH$, we have on resolving the force system along the vertical and horizontal directions :

$$\text{R} \cos\theta = g\rho.AH \cos\theta \text{ and R} \sin\theta = g\rho.AH \sin\theta,$$

$$i.e., \text{ R} \cos\theta = \text{W and R} \sin\theta = \text{F, say.}$$

Thus the vertical component of the base reaction supports the weight of the liquid, while the horizontal component prevents the base from moving aside. Similarly, the weight W of the liquid together with the lateral thrust F of the liquid on the base of the vessel produces the resultant thrust T. If the shape of the vessel BCDE be as represented in Fig. 152, (the base CD being horizontal) and if the liquid is filled to a height H, the reaction R of the inclined part EK has got a vertical component, say V. The resultant thrust at the base of the vessel is the sum of the liquid thrust and the vertical component V. In this case also the thrust at the base exceeds the weight of the liquid.

It is now easy to understand why in Pascal's vases although masses of the liquid in different vessels are different, equal upward thrust maintains the same liquid height in different vessels.

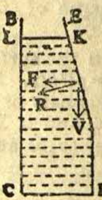


Fig. 152

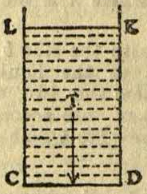


Fig. 153

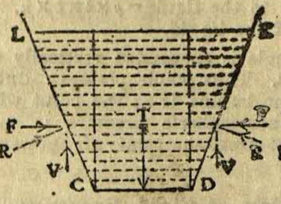


Fig. 154

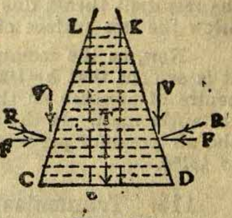


Fig. 155

Consider the uniform cylinder LCDK with vertical walls (Fig. 153). The reaction of the walls is everywhere horizontal and has no vertical component. Hence the thrust at the base is only due to the weight of the liquid being proportional to the height of the column LC. When the vessel is wider at the open end (Fig. 154), consider a vertical column of liquid standing upon the base CD. The thrust at the base is due to the weight of this column only. The two other columns on two sides of the vertical column are being supported by the vertical component of the wall reactions and have nothing to do with the thrust on the base. For the vessel with a tapering end (Fig. 155), consider the column LCDK. Clearly the thrust on the part CD of the base is due to the column of water standing upon it. The thrust on the parts LC and DK are partly due to the weight of water standing upon them and partly due to the vertical components of the wall reaction. The parts LC and KD in this figure are inverse of those of the preceding figure and consequently the vertical component of the wall reaction on LC or KD is equal to the weight of the column LCP or KDQ. Hence the thrust at CD is due to a uniform column of liquid with CD as base. If now the base areas be equal, the thrusts at the base of all the vessels due to liquids standing at equal heights are equal, although the actual quantity of liquid differs from vessel to vessel.

Examples :

1. The pressure at the bottom of a well is 9 times that at a depth of 2 ft. What is the depth of water in the well, if the pressure of the atmosphere is equivalent to 30 ft. of water? (Utkal U.—1951)

Ans. Let the depth of water be h ft, The pressure at the bottom of the well$=g\rho(h+30)$, where ρ stands for density of water in F. P. S. units. The pressure at a depth of 2 ft. is$=g\rho\times(30+2)$. Now by hypothesis,

$$g\rho(h+30)=g\rho\times 2(30+2),$$ whence the desired height $h=34$ ft.

2. A cube, each of whose sides is 20 cm. is suspended in a liquid of density 1·2 with its sides vertical and its upper surface at a depth of 30 cm. below the free surface of a liquid. Find the thrust in grammes weight on each of its faces. (Neglect atmospheric pressure.)

Ans. The upper surface of the cube is at a depth of 30 cm. and so its lower surface is at a depth of 50 cm. The centre of gravity of each of the vertical face is 40 cm. below the suriace of the liquid. Therefore,

the thrust on the upper surface $= (20 \times 20 \times 30 \times 1{\cdot}2)$ gm.-wt. $= 14400$ gm.-wt.
the thrust on lower surface $= (20 \times 20 \times 50 \times 1{\cdot}2)$ gm.-wt. $= 24000$ gm.-wt.
and thrust on each vertical face $= (20 \times 20 \times 40 \times 1{\cdot}2)$ gm.-wt. $= 19200$ gm.-wt.

2. A rectangle of length l and breadth b is immersed vertically in water with the side b on the surface of water. Calculate the thrust and the centre of pressure on the rectangle. (Mad. U.—1950)

Ans. The area of the rectangle $= lb$. If the density of water be ρ, we find that the resultant thrust due to the liquid $= \rho \times l \times b \times h$, where h is the depth of C. G. below the free surface of the liquid.

Now, as the rectangle is suspended vertically with the side b on the surface, it is evident that $h = l/2$. Thus whole pressure due to the liquid $= \rho l^2 b/2$. The centre of pressure on the lamina is at the point where the two diagonals of the rectangle meet. If the atmospheric pressure is taken into account, and if P be the air pressure per unit area, the total air pressure $= Plb$ which is to be added to $\frac{1}{2}\rho l^2 b$.

113. Transmission of Fluid Pressure—A change of pressure at *any* point of a liquid or a gas causes an equal change in pressure at all other points within it. The transmission of fluid pressure, as given by **Pascal's Law**, may be stated as follows:

The pressure exerted anywhere in a mass of a confined fluid is transmitted by the fluid in all directions, so as to act with undiminished force per unit area and at right angles to the surface exposed to the fluid.

To verify Pascal's Law in the case of a liquid we take a vessel with a number of openings of different cross-sectional areas a, b, c, d, etc., (Fig. 156), all of which are fitted with water-tight movable pistons. The vessel is completely filled with water and the pistons kept in position as shown in the figure. If any one of the pistons (say, the topmost one), be now pressed inwards, all other pistons will be found to move outwards, which shows that *pressure is transmitted in all directions.*

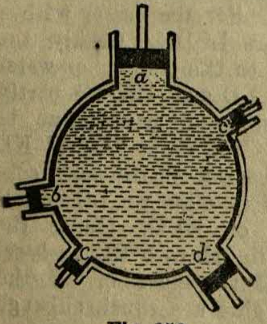


Fig. 156

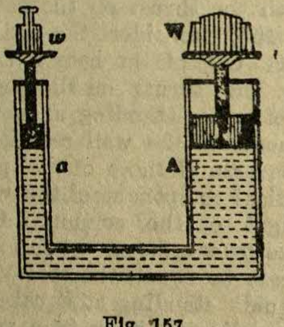


Fig. 157

Again, if a force F_1 be applied on the piston whose area of cross-section is a, the pressure p applied per unit area is F_1/a. It will be found that in order to maintain the positions of the pistons on areas of cross-sections b, c, d......., additional forces F_2, F_3, F_4.......

are to be respectively applied to them, so that F_2/b, F_3/c, F_4/d,...are all equal. In other words, the pressure on each piston is the same. This shows that the pressure is transmitted *undiminished* in all directions.

To verify Pascal's Law another simpler apparatus may be used. Two cylinders of sectional areas A and a respectively are connected with a pipe, the cross-section A being larger than a (Fig. 157). They are filled with water to some height and fitted with water-tight pistons P and p. Assume the pistons to be of negligible masses.

On the smaller piston p, let a weight w be placed. Then the pressure applied on this piston is given by $p = w/a$. This pressure is transmitted through the liquid to the larger cylinder and the piston P is pushed upwards. It is found that to maintain equilibrium, *i.e.*, to keep P in position, a weight W is to be placed on this piston, so that

$$W = \frac{w}{a}A. \text{ Hence } \frac{W}{w} = \frac{A}{a}, \text{ or, } \frac{W}{A} = \frac{w}{a}. \qquad \text{...(113,1)}$$

This shows that the force transmitted per unit area is equal.

For example, if the cross-section a and A be 1 sq. cm. and 50 sq. cm. respectively and if a mass of 1 kgm. be placed on the piston p, a mass of 50 kgm. is to be placed on P for equilibrium. This illustrates the principle of *multiplication of forces* by transmission of fluid pressure.

But the working of the apparatus does *not violate the principle of conservation of energy*. If L be the distance through which W descends, the work done by O is WL. If w ascends through l, then work done upon w is wl. For coming down through L, the liquid that is transferred to the narrow limb is LA which must be equal to la, since the liquid is incompressible. Therefore $LA = la$.

or, $\frac{A}{a} = \frac{l}{L} = \frac{W}{w}$ from equ. (113,1)

or, $WL = wl$. ...(113, 2)

So work done and work received are equal and conservation of the principle energy is verified.

Examples :

A bottle of length 40 cm. full of water with a bottom of 30 sq.cm. area has a cork fitted to it. The cork has 1 sq. cm. as its area of cross-section and is pressed in with a force of 40 gm. wt. What is the total thrust on the bottom?
(Del. H. S.—1952)

Ans. The total thrust at the bottom of the bottle is the sum of the thrusts due to water within it and the thrust due to the cork. The thrust due of atmospheric pressure is cancelled, because it acts at the bottom of the vessel vertically downwards as also upwards on the cork.

The pressure due to the cork is transmitted undiminished to the bottom. The pressure exerted by the cork = 40 gm. wt. per cm.^{2}. Hence the thrust due to this at the bottom = (pressure) × (area) = (40 × 30) gm.-wt. = 1200 gm.-wt.

If the vertical height of the bottle be 40 cm., the pressure at the bottom due to water = 40 gm. wt. per cm.^{2}. Hence the thrust due to water = 30 × 40 gm.-wt.

$\therefore$ the total thrust = (1200 + 1200) = 2400 gm.-wt.

114. Hydrostatic Bellows—The principle of transmission of fluid pressure is also illustrated by the *Hydrostatic Bellows* designed by Pascal. It consists of a long glass tube C (Fig. 158) attached to stout India-rubber or leather bellows B. The bladder is placed on a table while the tube is held vertically. The bladder and part of the tube are filled with water. A piece of light board may be placed on the bladder. A heavy weight W placed on the board, can be supported simply by the weight of the column of water in the tube. The pressure exerted by the column of water in the tube is transmitted undiminished into the bladder and produces on the entire upper surface a vertical upward thrust proportional to its area. This thrust supports the weight placed on the board. The board is used for uniform distribution of the pressure exerted by the weight placed on the bladder.

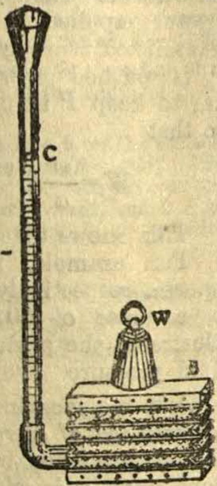


Fig. 158

Hydraulic Press—The principle of multiplication of forces by transmission of fluid pressure is utilised in the working of a Hydraulic Press. This is used for exerting enormous force, as in compressing bales of paper, jute, cotton or cloth, in extracting oil from seed, testing the strength of iron beams, in punching holes through metal plates, etc.

It consists of two iron cylinders C and T communicating with each other through a stout metal pipe (Fig. 159), the cross-section of C being many times larger than that of T. In each cylinder there is a solid rod R or P acting as the water-tight piston. The piston in T is worked by a lever L, having the fulcrum at the end. The piston in C is a large iron 'ram' R which carries a platform at its top. Above the platform there is a fixed plate supported on iron pillars S. The object to be pressed is placed on the platform on R. A reservoir filled with water (or any other liquid) is in communication with the cylinder T through a pipe fitted with a valve V which allows water to flow only in one direction from the reservoir to the cylinder. There is another valve V fitted to a side tube to allow water to pass only in one direction from the smaller cylinder to the bigger one when the piston P is worked. The above improvement was devised by an engineer Bramah (1701-1773) and the press is, therefore, sometimes called *Bramah's Press.*

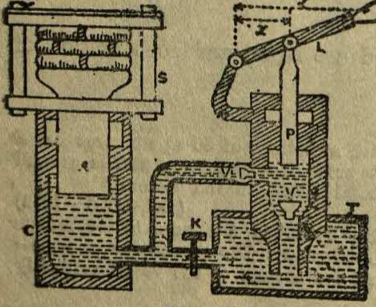


Fig. 159

As the piston P is raised by the lever L, the value within the side tube is closed and the inlet valve becomes open, whereby water from the reservoir enters the cylinder T below the piston. If now the piston is lowered, the pressure inside increases, due to which the valve within the side tube is forced open and the inlet valve is closed. Consequently water passes into the cylinder C. Thus during each upward stroke of the piston a quantity of water is drawn into T, and during the next downward stroke this water is pushed into the cylinder C. The pressure applied on the piston is thus transmitted to the water in C and produces on the ram R an upward force which is many times greater than the force applied at the piston P.

When the ram R is to be lowered, the tap K attached to the connecting pipe is opened, and water due to the weight of the ram is forced back into the reservoir. This tap must be closed before the press is operated. By closing the tap K the ram may be kept under a pressure for a considerable period. Let the areas of cross-section of T and C be α and β respectively. To the end of the lever L apply a force F_1. If the thrust generated on the piston P be F_2, we find from Statics in the Chapter on lever that:

$$\text{Mechanical Advantage } m=\frac{F_2}{F_1}=\frac{\text{power arm}}{\text{resistance arm}}$$

$$=\frac{\text{longer arm}}{\text{shorter arm}}=\frac{y}{x} \qquad \therefore\ F_2=mF_1=\frac{y}{x}F_1 \qquad \ldots(114{,}1)$$

Let the total force generated on the ram be F_3.

$$\text{Then} \qquad F_3=\frac{\beta F_2}{\alpha}=\frac{\beta}{\alpha}mF_1,\ \text{by } (114{,}1) \qquad \ldots(114{,}2)$$

Further the *mechanical advantage* of the machine

$$=\frac{F_3}{F_1}=m\frac{\beta}{\alpha}=\frac{\beta y}{\alpha x}. \qquad \ldots(114{,}3)$$

We neglect here the effect of friction at different parts of the machine. So by applying a small force at the lever, we get a much larger force exerted on the ram. Thus if the longer arm of the lever be 10 times that of the shorter one and the area of cross-section of the larger piston be 100 times that of the smaller one, the force on the ram becomes (10 × 100) = 1000 times the force applied at the lever, so that the mechanical advantage of the machine comes out to be 1000.

As explained below, the principle of Conservation of Energy may be applied in this case. The decrease in the volume of water in T is evidently equal to the increase in the volume of water in C. If the ram is raised through a distance x_2 and the piston in T is pushed down through x_1, it is easy to see that $\alpha x_1=\beta x_2$.

$$\therefore\ \frac{x_1}{x_2}=\frac{\beta}{\alpha}=\frac{F_3}{F_2},\ \text{whence } F_2x_1=F_3x_2. \quad \cdots \quad \ldots(114{,}4)$$

Now work done on the smaller piston $=F_2x_1$
and work done by the larger piston $=F_3x_2$.

From (114,4) it follows that the work done by the larger piston is equal to that done on the smaller piston when frictional loss is neglected. This is according to the law of conservation of energy.

Examples :

1. The diameters of two pistons of a Hydraulic Press are 1 in. and 6 in. respectively. What multiplication of force will be produced ? (Del. H. S.—1949)

Ans. By Pascal's Law the pressure acting on the two pistons must be equal. Now force = (pressure) × (area of cross-section). Let F_1 and F_2 be the forces acting on the shorter and the longer pistons of radii 0·5 in. and 3 in. respectively.

Equality of pressures now gives

$\frac{F_1}{\pi \times (\cdot 5)^2} = \frac{F_2}{\pi \times 3^2}$, whence $\frac{F_2}{F_1} = \frac{\pi \times 3^2}{\pi \times \cdot 5^2} = 36$. Thus the force is multiplied 36 times.

2. The diameters of the pistons of a Hydraulic Press are 4 in. and 40 in. respectively. The shorter arm of the lever working the smaller piston is 8 in. and the longer arm is 4 ft. in length. Find the total force produced on the larger piston when a force of 75 lbs. is applied at the end of the longer arm of the lever.

Ans. Let the effective force at the smaller piston $= p$ lbs.

Then from principles of lever, $p = 75 \times 4 \times 12 \div 8$, so that $p = 450$ lb.

Now the areas of the cross-sections of the smaller and the larger pistons $= 4\pi$ sq. in. and 400π sq. in. respectively.

Let the total force produced on the larger piston be $= P$ lbs.

Consideration of equality of pressures we get the equation

$\frac{P}{400\pi} = \frac{450}{4\pi}$, whence $P = 45000$. Thus the required force is 45000 lbs.

115. Conditions of Equilibrium of a Liquid—When a quantity of liquid is *at rest*, the following conditions hold good.—

(*i*) *An elementary volume of liquid considered within the liquid is acted on by a vertical upward thrust which is equal to the weight of that elementary volume.*

The explanation is obvious, since an elementary volume of the liquid has some weight and this weight always acts downwards. As because this element is at rest, there must be an upward thrust which balances the weight. Were it not so, this element would have tended to move in the direction of the greater force and the equilibrium would have been lost. This is contrary to the hypothesis.

(*ii*) *The free surface of a liquid at rest in horizontal.*

Suppose, if possible, that the liquid surface AC it not horizontal, as shown in Fig. 160. Take two points B and D in the same horizontal line within the liquid. Let the vertical depths of these two points below the free surface of the liquid be h' and h. At D the pressure $= \pi + g\rho h$, and at B the pressure $= \pi + g\rho h'$, where π is the atmospheric pressure. If h' is greater than h, the pressure at B would be greater than that at D. Now we know that liquid flows from a higher pressure to a lower pressure. By reason of our supposition therefore, an element of liquid would move away from B to D and equilibrium would be

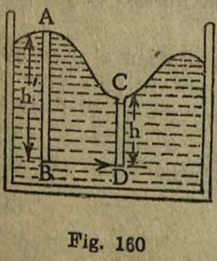


Fig. 160

lost. This flow would persist in the direction BD so long as the pressures at B and D are not equalised. So for a stable condition, the pressures at B and D must have to be equal, *i.e.*, the depths of the two points from the free surface must be equal. As B and D are on a horizontal plane, points A and C must then have to be on another horizontal plane, at an upper level.* Since A and C are *any* two points on the surface, it proves that *a liquid surface at rest remains horizontal.*

Equilibrium of a liquid in Communicating Vessels—When a liquid is poured into a number of vessels interconnected with one another, as in Fig. 161, then at rest the liquid stands to the same height in all the vessels, so that the free surface of the liquid in all the vessels are on the same horizontal plane. Irrespective of the shape and size of the vessels this condition holds good.

If a horizontal plane be drawn within the liquid, cutting all the vessels, the pressure at the points on this plane must be the same. As the pressure at any point depends on the depth of the point from the upper surface of the liquid, it shows that the pressure on the same horizontal plane must be the same. So *the free surface of the liquid in different vessels are on the* same *horizontal plane.* The fact is usually expressed by saying that a *liquid finds its own level.*

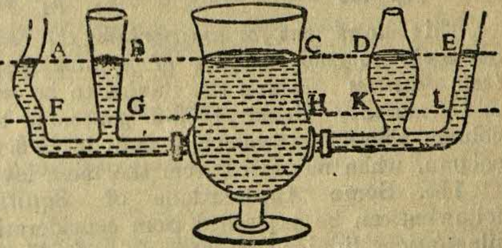


Fig. 161

Equilibrium of Unmiscible Liquids in the Same Vessel—If several liquids of different densities, which do not mix with one another, be at rest in the same vessel, they would arrange themselves in the order of their densities, the heaviest liquid occupying the lowest position. Further, the free surface as also the surface of separation in each case are found to be horizontal. If small quantities of mercury, water (coloured, if necessary) and paraffin oil be poured into a narrow bottle or a test tube, the mercury occupies the lowest position, and the oil the topmost. If the liquids be well shaken, they appear to mix; but on standing they separate and arrange themselves as before.

Equilibrium of two different Liquids in a U-tube—If two liquids of different densities, which do not mix with each other, be poured into the two limbs of a U-tube or any other kind of communicating vessel, then for equilibrium the free surfaces of the two liquids do not *generally* stand at the same height. ABC is a glass U-tube (Fig. 162). Pour mercury into the limb BC. In both the limbs mercury is found to attain the same level. Next pour into the limb AB a quantity of a liquid, which does not mix with mercury. Let the common surface of contact of the liquids stand at D, while the free surfaces of the liquid and mercury stand at A and C respectively. Let the horizontal plane through D meet the other limb at E.

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*This can be verified mathematically as shown below. Since the pressures at B and D are equal, we must have: $\pi+g\rho h'=\pi+g\rho h$, so that $h'=h$. Elementary geometry now asserts that AC must be parallel to BD. Since BD is horizontal, so also is AC.

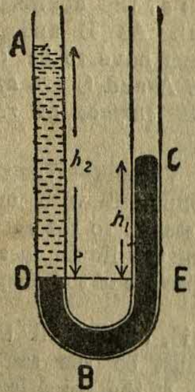


Fig. 162

Now the pressure at D is $= \pi + g\rho_2 h_2$ and the pressure at E is $= \pi + g\rho_1 h_1$, where π is the atmospheric pressure, $h_2 = AD$, $h_1 = CE$ and ρ_2 and ρ_1 are the densities of the liquid and mercury respectvely.

Inasmuch as the pressures at E and D are equal, we derive $\pi + g\rho_1 h_1 = \pi + g\rho_2 h_2$, whence $\rho_1 h_1 = \rho_2 h_2$.

$$\text{Hence } \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} \qquad \dots (115{,}1)$$

This shows that *for equilibrium, the heights of two liquid columns above the common surface of contact are inversely proportional to their densities*. Taking the liquids to be water (density = 1 gm./c.c.) and mercury (density = 13·6 gm./c.c.), the height of the water column above the common surface is 13·6 times that of the mercury column, when measured from the same level.

116. Some Applications of Equilibrium of Liquids—The following can be explained from considerations of the equilibrium of illustrations liquids.

Spirit Level—This instrument is used to test the level, *i.e.*, to examine whether the plane surface, on which it is placed, is horizontal or not. It consists of a glass tube slightly curved and closed at both ends. The tube is nearly filled with spirit so as to leave room for a small bubble of air. Such bubble naturally occupies the highest part of the tube [Fig. 163(*a*)]. The tube is mounted in a metal case with its convex side uppermost, the base of the case being made very accurately plane. When the instrument is placed on a perfectly horizontal surface, the air bubble occupies exactly the middle position between the two marks on the convex side. If the surface is not horizontal, the bubble moves to a different position. Figure 163(*b*) shows a spirit level of the circular type.

(*a*) Fig. 163—Spirit Level (*b*)

Water-supply in Towns—Water duly filtered is pumped up into a suitable reservoir placed at a high level. Main pipes run from the reservoir while smaller service pipes branch off from the main to different houses. Water runs down from the reservoir and in finding its own level, it is likely to be available in all places at lower levels than that of the reservoir. The 'head' of water, which means the vertical height of the water surface in the reservoir, thus furnishes the necessary pressure for the water-supply. But owing to friction of water against the sides of the pipes and the viscous resistance between different layers, the water falls short of the expected height.

It is to be mentioned here that for supply of water in Calcutta, water is filtered at Palta, 20 miles north of Calcutta and is stored in Tala tank at a height of about 100 ft. above the level of the city. But even with this arrangement the required pressure is not available at all places.

EXERCISES ON CHAPTER VIII

Reference

1. Describe an experimental arrangement to show that water exerts pressure in all directions. (C. U.—1959) Art. 113

2. Obtain an expression for the total liquid thrust in absolute units on the plane horizontal bottom of a vessel containing a liquid and describe briefly an experiment to show that this thrust does not depend on the shape of the vessel. (P. U.—1962 ; C. U.—1960) Arts. 109 & 111

3. Distinguish between Thrust and Pressure. Give the units in which they are expressed. (*Cf.* Utk. U.—1964 ; Del. H. S.—1962) Arts. 109 & 110

4. Define pressure at a point within a liquid at rest and obtain an expression for it. Art. 109
(C. U.—1959 ; Mad. U,—1960 ; Anna. U.—1961 ; Gau. U.—1955)

5. The density of sea water is 1·025. Find the pressure at a depth of 10 feet below the surface (in pounds per square foot), given that one cubic foot of water weighs 62·5 lbs. Art. 109

Ans. 640·624 lbs. per sq. ft.

6. If we have a vertical pipe half inch in diameter and 100 ft. high, filled with water, what will be the hydrostatic pressure at the lowest end? What is the hydrostatic pressure at a point 100 ft. below the surface of water in a lake? In both cases, assume that the density of the water is 64 lbs. per cu. ft. and express the pressure in lbs. per square inch. (P. U.—1959) Art. 109

Ans. $44\frac{4}{9}$ lbs. per sq. in. in both cases.

7. A hole, 6 inches square, is made in a lock-gate at a depth of 20 ft. below the surface of water. If the density of the water contained be 1·02, find the force that must be exerted on a plate to prevent the water from going out by holding the plate against the hole, given that 1 cu. ft. of pure water weighs 62·5 lbs. Art. 109

Ans. 322·575 lbs.-wt.

8. A vertical tube 3 ft. long and having an internal diameter of 1 inch is filled with equal volumes of water and mercury. Assuming that weight of mercury is 13·56 times the weight of water, calculate the pressure in lbs. per sq. in. at the bottom of the tube due to the head of the liquid. Also, what is the weight of water in the tube? Art. 109

Ans. 9·45 lbs./sq. in. ; 0·51 lb.

9. A lock-gate is 35 ft. wide and the heights of water above the bottom of the gate on the two sides are 26 ft. and 13 ft. respectively. Find the resultant pressure and the height, measured from the bottom of the gate at which it acts, the mass of water per cu. ft. being 64 lbs. Art. 110

Ans. 253·5 tons ; 10·11 ft.

10. Explain why the thrusts on the bases of Pascal's vases are not equal to the weight of the contained liquid. (Utkal U.—1951, '64) Art. 111

Reference

Art. 112 11. A hollow right circular cone of base diameter 20 cm. and height 15 cm. is completely filled with a liquid of density 1·2 gm./c.c. Find the mass of the liquid and the thrust at the base of the cone.

Ans. Mass = 1884 gm. ; thrust = 5652 gm.-wt.

Art. 113 12. State Pascal's Law as to the transmission of pressure in a liquid.

Art. 115 Describe the principle and action of a Hydraulic press giving a sectional diagram. (Utk. U.—1954 ; Del. U.—1960, '61 ; Del. H. S.—1959 ; Pat. U.—1955, 62 ; P. U.—1955 ; Dac. U.—1954 ; C. U.—1957 ; V. U.—1955)

Art. 115 13. Draw a diagram showing the construction of Hydraulic Press. A force of 50 kgm. is applied to the smaller piston of such a machine. Neglecting friction, find the force exerted on the larger piston, the diameters of the pistons being 2 cm. and 10 cm. respectively. (P. U.—1955)

Ans. 1250 kgm.-wt.

CHAPTER IX

ARCHIMEDES PRINCIPLE

117. Resultant Thrust on a Body immersed in a Liquid—When a body is immersed in a liquid, every point of its surface in contact with the liquid is subjected to a hydrostatic pressure. The pressure at any point is normal to the element of surface at that point and is proportional to the depth of the point below the free surface of the liquid. These pressures combine into a *resultant* thrust, which tends to move the body upwards.

Imagine a rectangular block ABCD of a solid to remain immersed in a vertical position in a liquid (Fig. 164). The total liquid thrust on the block consists of (i) thrust on the vertical sides AD and BC and (ii) thrust on the horizontal surfaces AB and CD. The pressure at any point on a side is horizontal and is exactly counter-balanced by an equal and opposite pressure at the point on the opposite side in the same horizontal line. The horizontal forces due to pressures at all points on the vertical sides are thus mutually neutralised. Only the vertical thrusts on the faces AB and CD are then to be considered. Let the depths of faces AB and CD from the free surface of the liquid be h and h' respectively, and the cross-sectional area of the face AB or CD be α.

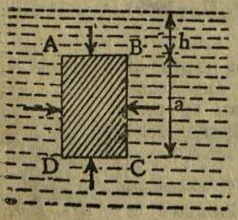


Fig. 164

Let the height of the block be a so that $h' = h + a$, and the density of the liquid be ρ.

$\therefore$ the thrust on AB $= g\rho\alpha h$ acting vertically *downwards*
and the thrust on CD $= g\rho\alpha h'$ acting vertically *upwards*.

Since $h' < h$, the upward thrust is greater than the downward thrust.

∴ the resultant thrust on the block $= g\rho\alpha(h'-h) = g\rho\alpha a$ acting *vertically upwards* = weight of the volume ABCD of the liquid displaced by the block.

Thus *the resultant thrust on a body immersed in a liquid is equal to the weight of the volume of the liquid displaced by the body and it acts vertically upwards through the centre of gravity of this portion of the liquid.*

If the solid is *not completely immersed* in the liquid, the upward thrust acts on the immersed portion of the solid. In this case the resultant thrust is equal to the *weight of the liquid displaced by the immersed portion of the solid.* The resultant upward thrust on a solid, immersed wholly or partly in a liquid, is called the *buoyancy* of the liquid. Its magnitude is, therefore, equal to the weight of the amount of the liquid displaced. The buoyancy acts vertically through the centre of gravity of the displaced liquid, and this point is accordingly called the **centre of buoyancy** or the *centre of floatation.* Evidently, the buoyancy does not ordinarily depend on the depth to which the body is immersed provided that the density of the fluid does not change with depth. Nor does it depend on the shape, mass or material of the body itself if the volume of the body does not alter with depth. It simply depends on the immersed volume of the solid and the density of the liquid displaced.

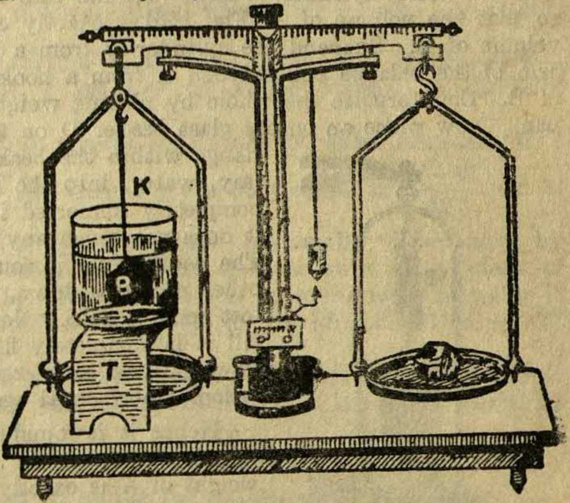


Fig. 165—Hydrostatic Balance

Apparent Weight—As the weight of a body acts vertically downwards, the weight of anybody immersed in a liquid must appear *less* than its true weight. If the true weight of the body be W and the buoyancy by W′, the *apparent weight* of the immersed body is equal to (W − W′). A body, therefore, appears lighter when immersed in a liquid. Thus it is easier to carry a heavy body inside water than when it is outside.

118. Principle of Archimedes—Long long ago a Philosopher of Syracuse in Sicily by the name of Archimedes (287-212 B.C.), while taking his bath in a tub, felt that his weight appeared less in water. A meditation on this ordinary observation led to the discovery of the Principle stated in the following lines :—*When a body is immersed* either wholly or partially *in a fluid at rest, it appears to lose a part of the weight which is equal to the weight of the fluid displaced.*

Experimental Verification of the Principle—To verify the principle stated above, take a *hydrostatic balance.* A hydrostatic balance is simply an ordinary balance with arrangements for getting conveniently the weight of a body in air and also when immersed in a liquid. In some special forms, one of the pans is suspended by a shorter frame and has a hook attached below, from which the body to be weighed can be suspended by a string. In the ordinary form an wooden bridge T (Fig. 165) is placed on the floor of the balance across one of the pans, so that a beaker containing the liquid may be placed on the bridge. (For experimental details *vide* J. Chatterjee's *Intermediate Practical Physics*)

Another simpler method of verifying the principle is as follows. Take a solid cylinder, which just fits into the hollow cylinder B, so that the volume of A (Fig. 166) is exactly equal to the internal volume of B. Suspend the cylinder B from a hook attached to one arm of the balance and suspend A from a hook fixed to the bottom of B. Counterpoise the whole by placing weight on the other scale pan. Now place an empty glass beaker D on the bridge C, so that A hangs within the beaker. Pour any liquid (say, water) into the beaker, till A hangs completely immersed in water. Ensure that it does not touch any side of the beaker. The equilibrium is found to be lost and the other pan goes down, showing as if A has lost some weight. Now pour water into B till it is completely filled. The equilibrium now is found to be restored, when the beam becomes horizontal again.

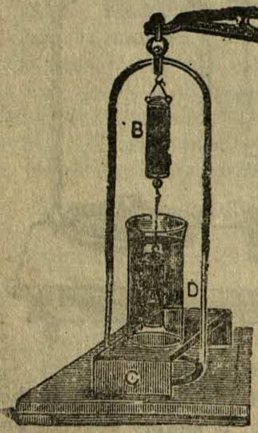


Fig. 166

Hence it is found that due to the upward thrust of water the apparent loss in the weight of A is exactly equal to the weight of water completely filling B. As the volume of B is exactly equal to the volume of A, *the apparent loss in the weight of A, is equal to the weight of water displaced by it.* The experiment may be repeated with any other liquid and spring balance may be used instead of an ordinary balance.

Apparent Loss—It is to be noted that the loss observed in the weight of an immersed solid is only *apparent.* Thus, if the beaker containing the liquid is placed on the scale pan, there would be no change irrespective of the cylinder A being inside the liquid or outside. When the cylinder is immersed in the liquid, it experiences an upward thrust which tends to raise the arm of the balance; but at the same time the cylinder exerts as reaction a downward force on water in the beaker. As by Newton's Third Law of Motion the reaction is exactly equal and opposite to the upward thrust, the balance remains always in the same condition.

Now if finger be dipped into the water in the beaker, the pan carrying the beaker at once goes down. Here the upward thrust on the finger is taken by the hand ; but the reaction acts on the water and hence on the beaker, causing the pan to go down. Similar effect is observed by suspending a solid into a liquid from a support placed outside.

119. Some Practical Applications of the Principle—The principle is utilised in the following cases :

(1) **Determination of Volume of Solid**—The volume of a solid, heavier than and insoluble in water, can be easily determined in the following way. Weigh the body in air ; let it be W gm. Then weigh the body, when suspended in water with a hydrostatic balance ; let it be W′ gm. Let the density of the water be ρ gm. per c.c. Then the mass of water displaced by the solid is clearly (W − W′) gm.

Hence, the volume of the displaced water $= \dfrac{W - W'}{\rho}$ c.c. ... (119,1)

which is equal to the volume of the body. In the C.G.S. system ρ may be taken to be 1 gm./c.c. Thus the volume of the solid becomes (W − W′) c.c.

If weights are given *in pounds*, the density of water is usually 62·5 lbs. per cu. ft. and so the volume of the body equals to $\dfrac{W - W'}{62{\cdot}5}$ cu. ft.

The volume of a solid of *irregular shape* is usually determined by the above way. For example, suppose that the mass of a piece of stone is 52·12 lbs. in air and that is apparent mass in water is 31·54 lbs. Then its volume come out to be $\{(52{\cdot}12 - 31{\cdot}54) \div 62{\cdot}5\}$ cu. ft. = 0·33 cu. ft.

(2) **Determination of Density of a Solid**—As density is defined to be the mass per unit volume, the density ρ' of the solid is given by

$$\rho' = \frac{\text{mass}}{\text{volume}} = W \div \frac{W - W'}{\rho} = \frac{W\rho}{W - W'}.$$

If expressed in C. G. S. units, the density ρ' of the solid

$$= \frac{W}{W - W'} \text{ gms. per c.c., taking } \rho = 1.$$

If expressed in F. P. S. units, the density ρ' of the solid

$$= \frac{W}{W - W'} \times 62{\cdot}5 \text{ lbs. per cu. ft.} \qquad \text{...(119,2)}$$

For a specimen of stone whose mass in air is 52·12 lbs. and mass in water is 31·54 lbs. its density in the F. P. S. units from the data supplied comes out to be 2·53 × 62·5 lbs. per cu. ft. and in the C. G. S. system it is simply 2·53 gms./c.c.

Examples :

A crown made of gold and containing silver as impurity weighs 200 gm. When it is dipped in water, it weighs 185 gm. Find out how many grammes of gold and silver are respectively present in the crown. Given density of gold = 19·3 gm./c.c. and that of silver = 10·3 gm./c.c. [Vis. U.—1954]

Ans. Let the quantity of silver in the crown be x gm. Then the quantity of gold present is $(200 - x)$ gm. The volume of gold is its mass divided by density and is therefore $(200-x)/19{\cdot}3$ c.c. Similarly the volume of silver is $x/10{\cdot}3$ c.c.

Thus the volume of the crown $=\left(\frac{200-x}{19{\cdot}3}+\frac{x}{10{\cdot}3}\right)$ c.c.

When dipped in water, the crown would displace its own volume of water and the volume of displaced water would have a mass by Archimedes principle, equal to the difference of the masses of the crown in air and in water. The difference of masses is (200 − 185) gm. = 15 gm. Since 1 c.c. of water weighs 1 gm., we have $\frac{200-x}{19{\cdot}3}+\frac{x}{10{\cdot}3}=15$, whence the quantity of silver $x=102{\cdot}43$ gm.

So the quantity of gold = (200 − 102·43) gm. = 97·57 gm.

120. Equilibrium of Immersed and Floating Bodies—When a body is immersed in a liquid, it is simultaneously subjected to the forces—(*i*) the weight W of the body acting downwards and (*ii*) the buoyancy W′ of the displaced liquid acting vertically upwards through the C. G. of the displaced liquid. The equilibrium of the body depends upon the relative magnitudes of the two forces. Consequently *three* cases may arise, according as the weight W is greater than, equal to, or less than, the buoyancy W′.

(1) If $W>W'$, *i.e.*, if the weight of the body is *greater than* the vertical upward thrust, the body *sinks*. The weight of the body is in this case greater than the weight of an equal volume of the liquid. This means that the body is heavier than the liquid. This is the case with a piece of stone or iron sinking in water.

(2) If $W=W'$, *i.e.*, if the weight of the body is *equal* to the vertical upward thrust, the body *floats completely immersed* and may remain in equilibrium at any position within the liquid. Evidently, the density of the body in such a case equals that of the liquid. For instance, a drop of olive oil may stand anywhere within a mixture of equal quantities of water and alcohol.

(3) If $W<W'$, *i.e.*, if the weight of the body is *less than* the vertical upward thrust, the body *floats partly immersed* in the liquid. Here the body is lighter than the liquid. Such is the case with a piece of cork or wood floating on water, or with iron floating on mercury. In a case such as this, the weight of the body is clearly less than that of an equal volume of the liquid.

The last case gives the condition under which a body may float on a liquid. So the **conditions of equilibrium of a floating body are**—

(i) The floating body must displace a volume of the liquid, whose weight is equal to the weight of the body itself.

(ii) The C. G. of the body and the C. G. of the displaced liquid must lie on the same vertical line called the *central line* of the body and, in general, the former is *above* the latter. The centre of gravity of the displaced liquid is called the centre of buoyancy. In the case of a body floating just completely immersed, the C. G. of the body should be vertically *below* the centre of buoyancy.

Metacentre—Consider a body floating in equilibrium (say a ship floating on an even keel). As already stated, the C. G. of the ship and the centre of buoyancy lie on the same vertical line, the former being above the latter. If the body is now displaced (*e.g.*, when the ship heels over), her C. G. and the centre of buoyancy are both displaced in the direction in which the body leans. The point, where the vertical line through the new position of the centre of buoyancy intersects the central line of the body, is called the **metacentre** of the body.

The equilibrium of a floating body is stable or unstable, according as the metacentre is above or below the C. G. of the body. When the two coincide (as in the case of a floating spherical body), the equilibrium is neutral. By putting ballast in the ship, her C. G. is kept below the metacentre so as to have an increased stability.

The freshness of an egg is sometimes tested by dipping it in ordinary water and then in a strong solution of brine. If it floats on brine but sinks in water, the egg has density intermediate between the two liquids and it is fresh. If it is rotten or otherwise, it floats on water. A fully loaded boat on sea is to be partly unloaded for safety before entering a river of fresh water.

121. Application and Illustrations of the Principle of Floatation—The following cases furnish some illustrations of the principle of floatation.

Floating of Ice on Water—Water on freezing into ice increases in volume, such that 11 volumes of water at 0°C become 12 volumes of ice at the same temperature. Hence, the density of ice is less than that of water and so ice floats on water.

From the principle of floatation, therefore, it follows that,

$$\frac{\text{volume of ice under water}}{\text{total volume of ice}}=\frac{11}{12}.$$

Hence a mass of ice will float on water with $\frac{11}{12}$th of its volume under water and only $\frac{1}{12}$th volume exposed. The density of sea water is about 1·08 and so an iceberg floats on sea water with only one-ninth of its volume exposed. Again, glacier ice is somewhat porous and so it is lighter. An iceberg from a glacier floats on sea water with about one-seventh of its volume exposed.

Floating of a Ship—A solid heavier than a liquid may be given a suitable shape as to float on a liquid. The shape of the body would have to be such that the weight of the liquid displaced by the immersed portion of the body is equal to that of the body. Thus a porcelain saucer or an iron cauldron floats on water partly immersed, for owing to its concave shape it displaces sufficient water to make it float.

The body of a ship is constructed with iron plates; but owing to its concave shape the interior is hollow. Even when partly immersed, it can therefore displace a volume of water whose weight is equal to the weight of the ship with her contents. Hence the ship floats. If the ship is loaded more, she is depressed further in water to make the upward thrust due to the newly displaced volume of water equal to the additional load.

To carrying capacity of a ship is represented by her tonnage. A ship, whose tonnage is 50,000 tons, can carry this mass of load. It is to be noted that for the sake of safety there should be a suitable margin in the loading of a ship and sufficient *free board* must be left above water line. It has now been legalised that every ship should bear a mark called the **Plimsol Line** which indicates the limit to permissible immersion.

Bodies heavier than water can also be made to float by attaching lighter bodies of suitable size. This explains the working of life-betls, life-buoys, etc. **A life-belt** consists of a hollow air-tight tube floating on water. When a person tries to swim with its help, it sinks a little further to counter-balance the weight necessary for the man to float.

Swimming—The human body is lighter than water ; but the head is heavier. So the dead body of a human being floats with head leaning downwards. In swimming, therefore, we are to keep the head up by movements of the limbs, which produce sufficient reaction upwards to keep the body afloat. For a beginner his body may be attached to a life-belt. It is easier to float on water after taking a deep breath, since being inflated by air, the lungs like a life-belt help the man to float. As sea water is heavier than river water, the immersed portion of the body would experience greater upward thrust in sea water and so it is easier to swim in sea water than in river water.* In the case of quadrupeds, the front part is lighter than the hind parts and so they swim naturally with head on water. Birds, such as duck, swim easily owing to the thick layer of impervious downs on the lower parts of their bodies.

122. Cartesian Diver—The different kinds of equilibrium of a body floating on a liquid as well as the principle of transmission of fluid pressure can be strikingly demonstrated by a well-known hydrostatic toy known as the *Cartesian Diver*. It is so called after its inventor Rene Descartes (1596-1650), who devised it to illustrate the principle of Archimedes. The diver consists of a small hollow porcelain doll with a small tubular tail in communication with the inside and open at the end (Fig. 167). It is placed in water within a glass jar, the top of the jar being closed air-tight by a sheet of rubber. The doll is hollow within and is filled partly with air and partly with water. Its mass is so adjusted that it floats with a very small part exposed. In some cases the doll is solid and is attached to a hollow ball with a small aperture at the bottom (shown separately in the diagram), so that the two together float in equilibrium.

Now if the rubber sheet is pressed, the diver sinks down. But on releasing the pressure the diver rises again to the surface. By regulating the pressure the toy may be made to remain stationary at any level.

Explanation—When the rubber sheet is pressed, the air enclosed within the jar above water is compressed. This pressure is transmitted through water to the air inside the diver. The air is thus compressed to a smaller volume, due to which an additional quantity of water enters into the body of the toy through the opening in the tail and makes it

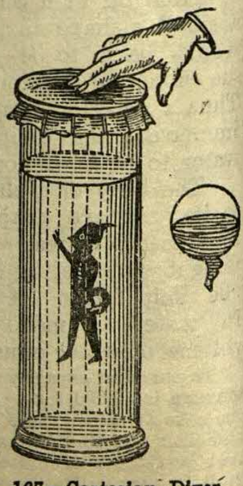


Fig. 167—Cartesian Diver

heavier than the water displaced. Hence the toy begins to sink. When the pressure on the rubber sheet is released, the air inside the toy expands again and drives out the additional water. The toy thus become lighter than the displaced water and floats in consequence.

It should be noted that as the diver goes down more and more, water is forced into the inside owing to increased liquid pressure. If the toy could be forced to such depth that owing to the large liquid pressure at that region the inside air is reduced to a very small volume, the toy would not rise up even when the pressure on the rubber sheet is removed. Most fishes have an air-bladder below the spine and by dilating or compressing it, they can rise up or dive in water at pleasure.

Submarine—The modern submarine works on the principle of a *Cartesian Diver* and may, therefore, be regarded as ship with variable and controlled mass. It is provided with several large ballast tanks T (Fig. 168) in the bow, middle and stern of the ship. These are fitted with valve or trap doors. Through these openings water can be admitted into the tanks. When the submarine requires to be sunk below the liquid surface, the requisite amount of water is drawn into the tanks, so that the ship becomes slightly heavier than the displaced water and she goes down. On the other hand, for rising to the surface, the water is forced out of the tanks by powerful pumps worked by compressed air, whereupon the ship becomes lighter than the displaced water and she rises to the surface.

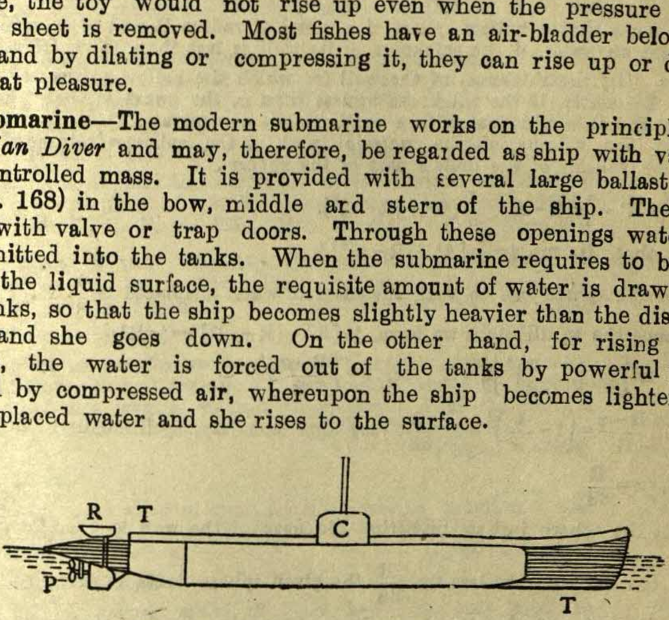


Fig. 168—Principle of a Submarine

The operation of filling or emptying the tanks may be done swiftly. Beside the vertical steering rudder there is a horizontal rudder and by suitably tilting these, the ship may be kept at any desired depth. The *periscope* is fitted to the *conning tower* C and remains above the surface of water for getting view of objects on or above the surface of water.

Examples :

1. A hollow spherical ball, whose internal and external diameters are 8 cms. and 10 cms., is found just to float in a liquid of specific gravity 1·5. What is the specific gravity of the material of the ball? (C. U.—1959)

Ans. When the ball just floats, it displaces its own volume of liquid. The external radius is 5 c.m. Hence, the volume of the ball $=(\frac{4}{3}\pi\times 5^3)$ c.c. $=523{\cdot}6$ c.c. which for equilibrium equals the volume of liquid displaced.

The mass of this liquid $=523{\cdot}6\times 1{\cdot}5$ gm. = mass of the ball.

If s be the specific gravity of the material of the ball, the mass of the ball = (sp. gr.) × (vol. of the material of the ball).

Now the volume of the material of the ball $=\frac{4}{3}\pi(5^3-4^3)$ c.c. Hence, we get $\frac{4}{3}\pi s(5^3-4^3)=\frac{4}{3}\pi\times 5^3\times 1{\cdot}5$, or, $s=\dfrac{5^3\times 1{\cdot}5}{5^3-4^3}=3{\cdot}08$. This determines the required specific gravity.

2. Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 28 gm. and its density is 5·6. If the mass of the other is 36 gm., what is the density? (Pat. U.—1956)

Ans. Let the required density be ρ gm. per c.c.
Now the apparent weight of the first body $=\{28-(28\div 5{\cdot}6)\}$ gm. $=23$ gm.
and the apparent weight of the second body $=(36-36/\rho)$ gm.
Since the apparent weights of the bodies are equal, we must have

$36-\dfrac{36}{\rho}=23$, whence $\rho=2{\cdot}77$. Thus the required density is 2·77 gms./c.c.

3. Show that a hollow sphere of a metal of radius R and of specific gravity S will float on water, if the thickness of its wall is less than R/3S. (Nag. U.—1952)

Ans. Let the thickness of the wall for which the hollow sphere can *just float* be x. Evidently, if the thickness be less than x, the mass of the sphere would be less, and it would float with a portion of it above water.

Now the volume of the *material* part of the sphere $=$ (volume of the outer sphere) $-$ (volume of the hollow part of the sphere).

$$=\frac{4}{3}\pi R^3-\frac{4}{3}\pi(R-x)^3=\frac{4}{3}\pi[R^3-(R-x)^3].$$

Further, its mass $=\frac{4}{3}\pi[R^3-(R-x)^3]S$.

For the sphere *just* to *float*, the mass of water displaced is $\frac{4}{3}\pi R^3$.

Hence the eqnilibrium we have, $\frac{4}{3}\pi[R^3-(R-x)^3]S=\frac{4}{3}\pi R^3$,

or, $R^3\left[1-\left(1-\dfrac{x}{R}\right)^3\right]S=R^3$, or, $1-\left[\dfrac{R-x}{R}\right]^3=\dfrac{1}{S}$.

or, $\dfrac{R-x}{R}=\left(1-\dfrac{1}{S}\right)^{\frac{1}{3}}=1-\dfrac{1}{3S}$ very approximately, by Binominal Theorem.

or, $x=\dfrac{R}{3S}$.

For the sphere just to float, the thickness of the wall is, therefore, $=\dfrac{R}{3S}$.

If the thickness be less than $\dfrac{R}{3S}$, the given sphere floats at ease.

4. A body of density δ is dropped gently on the surface of a liquid of density δ' (δ' being less than δ). Show that it will reach the bottom of the liquid after a time $\sqrt{\dfrac{2d\delta}{g(\delta-\delta')}}$, where g is the acceleration due to gravity and d is the depth of the liquid. (Pat. U.—1951)

Ans. If the mass of the body be m, its volume $=m/\delta$.

$\therefore$ The upward thrust due to the displaced liquid $=\dfrac{m}{\delta}\times\delta' g$.

So the resultant downward force on the body within the liquid $=mg-\dfrac{m}{\delta}\times\delta' g$

$=m\left(1-\dfrac{\delta'}{\delta}\right)g$.

Hence the acceleration f of the body, while going down $=\left(1-\dfrac{\delta'}{\delta}\right)g$.

Since from the surface of the liquid the body starts from rest, the time t to reach the depth d is given by

$$d=\tfrac{1}{2}ft^2,\text{ or, } d=\tfrac{1}{2}g\left(1-\frac{\delta'}{\delta}\right)t^2,\text{ whence } t=\sqrt{\frac{2d\delta}{g(\delta-\delta')}}.$$

To work out this sum, we have assumed that while the body is being accelerated down in water, it experiences no retarding force due to viscous friction. Such assumption is not, however, correct.

EXERCISES ON CHAPTER IX

Reference

1. State Archimedes principle and show how it can be verified experimentally. (Nag. U.—1961; *Cf.* Raj. U.—1958; Utkal U.—1959; Gau. U.—1962; C. U.—1958) Art. 117

2. Show how Archimedes principle can be applied to find the volume of a solid. (C. U.—1959) Art. 119

3. The crown of Hiero weighed 20 lbs. Archimedes found that it lost 1·25 lbs. when immersed in water. The crown was made of gold and silver. Find the weight of the metals. (Sp. gr. of gold =19·3; sp. gr. of silver=10·1) (Dac. U.—1951) Art. 119

Ans. The wt. of gold 15·078 lbs.; that of silver 4·922 lbs.

4. State and explain the conditions of equilibrium of a floating body. (Gau. U.—1952; Del. H. S.—1961; C. U.—1957; P. U.—1957; U. P. B.—1961; Raj. U.—1955; Nag. U.—1964) Art. 120

5. A piece of iron weighing 272 gm. floats on mercury of density 13·6 with ⅝th of the volume immersed. Determine the volume and density of iron. (Dac. U.—1957; C. U.—1950) Art. 120

Ans. 32 c.c.; 8·5 gm. per c.c.

6. A piece of paraffin wax of density 0·9 gm./c.c. floats on water. A layer of turpentine of density 0·87 gm./c.c. is added on the top of water until the wax is entirely submerged. Find the ratio of the volume of wax immersed in water to that in turpentine. (Nag. U.—1954) Art. 120

Ans. 3 : 10.

7. 52 gm. of an alloy of two metals of specific gravities 8 and 12 respectively is found to weigh 46 gm. in water. Find the masses of the metals in the alloy. (Utkal U.—1961) Art. 120

Ans. 40 gm.; 12 gm.

8. A hollow sphere made of glass ball just only sinks in methylated spirit of specific gravity 0·8. It floats on sulphuric acid (specific gravity=1·8). Explain why this is so and calculate the portion of the volume of the sphere immersed in the acid. (C. U.—1958) Art. 121

Ans. 0·44 of volume (nearly).

9. Two pieces of metals are suspended from the arms of a balance and are found to be in equilibrium when kept immersed in water. The mass of one piece is 32 gm. and its density is 8 gm./c.c. The density of the other is 5 gm./cc. What is its mass? (C. U.—1959) Art. 121

Ans. 35 gm.

10. A raft is constructed of four empty barrels lashed together, and planks weighing 40 lbs. are placed on them. If the barrels be regarded as approximately cylindrical and if the height, radius and mass of each be respectively 4 ft., 1·5 ft. and 50 lbs., find the load that the raft can carry. (1. cu. ft. of water weighs 62·5 lbs.) Art. 121

Ans. 6865 lbs. (nearly).

11. A wooden sphere of radius 1·0 cm. sinks in water. On coating it with a layer of 8 mm. of wax of uniform thickness Art. 121

Reference

and of density 0·8 gm. per c.c., it just floats. What is the density of wood ?

Ans. 1·24 gm./c.c.

Art. 121 12. A block of ice (sp. gr. = 0·91) floats in a vessel of water filled up to the brim. What fraction of the volume of the block will be above the surface of water ? Describe and explain the changes taking place when ice melts. (U. P. B.—1941)

Ans. ·09 of the volume.

Art. 122 13. A pontoon with perpendicular sides has a rectangular base 30 yds. by 10 yds. and is loaded so that the base is submerged to a depth of 5 ft., 4 ft. of the pontoon remaining above water. It commences to leak, taking in 10 gallons of salt water per minute. How long will it be before it sinks ? (A gallon of salt water weighs 10·8 lbs.)

Ans. 112·5 hours.

Art. 121 14. Describe the Cartesian Diver and explain how it acts. Do you know of any modern appliance which is based on this principle ?

Art. 122 15. A solid body floating on water has one-sixth of its volume above the surface. What fraction of its volume will project, if it floats in a liquid of specific gravity 1·2 ?

Ans. $\frac{11}{36}$.

CHAPTER X

SPECIFIC GRAVITY

123. Specific Gravity—If *equal volumes* of different substances are weighed, they are found to have *different* masses. Thus, if two cubes of the same size, one of marble and the other of iron, be taken, the mass of the latter is found to be about three and a half times greater than that of the former. The *specific gravity* of a substance indicates how much a given volume of the substance is relatively heavier than an equal volume of a standard substance.

The **specific gravity** *of a substance is the ratio of the weight of any volume of the substance to the weight of an* equal *volume of some standard substance.* In measuring the specific gravities of *solids* and *liquids*, the standard substance is taken to be water at 4°C, whereas in case of *gases* it is hydrogen at N.T.P. (*i.e.*, normal temperature and pressure).

∴ the Specific Gravity (sp. gr.) of a substance

$$= \frac{\text{weight of any volume of the substance}}{\text{weight of equal volume of water at } 4^\circ\text{C}}$$

Since the mass of a body at a given locality is proportional to its weight, the specific gravity of the body is then

$$= \frac{\text{mass of a volume of the substance}}{\text{mass of an equal volume of water at } 4^\circ\text{C}}$$

$$= \frac{\text{mass of unit volume of the substance}}{\text{mass of unit volume of water at } 4^\circ\text{C}}$$

$$= \frac{\text{density of the substance}}{\text{density of water at } 4^\circ\text{C}}.$$

Hence the specific gravity of a substance is often called its **relative density**. The specific gravity is a **pure ratio**; and so it has no dimension.

Density and Specific Gravity—As already stated, the *density* of a substance is defined as its mass per unit volume and so it has some unit, whereas the specific gravity, being a ratio, is a *pure number*.

If S be the specific gravity of a substance, ρ' its density and ρ the density of water at 4°C, then from the above relation $\rho' = S\rho$. In the C.G.S. system of unit $\rho = 1$ gm. per c.c.; so the density ρ' of the body = S. Hence the *specific gravity of a substance in C.G.S. unit is* numerically *equal to its density*. In F.P.S. unit, the density ρ of water = 62·43 lb. per cu. ft. at 4°C, and so the density ρ of the body = (62·43 × S) lb. per. cu. ft. Or, in other words, the density of a substance in the F.P.S. unit is numerically equal to 62·43 × its specific gravity.

For example, in C.G.S. unit the density of mercury = 13·6 gms. per c.c. and so the specific gravity of mercury = 13·6 ÷ 1 = 13·6.

Again, in the F.P.S. unit, the density of mercury = 13·6 × 62·43 lb. per cu. ft., so that the sp. gr. of mercury $= \frac{13{\cdot}6 \times 62{\cdot}43}{62{\cdot}43} = 13{\cdot}6$.

Thus the statement that the specific gravity of mercury is 13·6 merely means that a certain volume of mercury is 13·6 times heavier than an equal volume of water at 4°C, whatever be the unit of measurement of the volume.

Temperature Correction—In determining the specific gravity of a solid or a liquid, water at 4°C should be taken as the standard substance for comparison. But water, usually available in the laboratory, has a temperature higher than 4°C. As the mass of unit volume of water at any other temperature is different from that at 4°C, a correction for this is necessary. Let the temperature of water used in the experiment be t°C.

$$\text{Then the observed sp. gr.} = \frac{\text{mass of 1 c.c. of a substance}}{\text{mass of 1 c.c. of water at } t^\circ\text{C}}$$

$$\text{But the true sp. gr.} = \frac{\text{mass of 1 c.c. of the substance}}{\text{mass of 1 c.c. of water at } 4^\circ\text{C}}$$

$$= \frac{\text{mass of 1 c.c. of the substance}}{\text{mass of 1 c.c. of water at } t^\circ\text{C}} \times \frac{\text{mass of 1 c.c. of water at } t^\circ\text{C}}{\text{mass of 1 c.c. of water at } 4^\circ\text{C}}$$

= observed specific gravity × *density* of water at t°C, The variation in the density of water for a moderate change of temperature is very small. So unless great accuracy is required the temperature correction need not be taken into account.

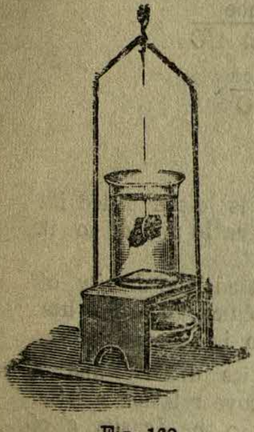


Fig. 169

124. Determination of the Specific Gravity of Solids—It is now evident that in measuring the specific gravity of a solid or liquid substance, by any method, we are to determine (1) the mass of a certain volume of the substance, and (2) the mass of an equal volume of water. The *specific gravity of a solid* may be determined by any one of the following methods :

(1) **By Hydrostatic Balance**—The Hydrostatic Balance has already been described in Art. 118. (For further details *vide* J. Chatterjee's *Intermediate Practical Physics*.)

(*a*) *Solid heavier than, and insoluble in water*—Take a suitable piece of the solid. Suspend the solid by a fine thread (Fig. 169) from the hook of the balance pan and weigh it. Then place a bridge on the table above the pan without touching it. Slide a beaker over the bridge, so that the body hangs freely within it without touching the walls. Now slowly pour water within the beaker, so that it hangs completely immersed in water. Then weigh it in immersed position.

Let the mass of the solid in air $= W$
and the mass of solid in water $= W_1$
Then the mass of water of the same volume as the solid
$=$ the upward thrust on the solid $= W - W_1$
Hence, the required specific gravity S of the solid

$$= \frac{W}{W - W_1} \qquad \ldots (124,1)$$

If the temperature of the water be t°C, the corrected specific gravity $= S \times$ density of water at t°C.

(*b*) *Solid lighter than, but Insoluble in water*—As a preliminary observation tie the given body with a small heavy unaffected by water so that the two together sink in water. The heavy body so used is called a *sinker*. Take a suitable piece of the *given solid* and weigh it in air. Then, with the solid *still on the pan*, suspend the sinker in water from that arm of the balance and counterpoise the combination with necessary weights. Next fasten the solid and the sinker together, suspend them in water and weigh them.

Let the mass of the solid in air $= W$
the mass of the solid in air with the sinker in water $= W_1$
and mass of the solid and the sinker, both in water $= W_2$
∴ mass of the solid in air − mass of the solid in water $= W_1 - W_2$

Hence the specific gravity S of the solid $= \dfrac{W}{W_1 - W_2}$ ... (124, 2)

If the water be at t°C, the corrected specific gravity

$$= S \times \text{density of water at } t^\circ\text{C}.$$

for lighter bodies $W_1 - W_2$ is greater than W. So S is less than unity.

(c) *Solid Soluble in Water*—In this case take some liquid, in which the body is not soluble. Then by the previous method find the relative density of the body with respect to this liquid. Let the relative density be S_1. Then measure the specific gravity of this liquid with respect to pure water. (See the following Article.) Let the specific gravity of the liquid be S_2. Then the *true specific gravity* S of the solid is equal to the product of S_1 and S_2 as shown below :—

$$\text{Sp. Gr. S of the solid} = \frac{\text{weight of volume V of the solid}}{\text{weight of volume V of water}}$$

$$= \frac{\text{weight of vol. V of the solid}}{\text{weight of vol. V of the liquid}} \times \frac{\text{weight of vol. V of the liquid}}{\text{weight of vol. V of water}}$$

$$\therefore \; S = \text{Relative density of the solid with respect to the liquid} \times \text{Sp. gr. of the liquid} = S_1 \times S_2 \quad \dots \quad \dots \quad \dots(124.3)$$

For example, take a piece of alum. Find its density S_1 relative to the kerosene and then find the specific gravity S_2 of kerosene. Then the specific gravity S of alum $= S_1 \times S_2$. (For experiments with a Hydrostatic Balance, *vide* J. Chatterjee's *Intermediate Practical Physics.*)

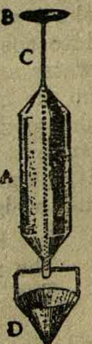
(2) By Hydrometers—The *Hydrometer* works on the principle of floatation and is so constructed that it can float vertically in a liquid. Various forms of the instrument may, by the method of its working, be classed under the following two heads :

(1) *Constant Immersion Hydrometer*

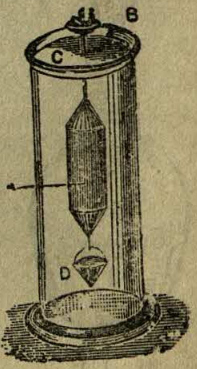
and (2) *Variable Immersion Hydrometer.*

Nicholson's Hydrometer—It consists of a hollow metal cylinder A (Fig. 170) ending in two cones. One of these is connected by a short thin stem C to a pan B above, while the other end is connected to a conical hollow closed pan D. The pan D is so weighed inside with lead shots that the instrument may float *vertically* in a liquid. There is a mark on the upper stem, which remains a little above, when the instrument floats on a liquid. During an experiment, the hydrometer is *always* made to sink up to this mark. It is, therefore, a type of a constant immersion hydrometer.

Float the hydrometer in water contained in a tall and wide glass jar, so that it does not touch the side of the jar (Fig 171). A bent piece of wire is sometimes placed round the stem C across the mouth of the jar to prevent the instrument from sinking too much. Place suitable weights on the upper pan B to sink the hydrometer up to the mark and note the weights. Remove the weights. Now place on B a *suitable piece of the given solid* and add weights on B to sink the instrument again up to the mark. Since in both the cases the hydrometer sinks to the same



Fig, 170



Fig, 171

extent, the downward forces on the upper pan *must be equal*. In the former case, it is the weight of the 'weights' placed and in the latter case, it is the weight of the body and some weight together. Hence, the difference between the two weights gives the weight of the solid in air.

Next place the solid on the lower pan in water. If the solid is lighter than water, tie the solid with the lower pan with a thin thread. Now place 'weights' on the pan B in order to allow the instrument to sink up to the mark. Note the total weight on B.

Let the weight to sink the hydrometer up to the mark $= W$
and weight to sink, when the body is on the upper pan $= W_1$
and also weight to sink when the body is on lower pan $= W_2$
Then the weight of the solid in air $= W - W_1$
and weight of the solid in water $= W - W_2$.
Hence, the weight of water displaced by the solid

$$=(W - W_1) - (W - W_2) = W_2 - W_1$$

$$\therefore \text{ the sp. gr. S of the solid} = \frac{W - W_1}{W_2 - W_1} \qquad \ldots(124{,}4)$$

Temperature correction may not be necessary here as the experiment is not so accurate.

(3) **By Specific Gravity Bottle**—The method is specially suitable for finding the specific gravity of a solid given in the from of powder or small fragments.

The specific gravity bottle consists of a small flat-bottomed glass bottle, having a narrow neck (Fig. 172). It is carefully fitted with a ground glass stopper having a capillary bore running through its length. If the bottle is filled to the brim with any liquid and the stopper is then carefully inserted, the excess liquid escapes through the hole in the stopper, thus leaving the bottle completely filled with the liquid.

Clean the bottle and dry it carefully. Weigh the empty bottle in a balance. Then put the solid inside the bottle and weigh again. If the solid is a lump too big to go into the bottle, crush it and introduce one or two chips of the specimen into the bottle. The difference between the two weights gives the weight of the solid put into the bottle. Now fill the bottle completely with water and replace the stopper carefully, so that no air bubble sticks inside. Wipe off any liquid that oozes out and weigh again. Next pour out the contents of the bottle, wash it thoroughly and fill it completely with water. Wipe the outside dry and weigh again.

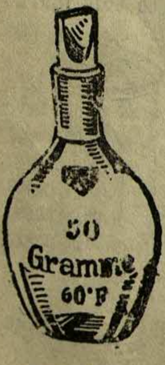


Fig. 172

Let the mass of the empty bottle $= W_1$
The mass of the bottle with solid inside it $= W_2$
The mass of bottle + solid and water to fill it $= W_3$
and mass of the bottle with water to fill it completely $= W_4$

$\therefore$ mass of the solid in air $= W_2 - W_1$

and mass of bottle completely filled with water + mass of the solid

$$= W_4 + W_2 - W_1.$$

Hence the mass of water of the same volume as the solid

$$= W_4 - W_3 + W_2 - W_1$$

Hence the specific gravity S of the solid $= \dfrac{W_2 - W_1}{W_4 - W_3 + W_2 - W_1}$ ...(124,4)

For a powder *soluble* in water, use a liquid in which the solid is not soluble and proceed as usual. The specific grvity S_1 of the solid with respect to this liquid is thus obtained. Next determine the specific gravity S_2 of the liquid with respect to water. Then the specific gravity S of the solid is given by $S = S_1 \times S_2$. Usual temperature correction may, however, be made as this method is accurate.

(4) **By Floatation**—If the solid is lighter than, and insoluble in water and if it is obtained in a regular form (*e.g.*, a cube or a cylinder), the specific gravity can be readily determined by the method of *floatation.* This is just a principle and is not used practically.

Measure the height and the cross-section of the solid. Let these be l cm. and α sq. cm. respectively. Float the solid on water and mark the height exposed (Fig. 173). If the density of water be ρ, the density of the solid is $S\rho$ and its mass is $l\alpha S\rho$. If the height immersed be h, the height exposed is $(l - h)$.

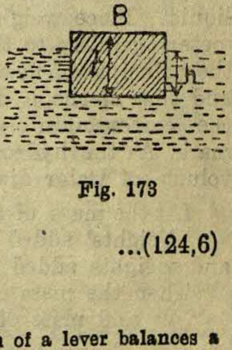


Fig. 173

Hence the volume of water displaced $= h\alpha$.
So the mass of water displaced $= h\alpha\rho$.

$\therefore$ For floatation, $l\alpha S\rho = h\alpha\rho$ whence $S = \dfrac{h}{l}$. ...(124,6)

Example :

A weight at a distance x from the fulcrum in one arm of a lever balances a solid suspended from the other arm in air. When the solid is immersed in water in a beaker placed under it, the weight has to be shifted at a distance y to restore the balance. Show that the specific gravity of the solid is x/y.
[Raj. U.—1953]

Ans. Let the mass of the solid in air be M_1 and let it be suspended from a point at a distance a on the side of the other fulcrum. Let the mass of the weight be m. Take moments about the fulcrum and for equilibrium we get

$$M_1 a = mx \quad \dots \quad (1)$$

If the mass of the solid in water be M_2, then because the 'weight' has to be brought nearer the fulcrum by a distance y so as to restore balance, we take moments about the fulcrum so as to obtain

$$M_2 a = m(x - y) \quad \dots \quad (2)$$

On division of (2) by (1) we find

$$\frac{M_2}{M_1} = \frac{x - y}{x}, \text{ or, } \frac{M_2}{M_1} = 1 - \frac{y}{x}, \text{ or, } 1 - \frac{M_2}{M_1} = \frac{y}{x}.$$

Hence the desired sp. gr. $= \dfrac{M_1}{M_1 - M_2} = \dfrac{x}{y}$.

125. Determination of specific gravity of Liquids—To determine the specific gravity of a liquid, methods as those of a solid are applicable. The usual temperature correction may, if necessary, be applied. The specific gravity of liquid may be determined in the following ways :

(1) By Hydrostatic Balance—Take a piece of glass (or similar solid) which is heavier than and chemically unaffected by the liquid as also by water. Weigh the solid in air, suspend it from the left arm of the balance and weigh it first in water and then in the given liquid.

Let the mass of the sinker in air $= W$
Mass of sinker in water $= W_1$
and mass of sinker in the given liquid $= W_2$
Then the mass of liquid displaced by the sinker $= W - W_2$
and the mass of water displaced by the sinker $= W - W_1$

$$\therefore \text{ the specific gravity S of the given liquid} = \frac{W - W_2}{W - W_1} \qquad ...(125,2)$$

(2) By Nicholson Hydrometer—Find the weight of the hydrometer in an ordinary balance. Float the hydrometer on water and add weights on the upper pan to sink it up to the mark. Note down the weights. Remove the hydrometer and float it on the given liquid. Place weights on the upper pan in order to sink the hydrometer again up to the mark and note down the weights. Then from the principle of floatation, the sum of the weights placed on the pan and that of the hydrometer is, in each case, equal to the weight of the corresponding liquid displaced by the hydrometer up to the mark. As the hydrometer sinks to the same point in both cases, the volume of water displaced is equal to the volume of liquid displaced.

Let the mass of the hydrometer $= W$
'Weights' added to sink it up to the mark in water $= W_1$
and weights added to sink it up to the mark in liquid $= W_2$
Then the mass of water displaced $= W + W_1$
and mass of liquid displaced $= W + W_2$

$$\text{Hence the specific gravity S of the liquid} = \frac{W + W_2}{W + W_1} \qquad ...(125,2)$$

Examples :

1. A Nicholson hydrometer sinks to a given mark in water when 3·32 gms. is placed on the upper pan and to the same mark in a liquid of specific gravity 1·02 when 9·41 gm. is placed on the upper pan. Find the weight of Nicholson's hydrometer. [Pat. U.—1951]

Ans. The specific gravity S of the liquid = 1·02 ; the weight W_1 to sink upto the mark in water = 3·32 gm. ; and the weight W_2 to sink up to the same mark in the liquid = 9·41 gm. If W be the weight of the hydrometer, we deduce from the standard formula :

$$S = \frac{W + W_2}{W + W_1} \text{ or, } \frac{W + 9{\cdot}41}{W + 3{\cdot}32} = 1{\cdot}02$$

or, $1{\cdot}02 \times W + 1{\cdot}02 \times 3{\cdot}32 = W + 9{\cdot}41$, whence the required weight $W = 301{\cdot}2$ gm.

2. A Nicholson's hydrometer floats on a liquid with a part of its stem immersed. On placing a weight of 40 miligrams on the upper pan, one centimetre more of the stem goes under the liquid. If the diameter of the stem is 2 mm., find the density of the liquid.

Ans. When the hydrometer floats on a liquid, it displaces a volume (say, V c.c.) of the liquid whose weight is equal to the weight of the hydrometer. When a load of 40 mgm. is put on the upper pan, the weight of the hydrometer virtually increases by the amount and in this case the volume of the liquid displaced is V c.c. *plus* the additional volume displaced. Thus the weight of this additional volume must be equal to 40 mgm.

Now the additional volume of liquid displaced = height × cross-section of the stem $= 1 \times \pi \times (\cdot 1)^2$ c.c. $= 0314$ c.c.

If the density of the liquid be ρ, the mass of this liquid $= \cdot 0314 \times \rho$. By hypothesis, this is equal to 40 mgm., *i.e.*, ·04 gm. Hence we get the equation:
$\cdot 0314 \times \rho = \cdot 04$, whence the density looked for $= 1{\cdot}274$ gm./c.c.

(3) **By Variable Immersion (or Constant Weight) Hydrometer**—In this case, the weight of the instrument as a whole remains constant, while the portion of the hydrometer immersed varies with the nature of the liquid. The instrument is made of glass. Usually it consists of a narrow hollow stem attached to a hollow body ending in a bulb at the bottom (Fig. 174). This bulb is weighed with mercury (or lead shots) so as to make the instrument float vertically. The stem contains a paper scale inside. The mark, at which the liquid surface meets the stem, directly reads off the specific gravity of the liquid under examination.

The instrument is graduated by floating it on a liquid of known specific gravity. With higher specific gravity of the liquid, the hydrometer floats out more. On the other hand, the lower is the specific gravity, the further it sinks. The graduations are, therefore, made from the top downwards. For different specific gravities ranging between 0·7 and 2 there is usually a set of hydrometers enclosed in a box. When made to float on water at 4°C, the reading is 1000, *i.e.*, specific gravity of water = 1·000; a reading of 1750 indicates that specific gravity is 1·750.

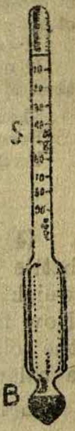


Fig. 174

These instruments are used for industrial purposes where the specific gravity of a liquid is to be readily determined. For special purposes instruments of different types are constructed and are differently named. The specific gravity of milk is tested by an instrument called a Lactometer, which may be graduated by immersing it in mixture of milk and water in various different proportions. Thus, if it sinks up to the mark ½, it indicates a mixture of milk and water in equal proportions. The specific gravity of ordinary milk is lighter than skimmed milk. The specific gravity of urine is tested by *urinometer* and that of alchol by *alcholometer*.

Examples:

1. A common hydrometer has a uniform stem on which the readings 1·00 and 1·10 are 8 cms. apart. Find the distance of the reading 1·06 from 1·10.
[Mad. U.—1951]

Ans. Let M be the mass of the hydrometer and V its volume from the bottom up to the graduation marked 1·10. When made to float on a liquid of specific gravity 1·10, then according to the law of floatation, $V \times 1{\cdot}1 = M$. If α sq. cm. be the area of cross-section of the stem, then when made to float on a liquid of specific gravity 1, $M = V + 8\alpha$.

Thus $M = V \times 1{\cdot}1 = V + 8\alpha$, whence $\alpha = \dfrac{V \times \cdot 1}{8}$ sq. cm.

Let l cm. be the distance of graduation 1·06. Then, from the law of floatation,
$M=(V\times l\alpha)\times1{\cdot}06=(V+lV\times0{\cdot}0125)\times1{\cdot}06$
and since $M=V\times1{\cdot}1$, $(V+lV+0{\cdot}0125)\times1{\cdot}06=V\times1{\cdot}1$
and $l=3$ cms. very approximately.

2. The stem of a common hydrometer is cylindrical and the highest graduation corresponds to a specific gravity of 1·0 and the lowest to 1·4. What specific gravity corresponds to a point exactly midway between the divisions?

[Utkal U.—1953]

Ans. Let M be the mass of the hydrometer and V its volume from the bottom up to the graduation marked 1·4. When made to float on a liquid of sq. gr. 1·4, then according to the law of floatation,

$M=V\times1{\cdot}4$ ... ... (1)

Suppose l be the length of the stem from graduation 1·4 to and α, the area of cross-section of the stem assumed uniform. When made to float on a liquid of sp. gr. 1·0, then from law of floation,

$M=(V+l\alpha)\times1=V+l\alpha$ ... ... (2)

If now S be the sp. gr. of the liquid on which it floats exactly $\frac{1}{2}l$ exposed, then $M=(V+\frac{1}{2}l\alpha)S$.

Then from eqn. 1, $V=0{\cdot}714\ M$
From eqn. 2, $M=V+l\alpha=0{\cdot}714M+l\alpha$ or, $0{\cdot}286M=l\alpha$
From eqn. 3, $M=(V+\frac{1}{2}l\alpha)S=(0{\cdot}714M+0{\cdot}143M)S$
or, $S=\frac{1}{0{\cdot}857}+1{\cdot}167.$

(4) By Specific Gravity Bottle—Weigh a clean, dry and empty bottle with the stopper. Then fill it completely with water and weigh it again. Pour out the water and dry the bottle carefully. Now fill it completely with the given liquid and weigh again. In the following case, *weights* mean masses of the bodies as measured in a balance.

Let weight of the empty bottle $=W$
weight of bottle filled with water $=W_1$
weight of bottle filled with the given liquid $=W_2$
Then the weight of water filling the bottle $=W_1-W$
and the weight of liquid filling the bottle $=W_2-W$

Hence the specific gravity S of the liquid $=\dfrac{W_2-W}{W_1-W}$ ...(125,3)

(5) By Hare's Apparatus or Watt's Hydrometer—The specific gravities of two liquids, which mix with each other, can be readily compared by this apparatus. It is merely an inverted U-tube, having a branch tube at the top end, to which a piece of rubber tubing is attached (Fig. 175). This is again fitted with a clip and has a short piece of glass tube attached at the other end. The U-tube is held vertically in a suitable frame and the lower ends of two limbs dip into the separate beakers, each containing a liquid. When by sucking at the open tube, air is partially withdrawn from inside, this pressure of air inside is decreased and the external atmospheric pressure forces the liquid to convenient heights, when the rubber tube is closed by the clip. The height of each liquid column above the surface of the corresponding liquid in the beaker is either read off the scale attached or measured by a scale.

Let the respective densities of the two liquids be $=\rho$ and ρ'
and corresponding heights of the liquid columns $=h$ and h'
and the atmospheric pressure at the time $=P$.

Now the pressure of air inside the tube is the same throughout; let it be p. Also the atmospheric pressure at the surface of the liquid in each vessel is P. Evidently then,

$$p + g\rho h = \mathrm{P} = p + g\rho' h' \text{ or, } \rho h = \rho' h',$$

$$\text{whence } \frac{\rho'}{\rho} = \frac{h}{h'}, \text{ or, } \rho' = \frac{h}{h'}\rho. \quad \dots \quad \dots \quad \dots (125{,}4)$$

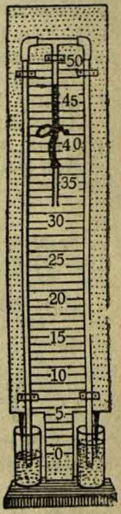


Fig. 175

Thus the densities of the two liquids vary inversely as the heights of the liquid columns, measured from the respective surfaces. Hence the densities of the liquids can be readily compared and if one of them be known, the other is obtained. For example, if one of the liquids be water, the density (and so the specific gravity) of the other is directly found. It is to be noted that the cross-sections of the tubes do not come into consideration and that the method can be applied to all liquids.

(6) By Balancing Columns (U-tube).—This is a convenient and ready method of comparing the specific gravities of two liquids (*e.g.*, mercury and water) which neither mix nor have any chemical action with each other. Take a glass U-tube ABC (as in Figure 162) and pour the heavier liquid into it. Then pour the lighter liquid carefully into the limb AD. The two liquids have a common surface of contact at D. Let the horizontal line through D meet the other limb at E. If the heights of the liquid columns from the common surface be h_1 and h_2 respectively and if ρ_1 and ρ_2 be their respective densities then proceeding as in Art. 115, it can be shown that

$$\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}, \text{ whence } \rho_2 = \rho_1 \frac{h_1}{h_2} \quad \dots (125{,}5)$$

whereby the specific gravities of liquids can be compared or the specific gravity of one liquid can be found if that of the other is known.

Specific Gravities of Common Substances

Substances	sp. gr.	Substances	sp. gr.	Substances	sp. gr.
Steel	7·7-7·9	Duralumin	2·8	Celluloid	1·4
Iron (pure)	7·87	Gunmetal	8·0-8·4	Cork	·22-·2
Brass	8·4-8·7	Manganin	8·5	Glycerine	1·26
Gold	19·32	Marble	2·0-2·8	Turpentine	·87
Copper	8·93	Quartz	2·66	Kerosene	·80
Silver	10·5	Sand	2·3-2·6	Castor oil	·97
Platinum	21·5	Glass, crown	2·4-2·6	Olive oil	·92
Lead	11·37	Sugar	1·52	Paraffin oil	·96
Nickel	8·9	Alum	1·52	Mustard oil	·82
Tin	7·3	Paraffin Wax	·87-·96	Petrol	·72
Copper Sulphate	2·1	Sealing Wax	1·8	Milk	1·03
		Common Salt	1·6		

Examples :

1. The specific gravity of ice is 0·918 and that of sea water is 1·03. What is the total volume of an iceberg which floats with 700 cu. yds. exposed ? [C. U.—1942]

Ans. Let the total volume of the iceberg = V cu. yds.
Then volume under water $= (V - 700)$ cu. yds.
∴ mass of the iceberg $= V \times 27 \times 0{\cdot}918 \times 62{\cdot}5$ lbs.
Also mass of sea-water displaced $= (V - 700) \times 27 \times 1{\cdot}03 \times 62{\cdot}5$ lbs.
∴ $V \times 27 \times 918 \times 62\ 5 = (V - 700) \times 27 \times 1{\cdot}03 \times 62{\cdot}5$
whence $V = 6437{\cdot}5$ cu. yds.

2. Find a mathematical expression for the density of a mixture when the densities and the masses of the components are known.

Calculate the quantity of pure gold in 100 gms. of an alloy of gold and copper of density 16. Density of gold = 19 and that of copper = 9. [Dac. U.—1940, '52]

Ans. Let the mass of the components be m_1 and m_2 and their densities, be ρ_1 and ρ_2 respectively. Let the required density of the mixture be ρ.

Now volume of the mixture = total volume of the components.

$$\therefore \quad \frac{m_1 + m_2}{\rho} = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}, \text{ whence } \rho \text{ can be calculated.}$$

Problem—Let the quantity of pure gold be x gm. ; then quantity of copper = $(100 - x)$ gms.

Now volume of the alloy $= \frac{100}{16}$, volume of gold $= \frac{x}{19}$
and volume of copper $= \frac{100 - x}{9}$

$$\therefore \quad \frac{100}{16} = \frac{x}{19} + \frac{100 - x}{9}, \text{ whence } x = 83{\cdot}125 \text{ gms.}$$

3. The densities of three liquids are in the ratio 1 : 2 : 3. What will be the relative densities of the mixture by combining *(a)* equal volumes, *(b)* equal weights ? [Gau. U.—1953 ; C. U.—1954]

Ans. Let D be the density of the first liquid. Then the densities of the second and the third liquids are 2D and 3D.

(a) If in the mixture the volume of each liquid be V, provided there is neither contraction nor expansion of the resulting volume, the volume of the mixture is 3V. If ρ be the resulting density, the mass of the mixture is obviously $3V\rho$.

Then the masses of the liquids in the mixture are VD, 2VD and 3VD. Hence from the principle of conservation of mass,

$3V\rho = VD + 2VD + 3VD$, whence $3\rho = 6D$, or the density required = 2D.

(b) If in the mixture the mass of each liquid be m, the total mass of the liquid is $3m$ and the total volume is $3m/\rho$.

The volume of the individual liquids are $\frac{m}{D}$, $\frac{m}{2D}$ and $\frac{m}{3D}$.

The principle of conservation of mass allows us to ensure that

$$\frac{3m}{\rho} = \frac{m}{D} + \frac{m}{2D} + \frac{m}{3D}, \text{ whence the desired density } \rho = \frac{18}{11}D.$$

4. A cylinder of wood whose specific gravity is 0·25, has another cylinder of metal (specific gravity 8·0) attached to one end. The cylinders are 2 inches in diameter. They have the same axis and are respectively 20 in. and 1 in. long. If the whole is placed in water, find how much of it will be above the surface. [Utkal U.—1952]

Ans. Let the height below the surface be h in.

Now the volume of the metal cylinder $= (\pi \times 1 \times 1)$ cu. in. $= \frac{\pi}{1728}$ cu. ft.

$$\therefore \text{ the mass of the metal cylinder} = \left(\frac{\pi}{1728} \times 62{\cdot}5 \times 8 \right) \text{ lbs.}$$

Also the volume of the wooden cylinder$=(\pi\times1\times20)$ cu. in.$=\frac{\pi\times20}{1728}$ cu. ft.

So the mass of the wooden cylinder$=\left(\frac{\pi\times20}{1728}\times62{\cdot}5\times0{\cdot}25\right)$ lbs.

Also the volume of water displaced$=(\pi\times1\times h)$ cu. in.$=\frac{\pi\times h}{1728}$ cu. ft.

$\therefore$ the mass of water displaced$=\left(\frac{\pi\times h}{1728}\times62{\cdot}5\right)$lbs.

Since the total mass of the cylinder is equal to the mass of water displaced, we get the equation $\frac{62{\cdot}5}{1728}\times\pi h=\frac{62{\cdot}5\times8}{1728}\times\pi+\frac{62{\cdot}5\times0{\cdot}25}{1728}\times20\pi$

or, $h=8+20\times0{\cdot}25=8+5=13$ in.

So the height above the water surface$=(21-13)$ in.$=8.$ in.

5. A silver ornament is suspected to be hollow. Its weight is 288·75 gm. and it can displace 30 c.c. of water. If the specific gravity of silver is 10·5, find the volume of the cavity. (P. U.—1946)

Ans. The volume of the ornament$=(288{\cdot}75\div10{\cdot}5)$ c.c.$=27{\cdot}5$ c.c.

But the outer volume of the ornament$=30$ c.c.

$\therefore$ the volume of the cavity$=(30-27{\cdot}5)$ c.c.$=2{\cdot}5$ c.c.

6. The cross-section of the arms of a U-tube are 2 sq. cm. and 1 sq. cm. and it is placed vertically. A quantity of mercury of sp. gr. 13·65 is poured into the U-tube. Find the height through which mercury will descend in the wider arm when 52 c.c. of water is poured into this arm. (Utkal U.—1954)

Ans. 52 c.c. of water within the wider arm of cross-section 2 sq. cm. would occupy a vertical column of 26 cms. Thus, a column of water of height 26 cms. would exert a pressure of $26\times g$ dynes per sq. cm. on the surface of mercury. With respect to this mercury surface, the free mercury surface in the narrower column would rise by h cm., where, $h\times13{\cdot}65\times g=26\times g$, whence $h=1{\cdot}9$ cms.

Since the cross section of the narrower column is 1 sq. cm. the rise in the narrower column is twice that in the wider one. Hence, depression of mercury surface in the wider column is $1{\cdot}9\div2=0{\cdot}95$ cm.

***126. Surface Tension**—It has already been stated that there is a cohesive force between the same kind of molecules of a solid or a liquid. The effect of such cohesive forces on the *free surface of a solid* or *a liquid* is a force of contraction acting along the bounding surface and this contractible force is different for different substances. Since a liquid can very easily take up any shape, this force of contraction along the surface may be demonstrated very conveniently with a liquid as shown in the following experiment:

ABCD represents a metal wire bent twice at right angles at points B and C, so that the arms AB and CD are parallel to each other (Fig. 176). Another straight piece of *thin* wire EF which is tied with a thin piece of thread at the middle point can slide over the parallel arms. A soap solution is made in a vessel and this wire combination

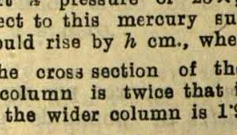


Fig. 176

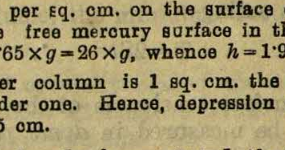


Fig. 177

is dipped into the solution. On putting the wire out, a soap film is formed within the space BCEF and on releasing the wire EF it will be found to be moving towards BC. The force, that moves the wire is due to the contractile force acting on the surface of soap solution. To maintain the film, a certain amount of force, although very small, would have to be exerted on the thread outwards. Unless this force is applied the wire EF would gradually approach BC and would ultimately coincide with it. The surface of the liquid, therefore, behaves as if it were in a state of tension like a thin stretched elastic membrane.

The phenomenon is explained by supposing that there is a system of forces acting along the surface of a liquid, Let PVQR be an element of the surface of a liquid and let PQ be an imaginary line drawn upon it (Fig. 177). At any point of this line, there are two forces equal and opposite and acting at right angles to this line. If we imagine a unit length along PQ, the force acting over this unit length is called the *force of surface tension.* It is to be borne in mind that such forces of tension act *over free surface of a liquid only and not within the interior of the liquid.*

Let the force applied along the outward direction just to balance the force of surface tension be T and if the length of the wire EF touching the film be l, then remembering that the soap film has got two free surfaces, upper and lower, on each of which the surface tension is acting—

$$T = 2Sl \quad \therefore \quad S = \frac{T}{2l} \qquad \dots \quad (126, 1)$$

The surface tension is of the dimension of a force per unit length and may be measured in *dynes per centimetre* or poundals per inch. Phenomena due to surface tension may alternatively be explained in terms of potential energy developed when a liquid surface is increased against the forces of surface tension. Suppose for instance, that the wire EF is pulled against the force of surface tension through a distance EG. The act of pulling should be so slow as *not to disturb the temperature* of the liquid. The work done to stretch the film surface is T × EG.

$$\therefore \; T \times EG = 2Sl \times EG = 2S \times (\text{area FEGH}).$$

The film surface, because of its two sides is actually increased by twice the area EFGH. Hence the work done per unit area is

$$\frac{T \times EG}{2 \times (\text{area FEGH})} = S.$$

Thus under isothermal condition the surface tension is *numerically* equal to the work done to increase the surface area of a liquid by unity. If the temperature changes, the force of surface tension also would suffer a change. The dimension of surface tension is a force divided by the length and may be expressed as $MLT^{-2} \div L = MT^{-2}$.

*127. **Illustrations of Surface Tension**—The following illustrations show the existence of the surface tension of a liquid. A round wire frame is dipped into a soap solution so as to form a film whereon is placed a short silk thread A with a knot at the ends. The closed loop with the thread may lie on the film in any manner as shown in Figure 178(*a*). On now touching any portion of the film inside the loop with a needle point so as to pierce the surface, this point of the film disappears and immediately the loop of the thread takes a round figure B [Fig. 178(*b*)]. This is due to the uniform surface tension of the film acting along the external boundary line of the loop of the thread. For such a system of forces, the equilibrium condition is a circle.

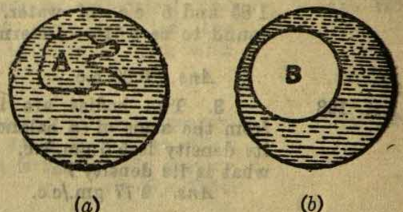


Fig. 178

It is always found that small drops of rain or dew or small floating particles of liquid in air are round in shape This can be explained on the basis of surface tension. To form a liquid surface some work is to be done. Consequently, some potential energy is associated with the surface of the liquid in question. It is a general law that for a stable equilibrium of a system, its potential energy would be minimum. Hence a liquid drop would assume such a surface as to have minimum potential energy. When forces of surface tension acting on the surface of the liquid drop are very great as compared to the weight of the liquid, the form of the liquid is round ; since for a given volume, the minimum surface area is that of a sphere.

A slightly greased sewing needle, if carefully placed on the surface of water in a vessel, is found to float although it is much heavier than water. It can be explained by supposing that due to the surface tension, the free surface of water behaves as an elastic membrane. Whenever the needle is placed on this membranous surface the part of it below the needle is depressed and consequently the surface as a whole is enlarged. The curved portion of the liquid surface below the needle in its attempt to straighten out offers a vertical force on the needle to support its weight. Spiders and insects are found to move and run on the surface of water without sinking. This is also due to the surface of water behaving as an elastic membrane strong enough to support the insects.

If a soap bubble, blown at the end of a pipe, be allowed to stand it gradually shrinks in volume. This shows that the surface tension tends to reduce the liquid surface to a minimum. If a camel-hair brush is dipped in water, the individual hairs project in different directions. But when taken out of water, the hairs are held together as if coated by a membrane. When the cohesive forces between the molecules of the liquid are less than the adhesive forces between the molecules of the liquid and the solid, the liquid wets the solid *e.g.*, water in contact with glass. But if the case is reverse, the liquid does not wet the solid, *e.g.*, mercury in contact with glass. Mercury sprinkled on a glass plate collects into small spherical drops, while water will spread out over the glass surface. If two glass rods are introduced in water and in mercury, then after withdrawal, the former is found to be moist while the latter is dry. (For other details of Surface Tension, *vide* Part III.)

EXERCISES ON CHAPTER X

Reference

Art. 123 1. Distinguish between density and specific gravity.
(E. P. U.—1961 ; Utk. U.—1959 ; C. U.—1958 ; Dac. U.—1950, '62)

Art. 123 2. A mixture is made of 7 c.c. of a liquid of specific gravity 1·85 and 5 c.c. of water. The specific gravity of the mixture is found to be 1·615. Determine the amount of contraction. (C. U.—1957)

Ans. 0·89 c.c.

Art. 123 3. Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 28 gm. and its density is 5·6 gm./c.c. If the mass of the other body is 36 gm., what is its density ? (Dac. U.—1952)

Ans. 2·77 gm./c.c.

Art. 123 4. A body weighs 121·5 gm. in air, and 106·11 gm. in one liquid and 102·25 gm. in another. Find the apparent weight of the body in a mixture formed by equal volumes of the two liquids.

Ans. 104·23 gm.

Art. 123 5. A cylinder of iron floats vertically and fully immersed in a vessel containing mercury and water. Find the ratio of the length of the cylinder immersed in water, to that immersed in mercury. (Sp. gr. of mercury = 13·6 ; sp. gr. of iron = 7·20) (Pat. U.—1955)

Ans. 97 : 113.

Art. 123 6. When equal volumes of two substances are mixed together, the specific gravity of the mixture is 4. But when equal weights of the same substances are mixed together, the specific gravity of the mixture is 8. Find the specific gravities of the two substances.

Ans. 6 and 2.

Art. 123 7. A piece of wax of volume 22 c.c. floats in water with 2 c.c. above the surface. Find the weight and the specific gravity of the wax.

Ans. 20 gm. ; 0·9.

Art. 124 8. A specific gravity bottle weighs 16·59 gms. when empty, 26·69 gms. when some quantity of salt is put into it. The bottle is then filled with kerosene oil and weighs 42·77 gms. The salt is now thrown out and the bottle is completely filled with kerosene oil, when it weighs 39·59 gms. If the specific gravity of kerosene oil be 0·8, find the sp. gr. of salt.

Ans. 2·06.

Art. 124 9. Describe any method of measuring specific gravity of a substance lighter than and insoluble in water. (Utkal U.—1949)

Art. 124 10. How would you proceed to find out the sp. gr. of (*i*) a solution of copper sulphate, (*ii*) a solid crystal of copper sulphate ? Find the sp. gr. of a solid from the following data :

Weight of solid in air = 0·4 gm.
Weight of sinker in air = 4·0 gms.
Weight of solid and sinker in water = 9·37 gms.
Sp. gr. of sinker = 8·00 (Dac. U.—1934)

Ans. 0·75.

Art. 125 11. A Nicholson's hydrometer sinks to a certain mark in a liquid of sp. gr. 0·6 ; but it takes 120 gms. to sink it to the same mark in water. What is the weight of the hydrometer ? (C. U.—1958)

Ans. 180 gms.

Reference

12. Explain the principle of a Nicholson's hydrometer. How would you use it for finding out relative density of cork. (*Cf.* Utkl. U.—1954; R. P. B.—1916, '48, *Cf.* Anna. U.—1951; *Cf.* E.P.U.—1953) Art. 124

13. A glass tube 30 cms. long and ½ sq. cm. in cross-section is closed at one end; its weight is 4 gms. and 10 gms. of mercury are poured into it. What will be the sp. gr. of liquid in which it floats vertically with 2 cms. length of its stem above the surface? (P. U.—1956) Art. 125

Ans. 1.

14. The stem of a common hyrdometer is cylindrical and the range of the instrument is from 1·0 to 1·25. Find the specific gravity of a liquid in which the instrument floats with one-fourth of its stem exposed. Art. 125

Ans. 1·05.

15. A specific gravity bottle weighs 14·72 gms. when empty, 39·74 gms. when filled with water, and 44·15 gms. when filled with a solution of common salt. What is the specific gravity of the solution? (C. U.—1954) Art. 125

Ans. 1·7.

16. A U-tube contains some mercury at the bottom. Water is poured into one limb and oil into the other. When the heights of the columns are respectively 7·6 cms. and 9·5 cms. the mercury stands at the same level in both the limbs. Find the density of the oil. Art. 125

Ans. 0·8 gms./c.c.

17. A hollow stopper made of glass of density 2·6 gms./c.c. is found to weigh 23·4 gms. in air and only 8·9 gms. in water. What is the volume of the air in the internal cavity of the stopper? (Utkl. U.—1962) Art. 124

Ans. 10·5 c.c.

18. Describe and indicate the use of a Hare's Apparatus. (Pat. U.—1954; C.U.—1953, '59; Anna. U.—1951) Art. 125

19. Explain with illustration the phenomena of surface tension. (And. U.—1951; *Cf.* E. P. U.—1952) Art. 126

20. What do you understand by the term 'surface tension' of a liquid? How do you account for it? Describe some simple experiments to demonstrate its effect. (P. U.—1962; *Cf.* E. P. U.—1963) Arts. 126 & 127

CHAPTER XI

PROPERTIES OF GASES

128. Gases and Liquids—As already mentioned, gases have several properties in common with liquids and both are commonly termed *fluids*. Gases like liquids exert pressure and possess elasticity of volume but *not* of shape. A gas does not possess a free surface as a liquid does. On the other hand gases being much lighter than liquids, are very compressible and have got the property of indefinite expansion. Any volume of a gas possesses weight. The fact that air has weight can be shown by the following experiment, first undertaken by Otto Von Guericke (1602–1686) in 1660. It is significant to note that a few years before this experiment, Guericke invented the air pump.

A fairly large glass globe 3 to 4 inches in diameter and provided with a stop-cock, is taken. By means of air pump the globe is evacuated as much as possible and the stop-cock is closed. The globe is now carefully weighed in a balance. The stop-cock is then opened and air is re-admitted into the globe. On weighing the globe again, it is found to be heavier. The difference between the two weights evidently gives the weight of the air admitted into the globe. With any other gas the above experiment can be performed which shows that all gases possess weight.

129. Pressure of Atmospheric Pressure—The air surrounding the earth extends up to a considerable height from the surface. This envelope is called the *atomosphere.* The atmosphere may be supposed to be divided into a number of horizontal layers parallel to the surface of the earth ; each layer bears the weight of all other layers above it and is thus subjected to a pressure due to their weight. This pressure is called the *atmospheric pressure.* Evidently this pressure is greatest at the surface of the earth and decreases as we move higher. Further, in the case of air at rest the pressure at all points on the same horizontal plane is the same. The idea of equality of pressure on the same horizontal plane first struck the mind of Blaise Pascal (1623-1662), a French scientist. As different layers are compressed to different extents, they possess different densities, the lowest one being densest.

The *existence of the atmospheric pressure* can be demonstrated by the following experiments. Take a glass tumbler filled with water to the brim and slide carefully a sheet of card-board on its rim. If the tumbler is now carefully inverted, the water does not fall, showing that the upward pressure of the atmosphere on the lower surface of the card-board supports the weight of water and the card-board.

Depress an empty glass tumbler into water with mouth downwards. The level of water inside the tumbler is found to be lower than that of outside. This is due to the pressure of enclosed air which depresses the level of water inside.

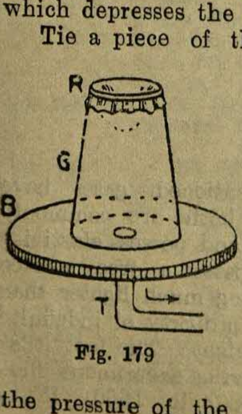


Fig. 179

Tie a piece of thin rubber sheet R airtight over a stout glass cylinder G open at both ends (Fig. 179). Place the other end on the plate B of an air-pump after having it well-greased. Now evacuate the air gradually from inside the cylinder. The rubber sheet is depressed more and more and finally it bursts with a loud report. This is due to the fact that when air is taken out, the inside pressure falls and the pressure of the outside air depresses the rubber sheet which ultimately bursts when this pressure becomes too much. If the upper end of the cylinder is pressed by the palm of the hand in place of the rubber sheet, then on working the pump, the pressure of the air can be felt. When the exhaustion has proceeded considerably, the hand is pressed so much that it is matter of effort to lift it up at this stage.

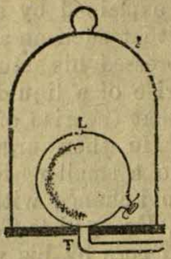


Fig. 180

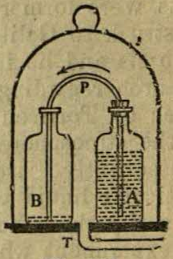


Fig. 181

Take a toy rubber balloon partially filled with air. Tie its mouth with a string and put it under receiver of an air-pump. On working the pump, the balloon gradually swells up as evacuation proceeds on (Fig. 180). Now readmit air into the receiver when the balloon again collapses to its original form. When the receiver is full of air, the air inside the balloon exerts pressure on the wall, which is balanced by the pressure of air within the receiver. But when the receiver is gradually evacuated, the outside pressure decreases, and the pressure inside the balloon becomes greater and so it swells up.

Take two bottles A and B connected together by a bent tube passing through *an air-tight* stopper, through which one end of the bent glass tube dips into the liquid (Fig. 181). The end of the tube within the other bottle passes very loosely, so that the inside of this bottle is in communication with the outside air. Put the two bottles under the receiver of an air-pump and gradually evacuate. It will be found that the liquid flows from the closed bottle to the open one through the bent tube. This happens, because when the receiver is evacuated, the pressure of air within the closed bottle, rises higher and forces the liquid out through the tube.

Magdeburg Hemispheres Experiment—The existence of atmospheric pressure was demonstrated with brass hemispheres by Otto Von Guericke in 1654. The apparatus consists of two hollow brass hemispheres (Fig. 182), whose edges are so grinded that they fit each other exactly. One of these is provided with a handle and the other with a stop-cock, a handle being screwed to the latter. The edges of the hemispheres are well-greased and put together, so that they fit air-tight. The air from inside is then pumped out. A very great force is now required to separate the hemispheres. The reason is simple and is stated below.

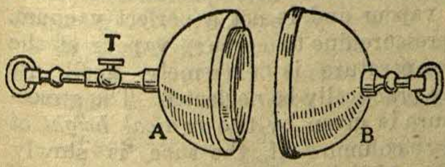


Fig. 182—Magdeburg Hemispheres

When there is air inside, the internal pressure balances the external pressure. But when the air from inside is removed, the atmospheric air presses the hemispheres from all sides and keeps them very tightly pressed against each other. On re-admitting air the hemispheres can be easily separated.

130. Rise of Liquid in an Exhausted Tube—When the lower end of an open tube is dipped into a liquid and air is sucked out through the upper end, the liquid rises up the tube. Similarly, water rises in a syringe in a pump as the piston is pushed up. All these facts were formerly explained by assuming that *nature abhors vacuum.* It struck to Galileo that such an assumption could not have a scientific basis and he impressed his student Torricelli to go on with the research regarding 'rise of a liquid in vacuum'. The explanation was given by Torricelli that the rise of the liquid is due to the pressure of the atmosphere. He then argued, that a heavier liquid should consequently rise to a smaller height and that the height for a particular liquid would change with a change of atmospheric pressure. To verify these facts, Evangelista Torricelli (1608-1647) in 1643 devised an experiment which goes by his name.

Torricelli's Experiment—Take a thick glass tube, about a metre long and closed at one end. Taking care that no air bubble sticks to the inside of the tube, fill it completely with pure mercury. Now close the open end tightly with the thumb and invert the tube vertically over a basin D containing pure dry mercury so that the open end is under the mercury surface (Fig. 183). On removing the thumb the mercury column sinks down to some extent but remains stationary at a certain height. The height of this level from above the mercury surface in the basin is found to be nearly 76 cms.

Explanation—The mercury surface in the basin is acted on by the pressure of the atmosphere which is transmitted undiminished in all directions. Again the pressure due to the mercury column in the tube at a point in level with the outside surface, is due to the weight of the mercury column of height equal to the vertical distance between the two mercury surfaces. As the mercury column is in equilibrium these two pressures balance each other and must, therefore, be equal in magnitude. Hence, the atmospheric pressure maintains the weight of the mercury column. If an opening be made at the top of the tube, the atmospheric pressure would also act upon the mercury inside the tube and so the column would come down to the level of mercury in the basin. Evidently then *the atmospheric pressure is equal to the pressure due to a column of mercury nearly 76 cms. in height.* Torricelli was the pioneer in devising an original form of a barometer.

The space within the tube above the mercury surface contains no air and is usually referred to as *Torricellian vacuum.* Really, however, the space contains a little of mercury vapour and is not a perfect vacuum. But as the pressure due to mercury vapour at the ordinary temperature is extremely small, its pressure may practically be neglected. The atmospheric pressure is given by the *vertical height* of the mercury column. If the tube is slowly inclined (Fig. 183) it is found that the space above mercury column gradually decreases but the vertical height measured from the mercury surface in the basin remains the same. When the tube is sufficiently inclined the mercury strikes the top of the tube with a smart metallic sound,

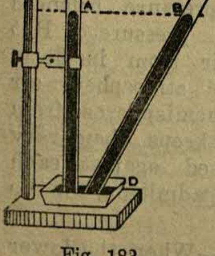


Fig. 183

showing that there is no air inside. The height of the mercury column changes from time to time and also at different places, which shows that the *atmospheric pressure is not constant.*

If there is a change of pressure above the free surface of mercury, the height of mercury column is also found to change. Place the basin under the receiver of an air-pump while the tube passes through an air-tight rubber-cork fitted to the open top of the receiver. As the pump is worked, the level of the mercury column gradually sinks down. On re-admitting air into the receiver, the mercury column gradually rises to its original level in the tube.

Take the tube as in figure 183 and *raise* it gradually with the open end always under mercury. The space above mercury increases, but the height of the mercury column measured from the surface of mercury in the basin remains the same. If a very deep cistern is taken and the tube is gradually *pushed* in it, the space above mecury decreases but the height of the mercury column above the outside surface remains the same. The mercury ultimately touchas the top of the tube and if the latter is pushed further down, the tube remains always filled with mercury. The height of the mercury column will not remain the same, if the space above mercury contains air or any other gas.

Pascal (1623-1662) suggested that since the atmospheric pressure decreases as we go up, the height of the mercury column also would decrease accordingly. In 1648, Perrier went up to the top of Puy-de Dome, a mountain in France about 1000 metres in height, and found that the mercury column actually came down through 8 cm. It was also found that with different liquids, the vertical columns stood at diffirent heights.

131. Magnitude of the Atmospheric Pressure—Since the pressure of the atmosphere supports a column of mercury, 76 cm. in height, the atmospheric pressure per sq. cm. is equal to the weight of a column of mercury of 1 sq. cm. in cross-section and of 76 cm. in height. Taking the mean value of g to be 981 cm./sec.^{2} and the density of mercury to be 13·6 gm. per c.c., the atmospheric pressure

$= \text{wt. of } 76 \text{ c.c. mercury} = 76 \times 13{\cdot}6 \text{ gm. wt. per cm.}^2$

$= (76 \times 13{\cdot}6 \times 981) \text{ dynes per sq. cm.}$

$= 1{,}013{,}961 \text{ dynes per sq. cm.}$

So the atmospheric pressure is **approximately** equal to one million dynes or 10^6 dynes per sq. cm.

Expressed in F. P. S. units the atmospheric pressure = weight of a column of mercury 30 inches in height and 1 sq. in. of cross-section = weight of 30 cu. inches of mercury.

$$= \left(30 \times 13{\cdot}6 \times \frac{62{\cdot}5}{1728}\right) \text{ lbs. wt. per. sq. in.} = 14{\cdot}8 \text{ lbs. wt. per sq. inch.}$$

$= (14{\cdot}8 \times 32) \text{ poundals per sq. in.} = 473{\cdot}6 \text{ poundals per sq. in.}$

Hence the average atmospheric pressure is approximately equal to *15 lbs. wt. per sq. inch* or nearly one ton per sq. foot. When any gas or liquid exerts an equivalent pressure, the pressure is often mentioned as being that of *one atmosphere.*

Since density of mercury is 13·6 times the density of water, the height of *water column* supported by atmospheric pressure
$=(30 \times 13{\cdot}6)$ in. $=34$ ft. (nearly).

Normal (or Standard) Atmospheric Pressure—The *normal* (or *standard*) atmospheric pressure is defined to be the pressure exerted by a column of mercury, 76 cm. in height, and of density 13·5951 gm./c.c. subject to the gravitational acceleration of 980·665 cm./sec.^{2} This is taken to be the standard, with which other pressure may be compared.

Hence the **standard atmospheric pressure**
$=(76 \times 13{\cdot}5951 \times 980{\cdot}665)$ dynes per sq. cm.
$=1{,}013{,}237$ dynes per sq. cm.

In meteorology, a pressure of one million dynes per sq. cm. is called a **bar.** This is also equivalent to a pressure of 29·52 inches of mercury. One thousandth part of *bar* is called a *millibar.* The standard atmospheric pressure is thus equal to 1013·24 millibars.

Pressure on the Human Body—Taking the average surface area of the body of a man of middle size to be about 16 sq. ft. the pressure on the body amounts to $(15 \times 12 \times 12 \times 16)$ lbs. or over 15 tons weight. It is very surprising to think how a human being can withstand such an enormous thrust without least discomfort while it is absurd for him even to carry a load of one ton ! The explanation is not difficult to find, for there are openings of innumerable pores on the skin of the body, through which air goes within the system and there is a free communication between inside and outside. The pressure of inside air counterbalances the pressure outside and consequently just as a fleece of cotton in air can fly about safely without being squeezed by atmospheric pressure, any fine part of a human system is equally safe. The pressure of blood within veins is matched against the outside atmospheric pressure. But if the outside pressure *considerably* alters, for example, in climbing up high altitude or in diving under sea, precautions should be taken to conpensate for the pressure of inside and outside. The space-suit, worn by the aeronauts in outer atmosphere is provided with double walled jacket for the compensation of outside pressure and consequently there is no feeling of discomfort even staying in vacuous space.

132. Barometers—A barometer is an instrument for measuring the pressure of the atmosphere at any time in terms of the height of the liquid column. Mercurial barometers are of two types—(*i*) *Cistern barometer* and (*ii*) *Siphon barometer.* There is another type of barometer, called *Aneroid barometer,* in which a liquid is not used.

Cistern Barometer—A glass tube containing mercury and standing vertically on a cistern of mercury constitutes a very simple form of the cistern barometer. The height of the mercury column within the tube measured from the mercury surface in the cistern measures the atmospheric pressure at the time. When the atmospheric pressure changes, that height of mercury column in the tube and consequently the level of mercury in the cistern also changes. Hence a scale cannot be permanently fixed with its zero mark in level with surface in the cistern. To get an accurate reading of the pressure, therefore, either the scale should be variable in position or the level of mercury in the cistern must necessarily be kept invariable. The latter condition is attained in a type of barometer as given below :

Fortin's Barometer—This is most accurate type of mercurial barometer of the cistern type and is, therefore, widely used. It essentially consists of a barometer tube which is completely filled with pure dry mercury with no air bubble sticking to the inside (Fig. 184) and the tube is inverted over a cistern of mercury called the *reservoir*. The mercury stands at a height depending on the amount of atmospheric pressure. The upper part of the reservoir consists of a glass cylinder A (Fig. 185) which allows the mercury surface to be seen. The cylinder is fitted at its lower end into a box-wood cylinder B which is surrounded by a brass case. The lower end of B is closed by a piece of flexible leather which thus forms the true base of the cistern. The leather base is provided with wooden bottom C at the lower surface. A screw S passes through the bottom of the cistern and presses against C. By turning the screw, the bottom can be raised or lowered, so that the level of mercury of the reservoir can be altered at will. The upper end of the cistern is closed by a strip of leather through the pores of which communication is maintained with the external air and thus the atmospheric pressure is transmitted to the inside. A small ivory *pointer* P (Fig. 184) is fixed vertically to the lid of the cistern such that its lower tip is on the same horizontal plane as the zero of the scale.

The *barometer tube* is made of fairly large bore at the upper side to avoid a tangible pressure effect on mercury column due to the surface tension. The tube is also drawn to a small opening at the lower end and is constricted a little above it to reduce any oscillation which may be set up in the mercury column during adjustment. The barometer tube is enclosed for safety within a long brass tube. In the upper part of this tube, two wide vertical slits are cut diametrically opposite to each other, so that the upper level of mercury may be seen through them.

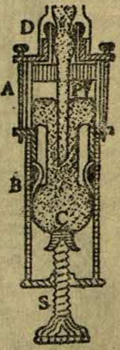


Fig. 185

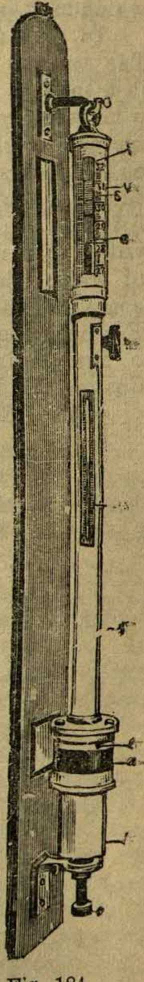


Fig. 184

The linear *scale*, S (Fig. 184) for measurement is engraved on the brass tube along the edges of the front slit, one edge being graduated in centimetres and the other in inches. Although the scale continues throughout the length of the case, it is usually graduated from about 27 to 32 inches or 68 to 80 cms. towards the top, the zero of each corresponding to the tip of the pointer. A brass vernier scale V can be slided up and down in the front slit by rack and pinion arrangement, worked by the screw H. The lower edge of the vernier is exactly coincident with the zero marks on either side To know the atmospheric temperature, a *thermometer* T is fixed to the lower part of the brass case. The barometer is attached to the wooden board which may be hung within a case or against the wall of the room.

To *read the barometer* the instrument is at first made vertical. The screw S is then worked until the mercury surface in the cistern just touches the tip of the ivory pointer. The adjustment is correct when the ivory pointer appears just to touch its image on the clean mercury surface. The vernier scale is then moved until the lower edge appears to be tangential to the curved mercury surface in the tube. To facilitate this adjustment, a piece of milk glass is usually fitted to the backboard of the instrument. Keeping the eye in level with the mercury surface, the vernier scale is moved until the piece of white glass is just cut off from view. When the adjustment is complete, the reading is taken from the main scale and the vernier. For accurate determination of the pressure, a *temperature correction* of the reading is necessary. (For details *Vide* Heat Art. 29 & J. Chatterjee's Intermediate Practical Physics)

***133. Siphon Barometer**—This is a convenient and portable

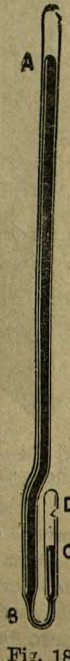


Fig. 186

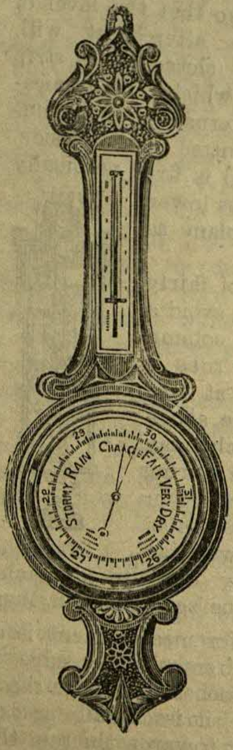


Fig. 187

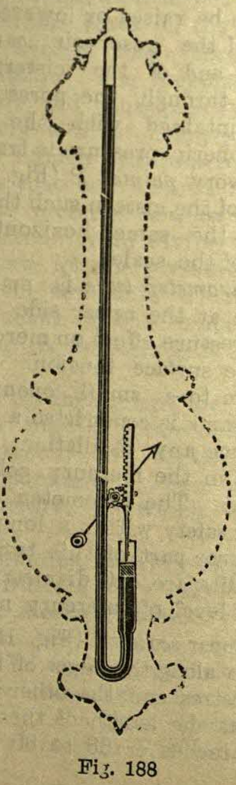


Fig. 188

tyye of mercurial barometer. It consists simply of an U-shaped tube having unequal limbs. The longer limb AB (Fig. 186) is closed at the top and is about 90 cms. long, while the shorter limb CD is open to the atmosphere. The space above mercury column within the limb AB is vacuous and the mercury column in CD is exposed to the atmospheric pressure. The difference of mercury levels in the two limbs gives the barometric height.

To protect the mercury surface at C, the limb CD is very nearly closed, leaving a very small opening at D through which communication with the external atmosphere is maintained. To prevent the entrance of air into the limb AB when the instrument is inclined, the two limbs are joined by an U-tube of very narrow bore, as shown in the figure. The instrument is fixed on a wooden board and a scale is attached to the board against each of the two limbs to read the difference of levels of mercury.

The weather glass or a *household barometer* (Fig. 187) is a form of Siphon barometer. In the shorter limb, there is a float on the mercury surface, which moves up or down with the surface. This float is connected through a rack and pinion arrangement to a wheel and to the centre of the float a pointer is attached (Fig. 188). The pointer moves over a dial mounted in front of the tube. The dial is graduated in inches and is also marked with indications such as *fair*, *variable*, *rainy*, *stormy*, etc. The position of the pointer, therefore, gives the atmpospheric pressure in inches and the probable state of the weather in immediate future. Hence this instrument is termed a *Weather glass.*

134. Aneroid Barometer—In this barometer no liquid is used (*a*, without ; *neros*, liquid). In the present form, it is sufficiently accurate and at the same time very light and easily portable. It consists of a shallow cylindrical metal chamber C (Fig. 189), which is partially evacuated and is hermetically sealed with a thin elastic metal diaphragm. The diaphragm is corrugated in circles to make it readily yield to small changes of atmospheric pressure. Any variation of pressure causes a deformation of the corrugated top, an increase of pressure depressing the top and any decrease raising it. The amount of depression or elevation is always proportional to the change of pressure. The deformation, which is always small, is magnified by a delicate system of levers and is ultimately transmitted to a pointer at the top moving over a circular scale. The scale is previously graduated in comparison to a standard mercury barometer, so that the pressure can be directly read on it in centimetres or inches. To prevent the instrument from being permanently deformed due to a sudden rise of pressure, the lid is supported on a strong spring which presses against the lever L acting on a fulcrum. Words *rain*, *fair*, *storm*, *change*, *etc.*, are written on the dial and the pointer at any position *correctly* indicates the

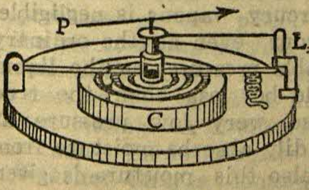


Fig. 189

corresponding state of the weather, provided the instrument is kept at approximately the same level for which it is meant to be used. A complete picture of the instrument with its outer casing and dial is shown in figure 190.

The aneroid barometer is lighter, more easily portable and less liable to damage than a mercurial barometer. An aneroid barometer may also be provided with a scale to indicate directly height in feet or in metres. Such barometers, which may be as small as watches and are used by mountaineers and aviators to record the heights attained, are called **altimeters.**

Barograph—A barograph is an instrument for automatically recording on a chart a continuous variation of the atmospheric pressure (Fig. 191). It consists of an aneroid barometer C which is kept well covered in a glass box. The top of the barometer is connected through a system of levers L and S. Any change in the atmospheric pressure is recorded by the end of the lever S rising up or down. A recorder is attached by the end of a long lever and its point presses against the drum slowly rotated by a clock-work arrangement and revolves once a week. The pressure variation is recorded on the graph paper as a continuous wavy line.

Advantages of Mercury as a Barometric Substance—The *advantages* of mercury as a barometric substance would be apparent from the following facts :

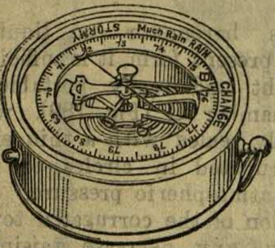


Fig. 190

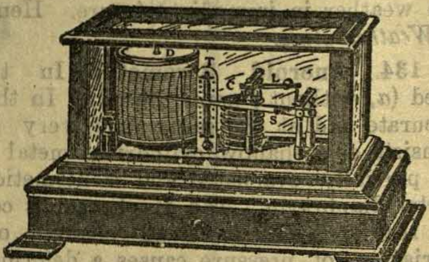


Fig. 191

(1) On account of the high specific gravity of mercury, a comparatively short column of it, about 30 inches long, balances the atmospheric pressure.

(2) In the Torricellian vacuum, there is very little mercury vapour, so the pressure due to this mercury vapour is negligible. On the other hand, water gives off vapour even at the ordinary temperature, and this would exert appreciable pressure on the liquid column and so the observed height would be less than the true height. Glycerine vapour has, of course, very low pressure at ordinary temperatures ; but as glycerine readily absorbs moisture from the atmosphere, its density changes and also this moisture is given off by the liquid within the tube.

(3) Mercury when pure does not wet glass.

***135. Uses of Barometer**—A Barometer may be used either for the measurement of atmospheric pressure or for the determination of height.

Determination of Height—At the sea-level, mercury barometer stands at about 76 cms. On ascending through the atmosphere, a certain height of the air column is left out, so the atmospheric pressure, and consequently the length of barometric column decreases. Let the barometer reading at lower and upper stations be h_1 and h_2 respectively.

Vertical height between the two stations = H cms.

Average density of air between the two stations = ρ gms./c.c.

$$\text{So } H \times \rho \times g = (h_1 - h_2) \times 13{\cdot}6 \times g, \text{ whence } H = \frac{h_1 - h_2}{\rho} \times 13{\cdot}6 \text{ cms.}$$

Taking the average density of air near the surface of the earth, it is found that for an altitude difference of 107·7 metres, the change of barometric height is 1 cm. This corresponds to a change of pressure of 1 inch for an altitude difference of 900 ft. But as air is highly compressible and as the temperature is lower in the upper regions, the density of air decreases as we go up. So the barometric readings do not fall in proportion to the height. A rigorous calculation of the *determination of height* is given in the part on Heat (Art. 81).

The height of a place above the sea-level in *metres* can be directly determined from the formula

$$H = K(\log_{10}h_1 - \log_{10}h_2) \qquad \ldots(135,\ 1)$$

where h_1 and h_2 are the atmospheric pressures *in centimetres* at the sea-level and at the required altitude and K is a numerical constant equal to 18900 nearly.

Forecasting of Weather—It is found that the atmospheric pressure at a place varies at different hours of the day, as also from day to day. The variations may be due to changes in the density of air consequent upon the changes of temperature and on the amount of water vapour in the atmosphere. Hence, the barometer readings would give us same idea as to the possible condition of the weather at any time and also in near future. Water vapour is lighter than air, its density being $\frac{5}{8}$ of that of dry air. So when the proportion of water vapour in the atmosphere becomes large, the density and hence the pressure of the atmosphere decrease and the barometer reading falls. A *considerable fall* in the barometer, therefore, indicates that *rain* is imminent. If there is a sudden decrease of pressure at the place, air from surrounding regions would rush to the place. *A sudden fall* in the barometer record therefore predicts an *immediate* storm. A *slow fall* for several days implies *continued bad weather*. On the other hand, a *sharp rise* indicates a fine weather but not lasting. A *steady rise* indicates a *continued fine weather* for some time. These forecasts are only approximate.

Weather Charts containing informations about pressure, temperature and humidity of atmosphere, strength and direction of the wind, the state of the sky, etc., are daily published by Meteorological

Offices. Air pressures in millibars are shown on the charts by continuous lines, called **isobars** each of which passes through places of equal barometric pressure at a certain time. The relative position of these lines indicate the state of expected weather. When the isobars are near, the variation in the pressure is rapid. The regions of low pressure are called **cyclones** and those of high pressure are called **anticyclones.** The instrument by means of which the strength and the direction of wind can be measured and recorded is known as **anemometer.** The latest method of weather forecasting is indicated by orbiting satellite and its radio-reception from meteorological stations.

136. Extent of the Atmosphere—The air at the surface of the earth would tend to expand indefinitely in space. But at a certain height, this tendency is expected to be balanced by the action of gravity and so the atmosphere may be taken to extend to a limited height. But the density of air gradually decreases with height.

Formerly balloon experiments were done to explore the limit of the earth's atmosphere. A record of previous history may be given here. On the 27th may, Prof. Piccard of Belgium rose to a height of about 13 miles. On September 30, the Soviet Balloon 'U. S. S. R.' rose into the stratosphere to an altitude of 19,000 meters or about $11\frac{7}{8}$ miles. On the 11th November, 1937 a stratosphere expedition was undertaken above Dakota in America under the National Geography Society by Capt. W. Stevens and Capt. Orville Anderson, who rose in balloon to a height of 72,395 ft. or nearly 13·7 miles.

An unmanned balloon carrying self-recording instruments went up in 1936 from U. S. A. to a height of 21·8 miles which proved evidence of sufficient air at a height of even 45 miles to reflect sun's rays at sunset. Reflection of radio waves has revealed that traces of atmosphere extend to even 200 miles and beyond. Recent experiments on artificial satellites (sputniks) conducted by scientists of U. S. A. and U. S. S. R. have further thrown light on the extent of atmosphere. It is now known that traces of atmosphere extend up to 200 miles approximately.

Difficulties of a High Ascent—It is to be mentioned here that great difficulties are experienced in a very high ascent as in the *Everest Expedition* for the following among other reasons :

(*i*) *The Density of Air*—The lower density causes difficulty of breathing as a person would have to make a greater number of inhalations per minute to take the requisite supply of oxygen. With artificial inhalation of oxygen, this difficulty may be minimised.

(*ii*) *The Low Atmospheric Pressure*—Owing to this the pressure within our body would be in excess ; this would cause a rush of blood towards the nose, ears and eyes, which would even sometimes bleed. This may be minimised by using pressurised garments. Further, the muscles should have to put forth greater force in supporting the limbs which would feel stiff and without any strength.

(*iii*) *The Low Temperature*—The temperature of the atmosphere becomes lower as we go up. At a height of 5 miles, the temperature of the air is found to fall to $-20^\circ C$ approximately, and at a height of 7 miles, it falls to about $-58^\circ C$. Unless special precautions be taken, the aeronaut may be frozen to death.

137. Isothermal Volume Changes of Gases—It has already been stated that gases are highly compressible so that the volume of an enclosed mass of a gas readily changes with changes of pressure. The relation between the pressure and the volume of a given mass of a gas at constant temperature was first discovered in 1662 from experimental results by Robert Boyle (1627-1691), and also independently in 1676 by Mariotte in France. The relation is enunciated in a law, called the Boyle's Law in England and the Mariotte's Law in the European Continent.

Boyle's Law—*The volume of a given mass of any gas at constant temperature varies inversely as the pressure to which it is subjected.*

Thus if V be the volume of a mass of a gas at a pressure P, then at constant temperature—

$V \propto \frac{1}{P}$ or, $V = k\frac{1}{P}$ where k is constant

whence $PV = k$, a constant. ... (137, 1)

Hence, if a given mass of a gas at a constant temperature has volumes V_1, V_2, V_3, etc., under pressures P_1, P_2 and P_3 respectively then we have $P_1V_1 = P_2V_2 = P_3V_3 =$ a constant. This constant depends on the nature, mass and temperature of the gas. The equation (137, 1) is known as the **isothermal equation of state.**

Pressure and Density—The relation given in Boyle's Law can also be expressed in terms of pressure and density of the gas. Thus let a certain mass m of the gas at a constant temperature possess a volume V_1 and density ρ_1 at a pressure P_1, and a volume V_2 and density ρ_2 when the pressure is P_2; then since the mass remains constant.

$$m = V_1\rho_1 = V_2\rho_2 \text{ whence } \frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} \qquad ...(1)$$

Again from Boyle's Law, $P_1V_1 = P_2V_2$ or $\frac{P_1}{P_2} = \frac{V_2}{V_1}$ ...(2)

$\therefore$ From equ., (1) & (2), $\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2}$ whence $\frac{P}{\rho}$ is constant (137, 2)

138. Verification of Boyle's Law—The apparatus generally used for this purpose consists of a long glass tube AE (Fig. 192) of uniform bore and closed at the top permanently by being sealed off or by means of two air-tight glass stoppers as shown in the figure. This tube is connected at its lower end by a piece of long flexible thick-walled rubber tubing to a wider glass tube DB open at both ends. Both the tubes can slide along vertical uprights and can be fixed at any desired position by clamps X and Y. The lower parts of both the glass tubes and the whole of the rubber tubing T are filled with pure dry mercury, while the upper part of A contains some amount of *dry* air or any other gas. There is a vertical scale S attached to the stand, which is graduated on each side in centimetres. The positions of the levels of the mercury in the tubes can be read from the scale S. As the tube AE is of uniform bore, the volume of the enclosed air is proportional to the length of the enclosed air column within the tube. For accurate observation, the tube is graduated in fractions of cubic centimetre.

For Higher Pressures—Raise the open end, such that the mercury level in B is higher than A. Fix it up somewhere and wait for some time to allow the compressed gas to attain room temperature. The *volume* of air in A is evidently proportional to the difference of the readings of the closed end E of the tube and mercury level at C. The *pressure* of air in A is obtained by *adding* the difference of the mercury levels D and C in the two tubes and the barometric height at the time. Take a few such readings of volumes and pressures with the mercury level in B higher in each case. *Here the pressure is always higher than the atmospheric pressure.*

For Lower Pressures - Slide B down till the level of mercury in the two tubes is the same again. Now lower B step by step. The *volume* of the air in A gradually increases and the level of mercury in B is always lower than in A. The pressure of the air in A is in each case obtained by *subtracting* the difference of the mercury levels in the two tubes from the barometric height. *Here the pressure of the enclosed air is always less than the atmospheric pressure.* Note the volume of the air and the corresponding pressure at each step. Find the product of the volume v and the corresponding pressure p in all cases. It is found that the product $p \times v$ *is approximately constant.*

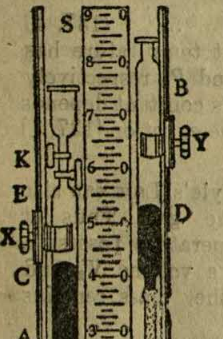


Fig. 192

Precautions—The mercury as well as the air enclosed in A must be perfectly dry. To get dry air the stop-cocks as the mouth of the tube A may be opened and some fused calcium chloride is kept in funnel tube. The tube B is then raised so that mercury level in A touches the lower stop-cock. Thus the air inside A is all forced out. Then B is slowly lowered, whereby mercury level in A in coming down sucks atmospheric air through calcium chloride making it dry. Thus any required volume of the gas may be taken in at the start.

To account for any variation in the atmospheric pressure during the experiment the mean of the barometer readings at the beginning and at the end of the experiment should be taken.

It is sometimes a difficult task to make glass stoppers absolutely air-tight in a glass tube. If there is any leakage from the closed end of the tube during experiment, then the experiment is spoilt. To ensure the safety, the upper end of the tube A is made permanently sealed.

The alteration in the volume of the enclosed gas must be made very slowly so as to avoid any change in temperature due to the sudden compression or expansion.

To represent graphically the relation between the pressure P and the volume V of a gas, the pressures are represented along abscissa and corresponding volumes along ordinate. The nature of the curve is shown in figure 193. The curve so obtained is a branch of a *rectangular hyperbola.* If the pressure P is plotted against $1/V$ which is reciprocal of corresponding volume, resulting graph is a *straight line.*

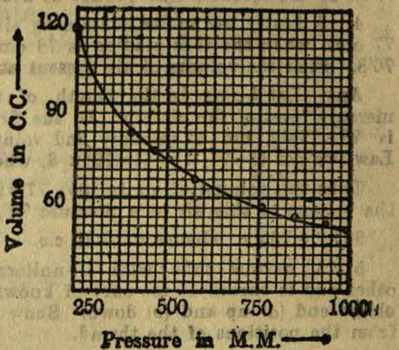


Fig. 193—Isothermal curve

The change of volume of a gas at *constant temperature* is known as *isothermal change* and the curve representing the change in the volume of a gas with the change of pressure at any constant temperature is called an **isothermal curve** or simply an *isothermal* at that temperature.

Examples :

1. A vertical cylinder of sectional area 10 sq. cm. is fitted with an air tight piston weighing 30 kgm. The cylinder contains some air and the height of the piston from the bottom of the cylinder is 28 cm. If the barometer reads 76 cm. at the time, find the tolal pressure inside. If an extra load of 30 kgm. be placed on the piston, find how far it will be depressed. ($g=980$ cm. per. sec.^{2}.)

Ans. Pressure due to the weight of the piston

$$=\frac{30\times1000\times980}{10}=2.94\times10^6 \text{ dynes per sq. cm.}$$

Atmospheric pressure per unit area $=76\times13.6\times980=1.01\times10^6$ dynes/cm.^{2}
∴ Total pressure inside $=(2.94\times10^6+1.01\times10^6)=3.95\times10^6$ dynes/cm.^{2}
The additional load produces a pressure of 2.94×10^6 dynes per sq. cm.

∴ Total pressure $=(3.95+2.94)\times10^6$ dynes $=6.89\times10^6$ dynes per sq. cm.
Let the piston be depressed through x cm

Then the $(28-x)\times6.89\times10^6=28\times3.95\times10^6$ whence $x=11.94$ cm. nearly.

2. The space above a mercury column in barometer tube contains some air. The mercury column is 28.40 inches long and the space above it is 3.05 inches long. The tube is then pushed downwards into the mercury so that the column is 28.14 inches long while the air space is 2.34 inches. What is the true height of the barometer. (Raj. U.—1955)

Ans. Let the true height of the barometer in the air-free state be P inches. So the initial pressure of the enclosed air $=(P-28.4)$ inches and the volume of the air $=3.05\times\alpha$; where α is the cross-section of the bore of the tube. Final pressure of the enclosed air $=(P-28.14)$ inches and the volume is $2.34\times\alpha$.

∴ From Boyle's Law, $(P-28.4)\times3.05\times\alpha=(P-28.14)\times2.34\times\alpha$
whence $P=29.26$ inches.

3. A tube 6 ft. in length closed at one end is half-filled with mercury and it is then inverted with its open end just dipping into a mercury trough. If the barometer stands at 30 inches, what will be the height of the mercury inside the tube ? (C. U.—1951)

Ans. Let the height of mercury column in the tube when inverted $=h$ in. Initially the air occupies (3×12) in. of the tube and its pressure $=30$ in. Final volume of the air is $(72-h)$ in. of the tube and the pressure $=(30-h)$ in.

∴ $(72-h)(30-h)=36\times30$, whence $h=12$ in, or 90 in.

But as the length of the mercury column cannot be greater than the tube itself so the second value cannot be accepted. Hence the required height is 1 ft.

4. A faulty barometer containing air and of 1 sq. cm. cross-section reads 77 cms. when the true reading is 78 cms. and it reads 70 cms. when true reading 70·8. Find the volume of air present at a pressure of 76 cms. (And. U.—1951)

Ans. Let l cms. be the length of the barometer tube measured from the mercury level of the cistern. In the first case air pressure within the barometer is $78-77=1$ cm of mercury and volume of air is $(l-77)$ c.c. Applying Boyle's Law, we get $(l-77)\times 1=(l-70)\times {}^{\cdot}8$, whence $l=105$ cms.

Thus the volume of air is $105-77=28$ c.c. under a pressure of 1 cm. If v be the volume of this air at a pressure 76 cms. then from Boyle's Law,

$28\times 1=76\times v$, whence $v=0{\cdot}37$ c.c.

5. A narrow tube with a uniform bore is closed at one end and near the other end is thread of mercury of known length. The tube if held vertical with closed end (*a*) up and (*b*) down. Show how the barometric pressure is determined from the positions of the thread. (Gau. U.—1955)

Ans. Let l be the length of the mercury pellet within the tube. If ρ and g be the density of mercury and acceleration due to gravity, then the pressure exerted by this pellet on the enclosed air within the tube is $l\rho g$. When the tube is held with its open end upwards, then the pressure P due to atmosphere and that due to the pellet acts downwards and the total pressure is $P+l\rho g$. If the length of the enclosed air column be l_1, then since the cross-section is uniform, the volume of the air column is proportional of l_1 call it Kl_1.

When the tube is held vertical with its open end downwards, then the atmospheric pressure within the tube acts upwards while the pressure due to pellet acts downwards. So the new pressure is $P-l\rho g$. If now the length of enclosed air column be l_2 the volume is Kl_2. Now applying Boyle's Law, we get $(P+l\rho g)\times Kl_1=(P-l\rho g)\times Kl_2$ or, $P(l_2-l_1)=l\rho g(l_2+l_1)$.

$$\text{whence } P=\frac{l\rho g(l_2+l_1)}{l_2-l_1}.$$

139. Archimedes' Principle applied to Gases

139. Archimedes' Principle applied to Gases—Just as a body immersed in a liquid experiences a vertical upward thrust equal to the weight of the liquid displaced by the body, so also a body in air is subjected to an upward thrust by the surrounding gas, which is equal to the weight of the gas displaced. The upward force exerted on any body by the surrounding air is called the *buoyancy of the air*. Hence the weight of a body in vacuum, which is its *true weight*, is greater than its apparent weight in air by the weight of the displaced air.

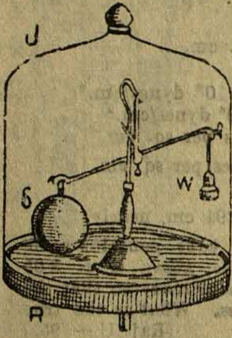


Fig. 194

This is clearly demonstrated by a **baroscope**, which consists of a small balance from the arms of which a small brass weight and a large cork sphere are suspended so as to counterpoise each other when in air. Therefore their apparent masses are equal. The whole is then placed under the receiver of an air-pump. If now the air is pumped out, the cork sphere is observed to sink down becoming actually heavier (Fig. 194). When both are suspended in air the cork sphere owing to its larger volume displaces greater volume of air than the brass weight and so it experiences a larger upward thrust. But as the apparent masses of the two are the same the true mass of the cork sphere must be greater than that of the brass weight. In the following calculation we use the word 'weight' to mean the mass of a body.

***Buoyancy Correction in Weighing**—The fundamental assumption in a buoyancy correction in weighing is that the apparent mass of the body to be weighed is equal to the apparent mass of the weights placed to counterpoise it.

Let the *true* weight of the body $= \mathrm{W}$
true weight of the counterpoising weights $= \mathrm{W}'$
density of the material of the body $= \rho$
density of the material of the weights $= \rho'$
density of air at the time $= d$

Then for equilibrium, $\mathrm{W} - \dfrac{\mathrm{W}}{\rho}d = \mathrm{W}' - \dfrac{\mathrm{W}'}{\rho'}d$

$$\text{whence } \mathrm{W} = \frac{\mathrm{W}'\left(1 - \dfrac{d}{\rho'}\right)}{\left(1 - \dfrac{d}{\rho}\right)} = \mathrm{W}'\left(1 - \frac{d}{\rho'}\right)\left(1 - \frac{d}{\rho}\right)^{-1}$$

$$= \mathrm{W}'\left(1 + \frac{d}{\rho} - \frac{d}{\rho'}\right) \qquad \ldots \quad (139{,}1)$$

since d is small in comparison to ρ and ρ'.

Example:

The weight of a body in air of density 0·00125 gm. per c.c. is found to be 399·8 gm. If the density of the material of the body be 2·5 gm. per c.c. find the weight of the body in air of density ·00129 gm., per c.c. and also its true weight.

Ans. Let the true weight of the body = W gm.

Then the volume of the body $= (\mathrm{W} \div 2{\cdot}5) =$ volume of the air displaced.

$\therefore \quad \mathrm{W} - \dfrac{\mathrm{W} \times 0{\cdot}00125}{2{\cdot}5} = 399{\cdot}8$ whence W = 400 gm.

Hence the volume of the body $= (400 \div 2{\cdot}5) = 160$ c.c.

$\therefore$ Its wt. in air of density $0{\cdot}00129 = (400 - 160 \times 0{\cdot}00129) = 399{\cdot}793$ gm.

140 Balloon—The buoyancy of the air is utilised in the ascent of a balloon, an airship or any other *air floating* machine. If the weight of the machine with its accessories be less than that of the displaced air the machine will rise up in air.

A balloon is essentially a large bag of silk or some other light material, the outside of which is thoroughly varnished to make it air-tight (Fig. 195). It is filled with some gas lighter than air such as hydrogen, helium or coal gas. The balloon is provided with,

(*i*) A *light car* to carry passengers which is suspended by cords attached to a large network of ropes covering the balloon.

(*ii*) A *safety valve* at the top. Opening it with a cord, the contained gas may be allowed to escape and the balloon may be lowered at will.

(*iii*) Some *bags of sand*, which being emptied would make the balloon ascend further or descend less rapidly.

(*iv*) A *parachute* which is a large umbrella-like contrivance with cords hanging from its circumference and carrying a light frame at the other end (Fig. 196).

The aeronaut may leave the balloon with the parachute, which then opens out like an umbrella due to the resistance of the air below and makes the descent slow. There is a small opening at its top through which the air slowly escapes whereby it descends upright.

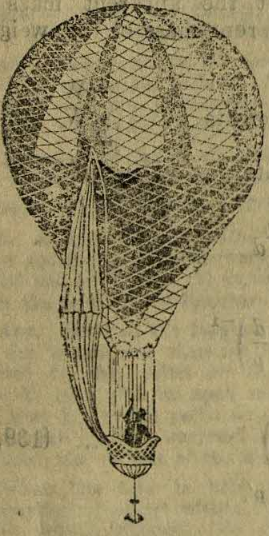


Fig. 195

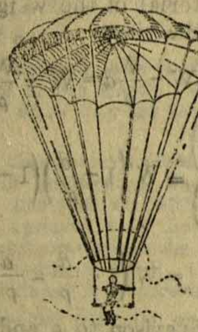


Fig. 196

Air Ship—It is a modification of the balloon with arrangement to steer the machine in any direction with changeable speed. After being lifted up in the air, it can be driven by *propellers* worked by motors and it can be steered in any direction by a suitable *rudder*. Zeppelin is an example. Hydrogen being extremely light is used in the air-ships; but later on helium was preferred as it is non-inflammable. It is due to hydrogen taking fire that the German air liner the *Hindenburg*, the biggest of its type and used in trans-Atlantic flight, was destroyed on 6th May, 1937. At present air-ship is obsolete since large jet planes, able to carry several tons of load and move with very high speed, have been built.

Examples ;

1. A litre of hydrogen and a litre of air weight about 0·09 gramme and 1·3 grammes respectively at a certain temperature (*t*) and pressure (*p*). What would be the capacity of a balloon, weighing 10 kilograms, which just floats when filled with hydrogen having the same pressure (*p*) and the same temperature (*t*) as the air.

Ans. Let the capacity or volume of the balloon $= V$ litres.
∴ Mass of the hydrogen enclosed in the balloon $= V \times 0·09$ gm.
And mass of air displaced by the balloon $= V \times 1·3$ gm.
Now wt. of the balloon + wt. of hydrogen enclosed = wt. of air displaced.
∴ $10000 + V \times 0·09 = V \times 1·3$ whence $V = 8264·46$ litres nearly.

2. A balloon, weighing 150 kgm., contains 1000 cu. metres of hydrogen and is surrounded by air. Calculate the additional weight it can lift. Also explain why the balloon will float in stable equilibrium at a constant altitude. Density of hydrogen = 0·00009 gm./c.c. and density of air = 0·00129 gm./c.c.

[Gau. U.—1952]

Ans. The mass of the balloon and the contained gas = 150 kgm. + mass of gas.
Now mass of 1000 cu. metre of hydrogen
$= 1000 \times 10^6 \times ·00009$ gm. $= 10^6 \times 0·00009$ kgm. $= 90$ kgm.
∴ Total mass of balloon and hydrogen $= 150 + 90 = 240$ kgm.
Mass of air displaced $= 1000 \times 10^6 \times ·00129$ gm. $= 1290$ kgm.

Hence from Archimedes' principle force of buoyance $= 1290 - 240 = 1050$ kgm. = its lifting power at the place where density of air is ·00129 gm./c.c. At a higher altitude the density of air is less, so that balloon cannot ascend with the given load. Again at a lower altitude the density of air is higher than ·00129, so that the balloon with the given load could be forced upwards due to greater buoyancy. So it takes up a constant height determined by the required density of air (·00129 gm./c c.).

3. A balloon is filled with a gas whose density is $\frac{1}{13}$ that of air at N.T.P. Compare the total lifting power of the balloon in air at a pressure of 75 cm. to that in air at 74 cm, the temperature reamaining always at 0°C and the volume of the balloon being supposed to be the same.

Ans. Let the density of air at N.T.P. $= d$; then that of enclosed gas $= \frac{d}{13}$.

Again density of air at 0°C, and 74 cm. $= d \times \frac{74}{76}$.

Let the volume of the balloon be V c.c. and the total lifting power in the two cases be W_1 and W_2 respectively.

$$\text{Then} \quad W_1 + \frac{Vd}{13} = Vd \quad \text{or,} \quad W_1 = Vd(1 - \tfrac{1}{13}) \qquad \dots (1)$$

$$\text{Again} \quad W_2 + \frac{Vd}{13} = Vd + \tfrac{74}{76} \quad \text{or,} \quad W_2 = Vd(\tfrac{74}{76} - \tfrac{1}{13}) = \tfrac{443}{494}Vd \qquad \dots (2)$$

From (1) & (2) $W_1 : W_2 = 456 : 443$.

4. A cylinder filled with air at atmospheric pressure is lowered in water with its mouth downwards till it is $\frac{1}{3}$ full of water. To what further depth should it be lower till it is $\frac{2}{3}$ full of water? Density of mercury is 13·6 and barometric height is 76 cm.

Ans. Let l be the length of the cylinder in cm. and α its area of cross-section in sq. cm. Then the volume of air is $l\alpha$ c.c. under the atmospheric pressure of $76 \times 13{\cdot}6 \times g$ dynes/sq. cm.

Let the depth of water be h cm. measured from the top surface of the cylinder at which the cylinder is $\frac{1}{3}$ full of water. The volume of air is then $\frac{2}{3}l\alpha$ c.c. and the total pressure within the cylinder is $76 \times 13{\cdot}6 \times g + (h - \frac{2}{3}l) \times g$ dynes/sq. cm.

Now, if temperature does not change, then according to Boyle's Law

$$l\alpha \times 76 \times 13{\cdot}6 \times g = \tfrac{2}{3}l\alpha \times [76 \times 13{\cdot}6 \times g + (h - \tfrac{2}{3}l) \times g]$$

$$\text{or,} \quad 76 \times 13{\cdot}6 = 2h + \tfrac{4}{3}l \quad \text{or,} \quad h = \frac{76 \times 13{\cdot}6}{2} \text{ cms., neglecting the length of the}$$

cylinder as compared to h.

When the cylinder is further lowered by h_1 cms., where is $\frac{2}{3}$ full of water the volume of contained air is $\frac{2}{3}l\alpha$ c.c. and the total pressure is $76 \times 1{\cdot}36 \times g + (h + h_1 + \frac{1}{3}l) \times g$ dynes/sq. cm. Then again applying Boyle's Law,

$$l\alpha \times 76 \times 13{\cdot}6 \times g = \tfrac{1}{3}l\alpha[76 \times 76 \times 13{\cdot}6 \times g + (h + h_1 + \tfrac{1}{3}l) \times g]$$

$$\text{or,} \quad 3 \times 76 \times 13{\cdot}5 = 76 \times 13{\cdot}6 + h + h_1 \text{ neglecting } l$$

$$\text{or,} \quad 2 \times 76 \times 13{\cdot}6 = h + h_1 = \frac{76 - 13{\cdot}6}{2} + h_1$$

$$\text{or,} \quad \frac{3 \times 76 \times 13{\cdot}6}{2} = h_1 = 1550{\cdot}3 \text{ cms.}$$

EXERCISES ON CHAPTER XI

Reference

1. Explain fully the meaning of the statement—the atmosphere exerts a pressure of 15 lbs. per square inch nearly. How would you verify the statement experimentally? (C. U. 1951) — Arts. 129 & 131

2. How will you show experimentally that the atmospheric pressure acts equally from all directions? Though the surface of the human body is large explain why we are not crushed by the pressure of the atmosphere. (Utkal U.—1952) — Arts. 131 & 132

Referenec

Art. 131 3. Express the normal pressure of air in absolute units (Density of mercury$=13·6$ and $g=981$ cms. per sec. per sec.)
(Del. U.—1963 ; And. U.—1952 ; Del. H. S.—1954 ;
C. U.—1957 ; Raj. U.—1955)

Ans. 1013961·6 dynes per sq. cm.

Art. 131 4. In a barometric column, mercury stands at 30 inches above the level of the mercury in a trough. Express the atmospheric pressure in C.G.S. units, it being given that 1 inch$=2·54$ cm. $g=981$, Sp. Gr. of mercury$=13·6$.

Ans. 1016929 9 dynes per sq. cm.

Art. 135 5. The reading of a barometer at the top and ground floor of a building are 29 905 and 29·949 inches respectively. The densities of mercury and air are 13·53 and 001293 gm. per c.c. respectively. Find the height of the building. (And. U.—1961)

Ans. 38·4 ft.

Art. 132 6. Describe with a neat diagram the principle and construction of a barometer which you use in your laboratory.
(Raj. U.—1963 ; Pat. U.—1962 ; Dac. U.—1952 ;
U.P.B.—1952 ; *Cf.* C. U.—1954 ; Utkal U.—1953 ;
P. U.—1962)

Art. 132 7. Give a brief description of Fortin's barometer and explain its adjustments. Calculate the atmospheric pressure in dynes per cm.^{2} when the corrected barometric height is 750 mm. (given $g=981$ and $p=13·6$). (Dac. U.—1961 ; *Cf.* Nag. U.—1953 ;
U. P. B.—1953)

Ans. 10,00,620.

Art. 134 8. Explain the principle of an Aneroid barometer.
(Dac. U.—1964 : R.P.B.—1955)

Art. 132 9. Explain why it is necessary to correct the reading of a Fortin's barometer for the temperature. Obtain an expression for this correction. (Anna. U.—1960 ; Pat. U.—1953 ; Nag. U.—1963)

Art. 137 10. A bottle whose volume is 800 c.c. sunk mouth downward below the surface of a pond. How far must it be sunk for 100 c.c. of water to run into the bottle ? The height of the barometer at the surface is 76 cms. and the sp. gr. of mercury is 13·6.
(Nag. U.—1950)

Ans. 9 201 metres.

Art. 137 11. The mercury in a barometer tube stands at 75 cms. and the space above it is 5 cms. in length. 2 c.c. of air at atmospheric pressure are admitted into the tube and the mercury now stands at 60 cms. Find the area of cross-sections of the tube.
(Nag. U.—1950)

Ans. ·5 sq. cm.

Arts. 137 & 138 12. State Boyle's Law. How is the law verified experimentally ? (Mad. U.—1961 ; Nag. U.—1950, '65 ; Raj. U.—1952, '59 ; Del. H. S. —1970 ; E.P.U.—1959 ; *Cf.* Gau. U.—1955 ; Pat. U.—1954)

Art. 137 13. A steel bulb of capacity 2 litres contains air at pressure of 5 atmospheres. What should be the resultant pressure if now 5 litres of air atmospheric pressure are pumped into the bulb.
(Nag. U.—1965)

Ans. 7·5 atms.

Arts. 137 & 138 14. State Boyle's law and describe an experimental arrangement for verifying it for pressures greater than and less than one atmosphere. (Gau. U.—1952 ; U.P.B.—1964 ; C.U.—1961 ;
Dac. U.—1941 ; P.U.—1950)

Art, 137 15. The volume of an air bubble increases tenfold in rising from the bottom of a lake to its surface. If the barometric height 30 inches, what is the depth of the lake ? (Sp. gr. of mercury $=13·6$) (Dac. U.—1961 ; Raj. U.—1951)

Ans. 306 ft.

Reference

Art. 137

16. At what depth in water will a bubble of air have the same density as water given that water is incompressible and air obeys Boyle's Law for all pressure ? (Density of air at ordinary atmospheric pressure is 1·35 gm. per litre) (R. P. B.—1959)

Ans. 27200 ft., taking water barometer = 34 ft.

Art. 137

17. A narrow tube with uniform bore is closed at one end and at the other is a thread of mercury 8 cm. in length. The tube is held vertically, once with the closed end up and then with the closed end down. The lengths of the air column enclosed between the mercury thread and the closed end are 42 cm. and 34 cm. respectively in the two cases. Calculate the atmospheric pressure. (Pat. U.—1964)

Ans. 75 cm. of Hg.

Art. 137

18. A glass tube closed at one end is 72 cm. long. The inside is covered with a soluble pigment. In sounding the depth of the sea the tube is, lowered vertically bellow the sea, open end down, till it just touches the bottom. When taken out, the pigment was found dissolved to within 4 cm. of the top. If the average density of sea water is 1·02, find the depth of the sea at the place,

(Density of mercury = 13·6 ; atmospheric pressure = 76 cm.). (U. P. B.—1944)

Ans. 172·88 metres.

Art. 137

19. The space above the mercury in a faulty barometer measures 12 cm. and the mercury column extends 72 cm. above the mercury in the cistern. On depressing the tube in the cistern, the atmospheric pressure in centimetres of mercury ? cm. What is

Ans. 75 cm.

Arts. 131 & 127

20. Explain fully how the height of a barometer affords us a measure of the pressure of the atmosphere. What is the weight of a litre of air collected at the top of a mountain when the barometer stands at 50 cm. ? (1 litre of air at a pressure of 76 cm. weigh 1·29 grams).

Ans. 0·3487 gm.

Art. 137

21. The density of air at N.T.P. is 0·000129 gm. per c.c. Find the alteration in the weight of 15 litres of air when the barometer falls from 76 cms. to 74 cms.

Ans. The weight decreases by 0·51 gm.

Art. 137

22. Due to the pressure of small quantity of air in the empty space above the top mercury in a Torricellian tube, the height of mercury column is 752 mm. when the atmospheric pressure is 762 mm and 750 mm. when the atmospheric pressure is 759 mm. What is the true atmospheric pressure when the height of the mercury column is 750 mm. ? (Del. H. S.—1960)

Ans. 746 mm.

Art. 137

23. A tube 5 fit. in length, closed at one end, is half-filled with mercury and then is inverted with its open end just dipping into a mercury trough. If the barometer stands at 30 inches what will be the height of mercury inside the tube ? (P. U.—1952)

Ans. 11·5 ins.

Art. 137

24. The mass of a balloon when empty 10·454 kgm. When inflated with hydrogen, its volume becomes $35·5 \times 10^6$ c.c. If the density of the enclosed hydrogen is 0·000085 gm. per c.c. and that of the surrounding air be 0·0012214 gm. per c.c. find the additional weight which can be lifted by the balloon.

Ans. 29·85 kgm.

CHAPTER XII

AIR PRESSURE MACHINES

141. Syringe—It is an instrument whose working depends on the effect of atmospheric pressure and it is generally used to raise a liquid from one vessel and then to transfer it to another. It consists of a hollow glass or metal tube with the lower end tapering to a nozzle. An air-tight piston work within the tube. To draw a liquid, the nozzle is dipped in it keeping the piston at its lowermost position. When the piston is drawn up, a partial vacuum is formed within the cylinder below the piston and so the atmospheric pressure acting on the liquid surface becomes greater than the inside pressure. The liquid is, therefore, sucked into the cylinder as the piston is raised up. When sufficient liquid has been drawn in, the syringe is taken up. Owing to the pressure of the atmospheric air acting at the nozzle the liquid cannot escape. If the piston is now pushed down, the pressure inside becomes greater than the pressure outside and so the liquid is forced out.

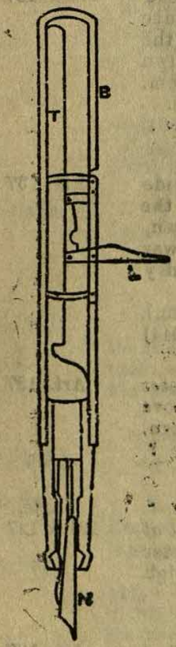


Fig. 197

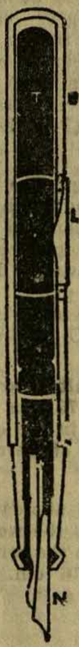


Fig. 198

An eye-dropper or a fountain-pen filler works on the same principle as a syringe but instead of a piston, there is a rubber bulb fitted air-tight at the top. On being pressed, the rubber bulb forces some air to go out of the tube through the nozzle. If now the nozzle is put under liquid and pressure on the bulb is released, the bulb regains its shape and expands the space inside. The pressure inside is thereby lowered and the requisite quantity of the liquid is sucked in so as to equalise the pressure. Figure 197 shows a pen with an automatic filler. Within the barrel B of the pen there is a rubber tube T in communication within the capillary tube leading to the nib. Fixed to the side, there is a lever L, which on being raised, presses the tube and forces air out of it. The pen is lowered in ink and the lever is released so as to inflate the tube, whereby the ink is sucked in and fills up the tube as shown in Fig. 198. Animals inhale air into the lungs on the same principle. During inhalation the muscles of the chest cause the lungs to expand, whereby the internal pressure is reduced and so air is drawn in. Also during exhalation the lungs contract and the carbon dioxide is forced out.

142. Principle of Pumps—Pump is a class of machines which can transfer fluids from a region of lower pressure to a region of higher pressure. Almost every type of pump consists of a large strong cylinder into which a suitable piston works air-tight. A complete *to-and-fro* motion of the piston is called a *stroke*.

Valves—The air pressure machines are provided with devices, called **valves**, which act as very light trap doors placed at the openings of discs or in connecting pipes of pumps. A valve allows the fluid to pass in *one direction* only but completely blocks the passage in the opposite direction. Valves differ in forms according to the purposes for which they are meant; one may be a flat disc turning about a hinge on a plate with a hole (Fig 199, A & B) serving as the partition wall between two compartments. If the fluid pressure in the lower compartment exceeds that in the upper, there would be an excess force acting from below on the valve, which would then open out and allow some fluid to pass through the hole. But if the fluid pressure in the upper compartment be in excess, the door will be pressed air-tight on the plate and on fluid is allowed to pass through. In addition to the form of a flap door, a valve may be of the type of a conical piece moving in a conical slot (Fig. 199, C) or even a light ball moving in a tapering hole. The form of valves, generally used in air pumps, consists of a strip of oiled silk stretched over a hole (Fig. 199, D). A valve should be very light so that it can work at the smallest possible difference of pressures.

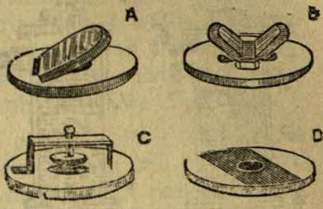


Fig. 199

143. Common Pump or Suction Pump—It consists of cylinder AB (Fig. 200), called the *barrel* within which a *piston* P works smoothly. The piston is made air-tight within the barrel by a collar of soft leather or rubber strip. The barrel is connected at its lower end with a long pipe whose other end is introduced into a reservoir from which water is to be raised. There are two *valves*, both opening upwards, one of which V is at an opening in the piston, while the other C is at the junction of the barrel and the pipe. Near the top of the barrel there is a *spout* E through which water flows out. In the case of a hand pump, the piston is worked by means of handle which works as a lever and is attached to the piston rod.

Action—Let us start with the piston at this lowest position. During the *upward stroke*, the piston is raised against atmospheric pressure and hence the valve V remains closed. The space below the piston therefore increases and the inside pressure falls. The valvet C is opened by the excess of pressure of the air inside the pipe and so this air partly enters into the barrel. The pressure of the air inside the pipe and the barrel becomes less that the atmospheric pressure and so water from the reservoir is sucked up into the pipe. The suction is continued till the piston reaches the topmost position. During the course of first up-stroke some water may even reach the barrel.

During the *down-stroke*, the air within the space below the piston is compressed due to which the valve C becomes closed. When the inside pressure becomes greater than the atmospheric pressure, the air inside the barrel pushes the valve V open and escapes through it. Air continues to escape through the upper valve till the piston reaches its lowest position, the valve C remaining closed all the while.

If some water has been raised within the barrel during the *up-stroke*, this wather finds its way above the piston through the valve V.

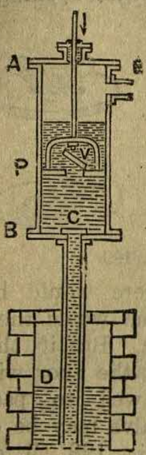


Fig. 200

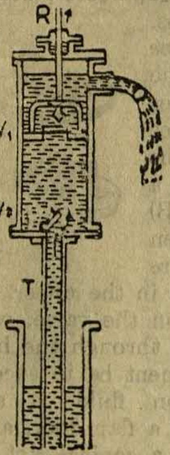


Fig. 201

During the next *vp-stroke* the valve remains closed due to the atmospheric pressure and the pressure of any water above it, while due to the vacuum produced above the valve C, it opens and water is forced into the barrel. The water above the piston flows out through the spout (Fig. 201). A few initial strokes may often be necessary to drive off air from inside the pump and hence to get the *first* supply of water. When once a supply of water is obtained, water is given off at every upward stroke of the piston. It is to be noted that although water is said to be 'sucked in', it is really 'pushed up' by the atmospheric pressure.

Limitation of the Suction Pump—Since water is raised by the atmospheric pressure, the height of the piston above the surface of water in the reservoir must not exceed that of the water barometer, *i.e.*, about 34 ft. But as the valve has some weight and the piston in not perfectly air-tight, the difference in heights is found to be *slightly less*. Moreover, the pump gives an *intermittent flow* of water, supply being made during every *up-stroke*.

Such pump is now being widely used in the tube-wells. If kept unused for sometime, owing to the drying-up of leather collar the valve in the piston may not remain perfectly air-tight and water would not rise up in the barrel. In such a case some water is to be poured into the top of the barrel before using the pump. This would make the piston move air-tight and thus help a quick working of the pump.

144. Lift Pump—This type of pump is used to raise water to a desired height. It is only a modification of the common pump having a third valve V_3 (Fig. 202) opening upwards at the junction of the delivery pipe and the barrel.

Action—To start with, suppose that the piston is at its lowest position and that water has already reached up to the top of lower pipe T after some initial strokes.

During the *up-stroke*, as the piston is raised, the valve V_1 opens due to the partial vacuum created above it within the barrel, while due to the excess pressure at the top part of the barrel, the valve V_2 remains pressed. As air within the cylinder above V_2 is compressed, the valve V_3 is opened and air escapes through the pipe into the atmosphere. At the same time, there is a fall of pressure inside the barrel and water rises.

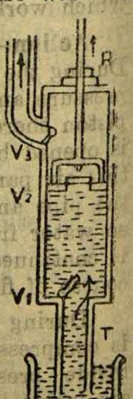


Fig. 202

During the *down-stroke*, the pressure inside increases and the valve V_1 becomes closed, while the valve V_2 opens and the water passes through the hole of this valve into the space above the piston. During the next *up-stroke*, a fresh quantity of water is drawn into the barrel, while the water above the piston opens the valve V_3 and passes into the pipe above. As the strokes are repeated, more and more water is forced up the tube through the valve V_3.

It is evident that during the *down-stroke* of the piston the valve V_3 supports the weight of the column of water within the pipe above and so relieves the valve V_2 of this weight during this period. Using this pump water can be raised to any height, provided the different parts of the pump are strong enough to bear the force caused by the weight of water above. It is important to remember that since the collection of water within the barrel depends upon suction, the barrel must be so placed as to be within 34 ft. from the water surface of the source.

145. Force Pump—It consists of the strong *barrel* AB (Fig. 203) fitted with an air-tight *solid piston* P and provided with a pipe at the bottom, which dips into the reservoir of water There is a strong valve C opening upwards at the junction of the barrel and the suction pipe. An outlet pipe E is fitted to a side close to the bottom of the barrel and at this junction there is another strong valve D opening into the pipe. It also works under a similar principle as a lift pump.

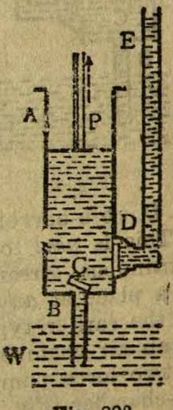


Fig. 203

Action -As the lower pipe by the first few strokes is exhausted of air, the level of water reaches the valve C. During the next *up-stroke* of the piston, pressure inside the barrel falls, the valve D remains closed by the atmospheric pressure while the valve C opens and water is forced into AB. During the *down-stroke* the pressure inside AB increases, the valve C closes while the valve D opens and water is forced up into the tube DE. Thus during each successive *up-stroke*, a quantity of water rises into the barrel, while the valve D remains closed due to the back pressure of the water within the pipe E ; and during each *down-stroke*, the water is forced into the outlet tube, the valve C remaining closed. Like the lift pump, this instrument also can raise water to any height, depending on the strength of the machine and the force applied at the handle. Here also the output in *intermittent*, water flowing out during the *down-strokes* only.

Arrangement for Continuous Flow—The output of a liquid in a pump can be made *continuous* by having an **air-chamber** R (Fig 204) into which outlet the tube directly passes. The delivery tube leads into the chamber at its upper portion and reaches nearly the bottom of the chamber, while the upper end of the tube leads to the required height.

Action—During the *up-stroke*, the water rises into the barrel while the valve C at the bottom of barrel remains open, the valve

D remaining closed. During the *down-stroke*, the pressure inside increases; so the valve C becomes closed, while the valve D opens and water is forced into the chamber R. If the pump is rapidly worked, then during the *down-stroke* along with the discharge of water from the delivery tube, some water collects into the chamber, so that the air in R is compressed. During the upward stroke of the piston, the valve D remains closed; but the compressed air in the chamber R expands and thus forces the water to flow out of the delivery tube. Thereby a *continuous flow* of water may be maintained.

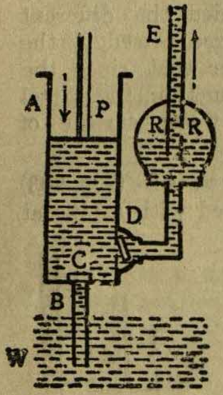


Fig. 204

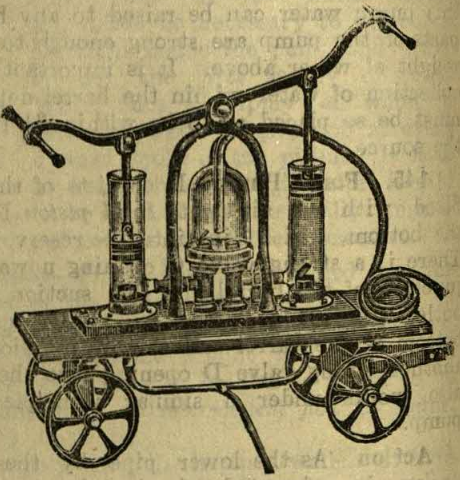


Fig. 205

Double-barrelled Pump—This is *used* to produce a continuous stream of water at a place at *any* height or at *any* distance from the source of water. In the earlier type, a continuous flow was maintained with the help of an air chamber, as just described. In the present form, it consists of two force pumps, the barrels of which are connected to a common air-chamber (Fig. 205). Both the pumps then draw from a common reservoir. When the piston in one moves down, that in other moves up and *vice-versa*. The delivery tube is fitted to lateral aperture in the air-chamber. The action of the piston is likely to stop momentarily at the time of reversal; but the continuity of the flow is maintained by the air chamber. Of such a pump the latest type is operated by a petrol motor or electric power. A double-barrelled water pump is often used by fire-fighters and they call this machine a **Fire Engine**.

Example :

Water is pumped up from a well through a height of 30 ft. by means of a 5 H. P. motor. If the efficiency of the pump is 85%, find in gallons the quantity of water pumped up per minute. (One gallon of water weighs 10 lb.)

[Gau. U.—1952]

Ans, Since 1 H. P. = 33000 ft. lb. of work per minute, a 5 H. P. motor can do 165000 ft. lb. of work per minute on the pump.

But $\frac{\text{work done by the pump}}{\text{work done on the pump}} = \text{the efficiency of the pump} = 85\% = \frac{85}{100}$.

$\therefore$ work done by the pump $= \cdot 85 \times 165000$ ft. lb. $= 140{,}250$ ft. lb.

Let x be the number of gallons of water raised through 30 ft. in one minute by the pump. Then work done $= 10x \times 30$ ft. lb.
or, $300x = 1{,}40{,}250$ whence $x = 467{\cdot}5$ gallons per minute.

146. Siphon—It is *used* for transferring a liquid from one vessel into another when the whole mass of the liquid is not to be disturbed or when the ordinary method of pouring the liquid is not convenient or desirable.

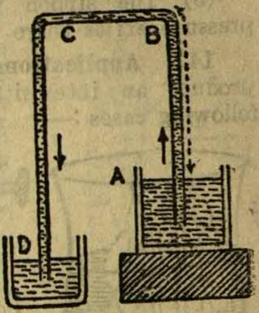


Fig. 206

The **siphon** consists of a tube CB usually of glass (Fig. 206) with two open ends bent down and with one limb longer than the other. The tube is first filled completely with the liquid to be transferred. The method of filling liquid for an ordinary glass siphon is to invert the tube and pour liquid into it. Then temporarily closing both the ends with fingers, the shorter limb is introduced into the liquid contained in the vessel A, while the longer limb projects outside into another vessel with its end below the level D of the liquid. On now opening the ends, the liquid at once begins to flow out through the longer limb in a continuous stream. For a siphon, which cannot be moved, first filling of the liquid is made by temporary suction.

Principle of Action—The start of the flow may be explained in the following way: Let C and B two points on a horizontal plane in the upper part of the tube and A, D the levels of liquid in the vessels.

Let the atmospheric pressure at the time $= P$; vertical heights of C and B above D and A respectively $= h$ and h'; density of the liquid in the vessel $= \rho$.
Then static pressure at $C = P - h\rho g$ and that at $B = P - h'\rho g$.

Now as h is greater than h', pressure at B is higher than pressure at C. Hence, the liquid begins to flow from B at a higher pressure to C at a lower pressure. Due to the motion of some liquid from B a partial vacuum is created at that region and fresh liquid is forced up to B by the atmospheric pressure. Thus a continuous flow is maintained the shorter limb of course remaining under liquid. In case the end of the longer limb dips below a liquid, h' is measured from the surface of the liquid D in the vessel. When the liquid is in motion, the pressures at B and C *slightly* change, although the former remains higher than the latter (Vide Hydrodynamics Vol. III).

Conditions for the Working of a Siphon—

(1) The level of the liquid in the vessel to emptied must be *higher* than that in the other. For when $h = h'$, the pressure at C = the pressure at B and water will not flow from B to C. Hence, the siphon ceases to work when the levels of the liquid in both the vessels become equal.

(2) As atmospheric pressure forces up the liquid into the shorter limb, the height h' of the top of the siphon above thel iquid in the

vessel to the emptied must remain less than the height of the corresponding liquid barometer.

(3) The siphon will not work in vacuum, as no atmospheric pressure exists there.

147. Applications of Siphon—A siphon may also be used to produce an intermittent flow of a liquid. This is illustrated in the following cases :—

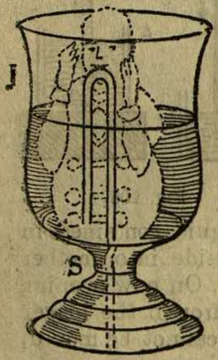


Fig. 207

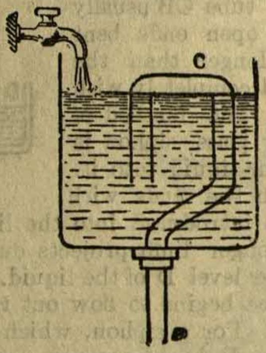


Fig 208

Tantalus Cup—It consists of a cup in which a siphon is placed concealed within the bust of a man, representing King Tantalus (Fig. 207). The siphon is so placed that its bend is just below the level of the lower lip of the figure. The shorter limb ends near the bottom of the cup, while the longer limb just passes out. As water is poured into the cup, the level gradually rises both in the cup and in the shorter limb. As soon as water reaches the top of the bend and is about to reach the lips, it begins to flow out. If the supply of water is then stopped or if it flows out at a rate faster than that at which it is supplied, the water level gradually sinks into the cup until it goes below the end of the shorter limb. The siphon now empties itself and the action stops. If water is supplied again, the same process is repeated. Water, therefore, never reaches the lips of Tantalus, thus keeping him as thirsty as ever.

Automatic Flush—The principle of a siphon is also utilised in *automatic flushes* often fitted in public latrines, etc. A siphon is fitted inside a tank and the longer limb passes out (Fig. 208) A stream of water flows into the tank and its level gradually rises. Ultimately when the level of water reaches the bend, the siphon begins to work and water flows out in a powerful gush till the level goes below the open end of the shorter limb. The ordinary flushing arrangement of bathroom is also a modified form of automatic flush.

Example :

A tall cylinder 90 cm. in length is full of mercury. What is the greatest depth of it which can be emptied out by means of siphon, the barometric height being 756 mm. of mercury ? [P. U.—1924]

Ans. The siphon can work till the height of the mercury column in the shorter tube above the mercury surface is not greater than 756 mm. Hence placing the bend of the siphon on the upper edge of the cylinder, mercury can be moved from the cylinder to a depth of 756 mm.

***148. Water Jet Pump or Filter Pump**—This is one of the simplest forms of an air-pump made ordinarily of glass. The apparatus consists of a glass tube T fused into a bigger jacket tube. The tube T ends in a nozzle J. The jacket tube is provided with a side tube C. Just against the nozzle, there is a funnel-shaped glass tube R which is led out through the bottom of the jacket. The side tube C (Fig 209) is connected with a rubber tubing to the vessel to be exhausted, while the end T is also connected by a rubber tubing to the water mains, the pressure of which should remain constant. As a strong jet of water is forced out of the narrow mouth of nozzle J at a very high speed, it entangles some of the air surrounding the narrow nozzle and carries it down the tube due to viscous friction. A partial vacuum is thus produced and so air is sucked from the vessel connected to it. The filter pump may produce a pressure only as low as 7 mm. of mercury and it is therefore not used when good vacuum is required. It is however very simple, economical and convenient requiring very little attention.

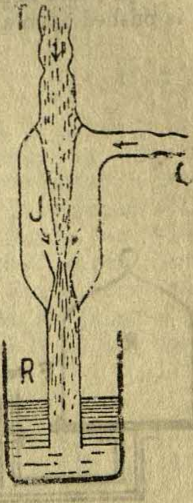


Fig. 209

A modified form of pump, known as Sprengel pump, had its origin from the principle of suction of a jet pump. This modified pump was devised in 1865. Mercury is allowed to fall through a delivery tube in drops being controlled by a pinch cock. From near the top of the tail tube, a side tube is connected to the receiver which is to be evacuated. By this means, as drops of mercury trickle down, small quantities of air entrapped between successive drops are continuously swept out. With proper care and modification a Sprengel pump is capable of producing a pressure of 10^{-4} mm. of murcury. Sir William Crookes (1832-1919) used such a pump in his memorable investigations on radiometers and discharge tubes.

149. Air-Pump (or Exhaust Pump)—The first mechanical air pump was constructed by Otto von Guericke (1602-1686) in 1650. The air-pump consists essentially of cylindrical metal *barrel* AB (Fig. 210), provided with an air-tight *piston* P, which can be moved up and down by means of a suitable handle. The pipe at the bottom connects the barrel with a vessel called the *receiver* R from which air is to be exhausted. The receiver may be well-greased and fixed air-tight on a flat circular disc EF, called the *plate* of the pump. There is a valve V in the piston and another at the junction of the barrel and the pipe both of which open into the barrel. To indicate the pressure within the receiver a manometer may be attached to the pipe connecting the receiver and the barrel through side tube M.

Action—Let us start with the position at its lowest position. During the *up-stroke*, as the piston is drawn up, the valve V is closed due to excess pressure at its top. The air in R in expanding opens the valve *a* and enters the barrel. During the *down-stroke*, the in the barrel below the piston is compressed. The inside therefore increases. Due to this valve *a* become closed. When the inside pressure becomes greater than the atmospheric pressure, the valve V is pushed open and the air inside the barrel goes above the piston.

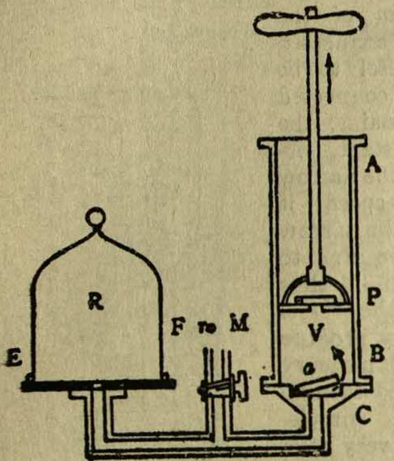


Fig. 210 Air Pump

During the next *up-stroke*, the valve V is closed due to the excess pressure outside. A partial vacuum is created between the piston head and valve *a*, and so the air remaining within the receiver again expands, opening the valve *a*. A fresh quantity of air from the receiver now fills the barrel. Thus during each up-stroke the barrel is filled with some amount of air from the receiver, which is let out during the next up-stroke. As the operation is repeated, the receiver gets exhausted more ane more. This goes on, till the pressure of the air left in the receiver bocomes too weak to open the valve *a*. The limit of exhaustion attained by the instrument is then reached and any further evacuation stops.

Degree of Evacuation—The pressure and the density of air within the receiver continuously fall off as the pump is worked. The following calculation shows how pressure and density change with the number of strokes.

Let the volume of receiver, including that of the pipe C = V

Volume of the barrel between extreme positions of piston $= \nu$

and density of air initially in the receiver $= \rho$

At the end of each up-stroke a volume V of air expands to occupy a volume $(V+\nu)$ and is thus reduced in density. Let the density of the air at the end of the first up-stroke be ρ_1.

Then, $V\rho = (V+\nu)\rho_1$, whence $\rho_1 = \left(\dfrac{V}{V+\nu}\right)\rho$.

At the end of the next down-stroke the air in the barrel escapes above the piston through its valve and a quantity of air of volume V and density ρ_1 is left in the receiver. At the end of the second up-stroke, this air of the receiver expands to a volume $(V+\nu)$ and thereby becomes reduced in density to ρ_2 say,

Then, $V\rho_1 = (V+\nu)\rho_2$ when $\rho_2 = \left(\dfrac{V}{V+\nu}\right)\rho_1 = \left(\dfrac{V}{V+\nu}\right)^2 \rho$.

Proceeding in this way, it can be shown that density of air left in the receiver after *n* strokes is given by,—

$$\rho_n = \left(\frac{V}{V+\nu}\right)^n \rho, \qquad \text{... (149,1)}$$

If the temperature of air within the receiver remains constant, Boyle's law holds good and the pressure becomes proportional to density. So if P be the initial pressure of the gass in the receiver and P_n be the pressure of the gas left in the receiver after n complete strokes then,

$$P_n = \left(\frac{V}{V+v}\right)^n P \qquad \ldots(149{,}2)$$

From mathematical stand-point, since $V/(V+v)$ is a quantity less than unity, the higher is the value of n, the less becomes the value of the powered quantity of eqn. (149.2), but it can never attain a zero value. A high degree of evacuation can, however, be attained with a good pump after a sufficient number of strokes. But in fact, with a mechanical air-pump, as described above the valves become more or less leaky at lower pressures. Moreover the valves have some weight and at a certain stage of lower pressure, the air pressure of the receiver is unable to open the first valve. It is at this stage that any further evacuation by the pump ceases.

Examples :

1. After four strokes the density of the air in the receiver of an air pump is found to the original density the ratio of 256 to 625. What is the ratio of the volume of the barrel to that of the receiver ? [C. U.—1953]

Ans. Let the volume of the receiver and that of the barrel be V and v.

then $\frac{256}{625} = \left(\frac{V}{V+v}\right)^4$ or, $\frac{V}{V+v} = \frac{4}{5}$, whence $\frac{v}{V} = \frac{1}{4}$.

2. The barrel of an exhaust pump has half the volume of the receiver to be evacuated. Find the pressure in the receiver after 4 complete strokes, if the original pressure was 76 cms. of mercury. [Mys. U.—1952]

Ans. If P_n be the pressure of air in the receiver after nth stroke, V and v the volumes of the receiver and the barrel of the pump, then neglecting the volume of the communicating pipe and assuming the starting pressure to be P,

$$P_n = \left(\frac{V}{V+v}\right)^n P = \left(\frac{V}{V+\frac{1}{2}V}\right)^4 \times 76 = \left(\frac{2}{3}\right)^4 \times 76 = 15 \text{ cms. of Hg.}$$

3. The volume of the receiver of an air-pump is 1250 c.c. and that of the barrel is 250 c.c. If the original density of the air in the receiver is 0·001293 gm. per c.c. find the density after 10 complete strokes. Find also the mass of air removed from the receiver.

Ans. Let the density of air in the receiver after 10 strokes be $= \rho'$.

Then $\rho' = \left(\frac{1250}{1250+250}\right)^{10} \times 0{\cdot}001293 = \left(\frac{5}{6}\right)^{10} \times 0{\cdot}001293 = {\cdot}8333^{10} \times 0{\cdot}001293.$

Taking logarithms, $\log \rho' = 10 \log {\cdot}8333 + \log 0{\cdot}001293$

$\therefore \log \rho' = \bar{1}{\cdot}2080 + \bar{3}{\cdot}1116 = \bar{4}{\cdot}8196$, whence $\rho' = 0{\cdot}0002087$ gm./c.c.

So the required mass $= 1250 \times (0{\cdot}001293 - 0{\cdot}000208)$ gm. $= 1{\cdot}356$ gm.

150. Guericke Pump—An improved piston type of an air-pump consists of a barrel C with a side tube E (Fig 212). The tube communicates with the barrel through number of port holes P.P. The piston L is provided with an oil valve at the bottom and is screwed rigidly with a rod D. This rod passes through a valve R and is connected to a lever, by which motion may be imparted to the piston. In some apparatus as shown in Fig. 213, the piston rod is connected externally to the flywheel W through a crank and by rotating the wheel a continuous up-and-down motion of the piston may be obtained. The junctions are made air-tight with a thick layer of heavy oil pressed on the piston and on the valve R. The valve R is kept pressed downwards by a spring HH (Fig. 212).

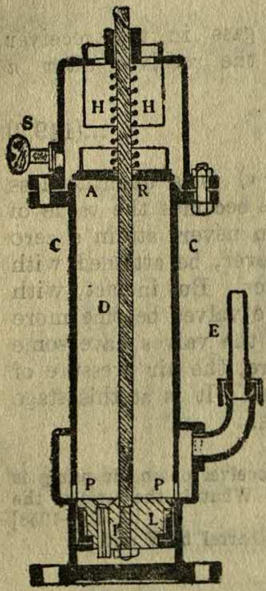


Fig. 212

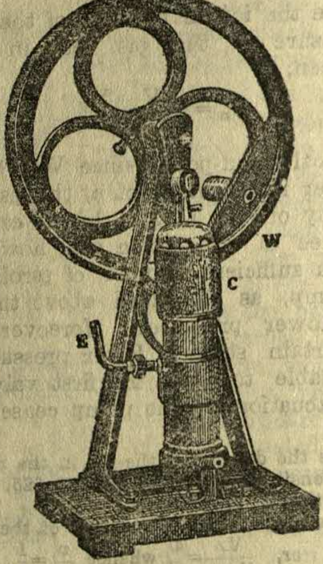


Fig. 213

The receiver is connected to the inlet pipe E. When the piston is at the bottom, the air within the receiver expands into the barrel through the port holes. But when the piston is raised up, the air within the barrel is isolated from the receiver and this air is gradually compressed till at the topmost position the piston pushes up the valve to make space for air to escape. When the piston is moved down a bit, the spring closes the valve A and the space within the barrel is cut off from atmosphere. When the piston is at the lowest position, the air with the receiver again finds its way into the barrel through the port-holes. In this way by the continuous up and down motion of the piston, air is intermittently taken out of the receiver. In a good working condition and with sufficient speed, this type of pump can bring down the pressure of the receiver to about 1 mm. of mercury.

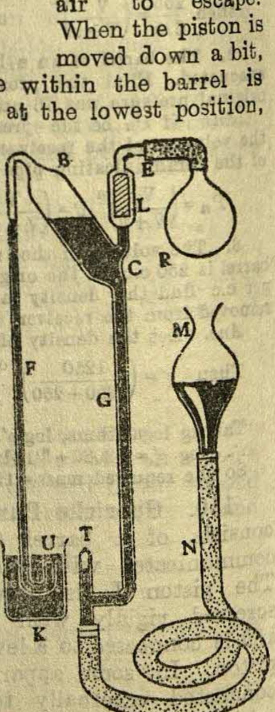


Fig. 214

***151. Teopler pump**—The apparatus originally designed by Teopler in 1862, consists of glass bulb B to the ends of which two vertical barometer tubes F and G, each about 32 inches long are connected (Fig. 214). The tube F is narrow and of about 1 millimetre bore and its lower end, which is usually bent upwards, is dipped into mercury contained in a vessel K. The wider tube G has got a bend at its lower part and is connected to a short piece of glass tubing T. One end of stout pressure tubing is connected to the base of this short tube while its other end is connected to a reservoir M containing mercury. A side tube connected with the bulb B and C leads to the receiver R through enlarged part in which a small glass bead L acts as a valve by not allowing mercury to run into the receiver. In original Teopler's machine there was no valve but the part CE was a vertical tube of glass of length of about 22 inches. This attachment made the apparatus about 8 ft. in height, but the provision of the valve has made the apparatus about 5 ft. in height with a greater case of operating the machine. The receiver is connected very carefully with leak-tight pressure tubing N.

At the start the receiver, a part of the tube G, the bulb B and the whole of the tube D are full of air at atmospheric pressure. The receiver M is gradually raised when the mercury level in G gradually rises slowly compressing air at different parts of the apparatus. When M is raised to such a height that the mercury level enters the tube B and is just above the point C, then the air within the receiver R is cut off from the air within B. On now raising the receiver still higher, the glass valve L floating on mercury closes the passage to the receiver while mercury level in B rises and forces air to pass to the atmosphere through the mouth U. Actually mercury is allowed to run down through F to drive off last traces of air in this region.

The reservoir is now lowered when the mercury level in B comes down to leave a vacuous space in it. When the level of mercury comes below C, the air within the receiver expands to fill the vacuous space in B. Again the reservoir is raised to cut off air in B and this air is ultimately driven out of U by raising the reservoir sufficiently. Thus the lowering of mercury level below C establishes a communication between B and R, and a part of the air expands into B and by raising the level of mercury to the extent that mercury falls through the tube F, this air is expelled to the atmosphere. Thus with a continued operation, more and more air from the receiver is discharged to the atmosphere and so the pressure within the receiver lowers down. The mercury in G plays the role of an air-tight piston, while mercury in the tall tube F plays the part of the valve in not allowing atmospheric air to enter the bulb A. As there is an intermittent stream of mercury running down through the tall tube, mercury gradually collects within the vessel K and has to be poured back into M from time to time. It is a very satisfactory type of pump, as we can get a good vacuum with it. But the action of the pump is very slow. Lord Rayleigh (1842-1919) stated that he required the whole of the morning to evacuate a receiver of capacity 2 litres to a sufficient degree. However slow might be the operation, the pump is capable of producing a pressure of the order of 10^{-4} mm. of mercury or even less.

152. Rotary Hyvac Pump—This type of the pump, originally designed by Gaede in 1807 depends for its working upon a continuous rotation of a cylinder within a suitable chamber (Fig. 215). The instrument consists of a hollow cylindrical body of iron having two openings near each other at the top. Of these, the one, called the *inlet port* I, is provided with a pipe projected vertically upwards which can be connected with a rubber tubing to the vessel to be evacuated. The other, called the *outlet port* O, is provided with a valve such that any gas contained within the cylinder may escape but nothing can come within. The two ports are located on either side of a movable air-tight partition S kept pressed by a lever L and a spring. The solid cylinder within the hollow cylinder is rigidly fixed to a shaft and can be rotated eccentrically about an axis. With the revolution of the inner cylinder, a rotatory contact is maintained between the solid and hollow cylinders and the vane is always kept pressed on the inner cylinder. The complete picture of rotary pump is given in figure 216 in which there are two pumps side by side, working on the same shaft and enclosed within a rectangular metal box.

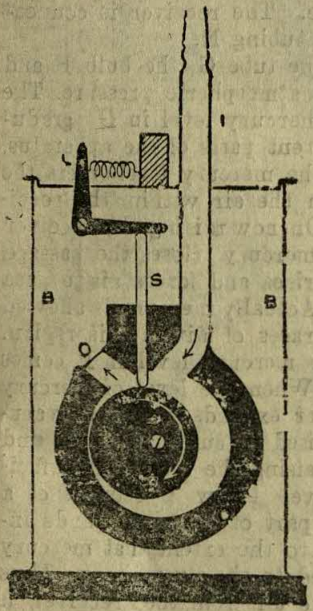


Fig. 215

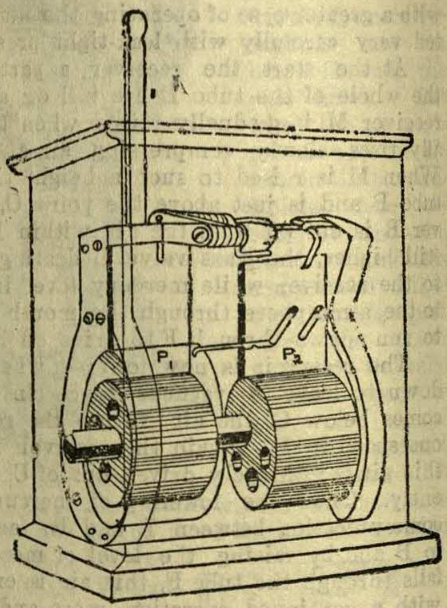


Fig. 216—Rotary Hyvac Pump

The principle of action of the instrument is as follows. When the solid cylinder in course of rotation occupies the position as shown in Fig. 217, it closes the outlet port and the inlet becomes just open. The gas from the receiver expands within the crescent-shaped space M. The next stage is shown in Fig. 218 when the expanded gas is

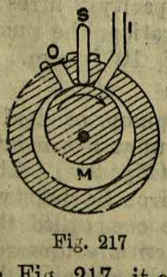


Fig. 217

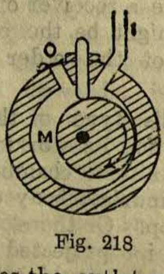


Fig. 218

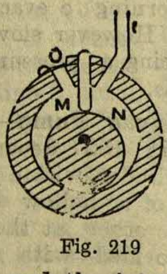


Fig. 219

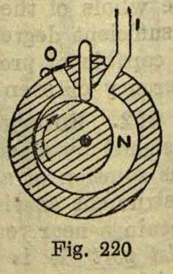


Fig. 220

ejected from the receiver into the space M. At the third stage, the cylinder gradually compresses the gas within the diminishing space, and also allows air from the receiver to fill in the space N (Fig. 219). At the fourth stage of operation, all entrapped gas is pressed out through the outlet port (Fig. 220). Thus with continuous rotation, the gas is gradually transferred from the receiver to the outside atmosphere. The whole apparatus is immersed in machine oil in a rectangular metal box B so as to make it air-tight from atmospheric air.

It is evident from the principle of pumping that with each revolution of the shaft, the amount of gas in space M is forced out into atmosphere. So it is not possible to make the receiver *absolutely vacuous*, but the limitation of pumping depends upon the pressure acting on the valve at the outlet. If the pressure outside the valve is the atmospheric pressure, as in the case when single pump is worked, it can evacuate a receiver up to a pressure of 10^{-2} mm. of mercury. But if the pressure outside the valve is kept lower than the atmosphere the degree of evacuation is much higher. For this reason a *two-stage* pump is frequently used. The first pump or the fore-vacuum pump creates a pressure of 10^{-2}mm. of mercury just before the outlet of the second pump which is connected to the receiver. Such a two stage pump as shown in figure 216, can evacuate down to pressure of 10^{-4} mm. of mercury.

Units for Low Pressure—Since any rough suction pump is able to evacuate a receiver to a pressure less than that due to column of mercury of height of 1 cm., it is customary to denote such a pressure in terms of height of mercury in millimetres. Thus one millimetre height of mercury corresponds to a pressure about 1333 dynes per sq. cm. taking density of mercury to be 13·59 at 0°C and $g=980{\cdot}6$ C.G.S. units. Moderately low pressure is measured in terms of *micron* (μ) of mercury which is 10^{-3}mm. of mercury (1·33 dynes/sq. cm.) and very low pressures are measurad in terms of a *millimicron* ($\mu\mu$) which is 10^{-6} mm. of mercury. For example, a pressure of ·000031 cm. is equivalent to a pressure of 31 $\mu\mu$ of mercury.

*153. **Vacuum Brake System**—Air pressure is conveniently utilised to arrest the motion of automobiles whenever necessary. Fig. 221 represents a *vacuum brake system* sometimes employed in

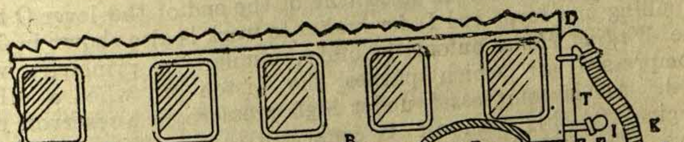


Fig. 221—Vacuum Brake System

railways and tram cars. Stout curved pieces of steel called brake *shoes* S are fixed near the rim of every wheel with a very small clearance between them under normal condition. These shoes can be operated by system of levers L and M and can be pressed hard on the wheels. The mechanism of action of the levers is as follows:

The chamber C, known as the vacuum chamber is normally placed under the carriage. There is a piston P dividing C into two compartments. A ball valve V fitted at one corner of the piston (shown enlarged in Fig. 222) allows air to pass from the upper to the lower compartment but not in the reverse direction. The piston moves over a rubber ring G and is so air-tight. The lower compartment is connected to a system of pipes TT running through every bogey composing the train. This pipe is called *train pipe.* One end of this air pump fitted at the engine. When the pump of the engine is worked continuously, a partial vacuum is produced within both the compartments of the vacuum chamber of every bogey. The weight of the piston and its accessories is sufficient to lower the piston P down. A downward displacement of the end M of piston rod causes the end O of the lever to move to the left side and to make the wheel free from brake pressures.

The valve E (Fig. 221), connected with the train pipe, can be opened by pulling a chain provided within the bogey, called the *alarm chain.* When the alarm chain is pulled the valve E is forced open and immediately the lower compartment of the vacuum chamber is filled with air pressing the piston up. The upward motion of the piston works the lever in the opposite direction forcing the brake shoes on the wheels. For an indication, a flat bar is coupled with the chain in every bogey and this bar projects out when the alarm chain has been pulled. Once the alarm chain is pulled from a train, it takes sometimes to re-evacuate the chamber before the engine can start again. A small displacement of the end of the lever O to the right pulling the lever rod L for brakes to work is also shown in figure.

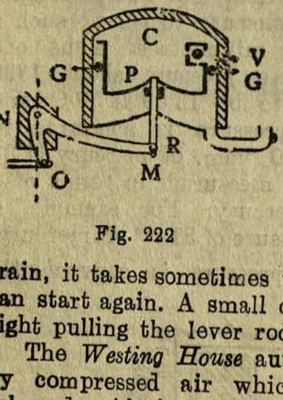


Fig. 222

The *Westing House* automatic brake, similar in principle, works by compressed air which pushes the piston up when a valve is released. Air is compressed under high pressure in a reservoir under the engine and this reservoir is connected to the train pipe through a valve. The opening of the valve fills the lower compartment of the chamber C with air at high pressure raising the piston and pressing the brake shoes on the wheels. To release the brake another valve is opened and the compressed air escapes to atmosphere lowering the piston. The usual *hissing* of the electric tram cars at the time of start is due to the escape of compressed air to the atmosphere through the release valve. In motor cars, there is a hydraulic brake system which has proved very efficient. It works on the principle of hydraulic press system which converts a small force into a large force by the displacement of a liquid from a narrow pipe into a large pipe.

154. Condensing or Compression Pump—This is used for compressing air into a closed vessel called the receiver. It consists of *barrel* AB (Fig. 223), provided with an air-tight piston P, which can be moved in or out by suitable handle. The end of a barrel may be connected through a stop-cock to the vessel into which air is to be compressed. The stop-cock is closed when the required degree of compression has been attained. The piston and the end of the barrel are provided with valves E and F both of which open towards the receiver.

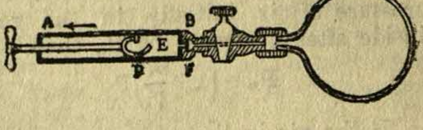


Fig. 223—Compression Pump

Action—Let us start with piston at the end of the barrel towards the receiver. As the piston is drawn out during the *backward* stroke, the space between the valves E and F increases, so that the pressure therein falls. The valve F is immediately closed owing to the excess of pressure of the air in the receiver. The atmospheric pressure opens the valve E and the barrel is filled with air at the atmospheric pressure.

As the piston moves inward during the next *forward* stroke, the air within the barrel between the valves is compressed. The inside pressure becomes higher than the almospheric pressure when the valve E is at once closed. When the pressure within the barrel becomes higher than that of air in the receiver, F is pushed open and the air in the barrel is forced into the receiver. Thus at the end of each outward stroke, the barrel is filled with some air at the atmospheric pressure, which during the next inward stroke, is forced into the receiver. Therefore the density of air within the receiver becomes higher and higher with number of strokes.

Degree of Compression—The density and pressure of air in the receiver after a given number of strockes can be calculated in the following way :

Let the volume of the receiver with that of the connecting tube $=V$

Volume of barrel between extreme positions of the piston $=v$

Density of atmospheric air at the time $=\rho$

Suppose the air in the receiver has initially the same pressure and density as those of atmospheric air. It is evident that at the end of each forward stroke a quantity of air of volume v and of density ρ and so of mass $v\rho$ is forced into the receiver.

Now mass of air initially within the receiver $=V\rho$

Mass of air forced tnto the receiver by n strokes $=nv\rho$

$\therefore$ total mass of air in the receiver after n strokes $=V\rho+nv\rho$

As the volume of the receiver remains equal to V, let ρ_n be the density after n strokes. Then

$$V\rho_n=V\rho+nv\rho, \quad \text{whence } \rho_n=\left(1+\frac{nv}{V}\right)\rho \qquad \text{...(154,1)}$$

The forcing of air within the receiver raises its temperature a little higher. After waiting for somewhile *the temperature is brought down to that of surrounding*. At this stage Boyle's law holds good and so the pressure is proportional to the density. Hence, if the pressure of air initially in the receiver be P and that of the air inside after *n* strokes be P_n then,

$$P_n = \left(1 + \frac{nv}{V}\right)P \qquad \ldots(154{,}2)$$

If the air in the receiver was initially at the atmospheric pressure, then the pressure of the air inside after *n* strokes is

$$P_n = \left(1 + \frac{nv}{V}\right) \text{ atmospheres.}$$

155. Different uses of a Condensing Pump—The ordinary football inflator (Fig. 224) is a common example of *condensing pump*. Within a barrel works a piston N connected with the rod R to the handle H. The valve at the piston head is replaced by a cup-shaped desc of leather, called *cup valve* or washer. The piston disc is a little smaller than the barrel of the pump. During the *inward* stroke, the washer expands due to the compressive reaction of inside air and fits air-tight within the barrel. When the pressure of the compressed air within the barrel becomes higher than the pressure within the receiver, a light ball valve V at the nozzle of the pump moves to the right to make room for air to pass into the receiver. A perforated diaphragm D does not allow the ball valve to move too far. During all the time of the inward stroke, air from outside enters into the left part of the barrel through the port hole of the handle. During the *outward* stroke (Fig. 225), the volume of the enclosed air within the receiver in trying to expand forces the ball valve to sit air-tight into the conical hole and so the entrapped air cannot leak out. The space between the piston and the nozzle becomes partly vacuous. The atmospheric air then forces its way through the sides of the washer filling up the space. For the next inward stroke, some air is again forced into the receiver.

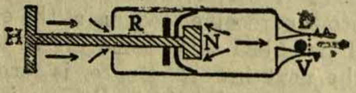


Fig. 224

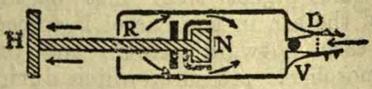


Fig. 225

Figure 226 shows a bicycle inflator, which is essentially the same as a football inflator, but instead of a valve at the nozzle O of the pump there is a separate attachment of an air-tight valve-tube as shown in figure 227. The compressor as usual consists of cylindrical

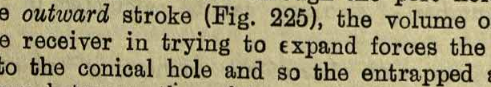


Fig. 226

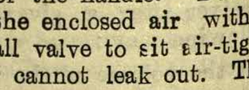


Fig. 227

barrel B, in which works a piston N with its leather washer W, a fixing disc G and the piston rod R with its handle H. The nozzle is provided with a screw thread to which the screw cap of a short rubber tubing can be fixed air-tight. The other end of this tubing is screwed to the end C of the valve tube permanently attached to the tube of the wheel. The valve tube (Fig. 227) consists of a narrow metal tube U closed at the extreme end and having an opening E at its side. It is preserved in an outer metal casing T. There is a rubber cylindrical jacket R covering the valve tube.

When the pressure of the compressed air forced by the pump into the space around C exceeds that on the other side of the rubber cylinder, the flap of rubber at the opening E is raised up and air is admitted into the tube. But when the handle H is drawn out, the pressure around C falls and air from the tube of the wheel, in an attempt to force its way through E, tightly presses the gap on the hole and closes the opening. Hence, with a continued operation of the handle, more and more air is pumped into the tube. This inflated tube rests within a thick India rubber tyre called the **Pneumatic tyre.** The compressibility of the enclosed air helps the wheels to pass undeviated over obstacles ; while due to the expansibility of air, the tyre regains its original form after being depressed.

Air Cushion—This consists of a hollow bag of rubber sheet with a small mouth at one side fitted with a stop-cock. By means of a *condensing pump* air is compressed into the bag to the required extent when stop-cock is closed. Owing to its compressibility and expansibility, the enclosed air serves as the cushion. An air-gun may be regarded as a *condensing pump* without any valve. By a mechanism, the air in the barrel is suddenly compressed. This air is being released works upon a spring which throws out the shot.

Oil-Stove—In stove and various lamps in which oil vapour burns, the oil pot is designed to leave some space above the oil level. A small *condensing pump* P is fitted with the pot (Fig. 228). There is an air-tight key K, which can keep air entrapped within the pot when necessary. Air being pumped into the pot bubbles through the oil and is stored at its upper part. The pressure of air forces the oil to rise along the pipe N to the jet-tube J. There is coiled wire gauge within the tube, through which the oil has to run up. If this wire gauge is sufficiently heated before the first circulation of oil, then the oil in course of reaching the jet is converted into vapour. The oil vapour rebounds at the burner B and circulates within the funnel. The heat of the burner is sufficient to ignite the vapour which burns at its mouth. Compressed air is used in spray painting, in driving boring machines, propelling torpedoes, etc. **The Soda-water machine**

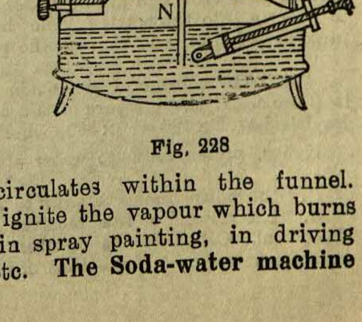
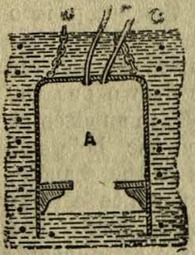


Fig. 228

is practically a *condensing pump.* Carbon dioxide gas is taken from the reservoir in which it is generated and then forced into a bottle containing water. This gas is absorbed by the water, which is said to be *aerated* and this water is called *soda water*. All the machines described in this Article work on the principle of utilisation of the **power of air.**

Diving Bell—It is used to enable a diver to go down to a great depth under water to lay the foundation of pires or do some such work there. It consists of a heavy cylindrical or bell-shaped iron vessel A with rigid walls, closed at the top and open at the bottom (Fig. 229). There is a platform attached inside the bell on which the worker can take his seat. It is lowered into water by means of chains C with the open end downwards and it sinks under its own weight. The air inside is compressed and water rises slowly inside. Thus taking the height of the water barometer as 34 ft. the air inside the bell would be under a pressure of 2 atmospheres at a depth of 34 ft, and so its volume would be reduced to half. Evidently, this will be inconvenient for the workmen inside.



Fig, 229

Nowadays to keep down the water at a convenient level inside the bell and to supply the workmen freely with fresh air, air is pumped into the bell through the tube P at its top by a *condensing pump* above. A second tube leading from the bell to the surface carries away the foul air from inside. In recent time a type of **diving suit** has been devised which can easily withstand water pressure down to the depth of several hundred feet without any discomfort of the diver. In going further lower to ocean bed, an improved type of diving bell, known as a **Bathyscope** is used.

Examples :

1. The barrel and receiver of a condensing pump have capacities of 75 c.c. and 1000 c.c. respectively. How many strokes will be required to raise the pressure of the air in the receiver from one to four atmospheres ? [C. U.—1945 ; Gau. U.—1952]

Ans. Let the required number of strokes be n.

Then $4=\left(1+\frac{75n}{1000}\right)$, whence $n=40$.

2. A bottle whose volume is 500 c.c. is sunk mouth downwards below the surface of a tank containing water. How far must it be sunk for 100 c.c. of water to run up into the bottle ? The height of the barometer at the surface of the tank is 760 mm. and the specific gravity of mercury is 13·6. (Pat. U.—1948 ; Gau. U.—1955]

Ans. The volume of air inside the bottle when 100 c.c. of water rushes in, is 400 c.c. If the pressure of the air within the bottle be now P then from Boyle's law $P\times400=76\times500$, whence $P=95$ cms. of mercury.

Hence, pressure due to water$=(95-76)=19$ cms. of mercury
$=19\times13{\cdot}6=258{\cdot}4$ cms. of water.

∴ the bottle must be sunk to a depth of 258·4 cms. under water.

3. The volume of the receiver in a condensing air-pump is 10 times that of the barrel. Calculate the number of strokes necessary to raise the pressure of air in the receiver to 5 times that of external air. [Utkal U.—1953]

Ans. If P_n be the pressure in the receiver after nth stroke and the atmospheric pressure is P then $P_n/P=5$. Again the ratio of the volume of the receiver to that of the barrel $=V/v=10$. Then from eqn. (154,2)

$$\frac{P_n}{P}=5=1+\frac{nv}{V}=1+\frac{n}{10},$$ whence n = number of strokes = 40.

4. A cylindrical diving bell 6 ft. high, is lowered into water until the top of the bell is 31 ft. below the surface of water. If the height of the water barometer at the time be 34 ft. find to what height water will rise into the bell. If the cross-section of the bell be 10 sq. ft. find how much air at atmospheric pressure must have to be introduced into the bell so as to keep away water from entering into it.

Ans. Let the water rise into the bell up to x ft. from the top.

Then the pressure of the air inside is equal to that of a column of water of height equal to $(34+31+x)$ ft. $=(x+65)$ ft.

The original and final volumes of air in the bell are proportional to 6 and x respectively.

$\therefore$ $6\times34=x(x+65)$ or, $x^2+65x-204=0$, whence $x=3$ ft. or, -68 ft. Since a negative value of x is absurd, we have to accept the positive value of x to be 3 ft.

Original volume of air in the bell = 60 cu. ft. and initial pressure = 34 ft.

When the top of the bell is 31 ft. under water and all water entering it is kept away, pressure of air inside $=(34+31+6)=71$ ft. water.

Let the volume of air introduced under atmospheric pressure $=v$ cu. ft.

Then $34(60+v)=71\times60$, whence $v=65{\cdot}3$ cu. ft.

***156. Pressure Gauges**—In order to know how much an air-pump can evacuate or compress air into a receiver, we have to use a class of apparatus which indicates the pressure of the residual gas within a receiver. Such a pressure measuring device is called a *pressure gauge*. There are various types of the instruments designed for different ranges of pressure

Siphon Gauge—It consists of a glass tube, ABCD, one limb CA of which is straight while the other limb D is bent at the top and is connected to the receiver (Fig. 230) The U-tube is about half-filled with a suitable liquid such as mercury oil or water and is fixed vertically to a stand fitted with a scale. When the pressure to be measured is either a *little above or below the atmospheric pressure*, the end A is kept open. If the pressure, within the receiver is equal to the atmospheric pressure, the column of liquid has at its end the same pressure and hence the free surfaces of the liquid within the U-tube are at the same level. But if the pressure within the receiver becomes higher, it would push down the level at D causing the other levles at C to rise up. If, in the steady condition, the difference of level in C and D be h cm. as recorded by the scale attached, then from the law of hydrosatic pressure—

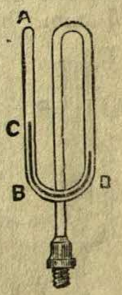


Fig. 230

Pressure within receiver $=h\rho g+$ atmospheric pressure ; where the density of liquid used is ρ and acceleration due to gravity is g.

Thus the difference between the gas pressure and atmospheric pressure is proportional to the difference of the levels. When the gas pressure falls below the atmospheric pressure, the level at D goes up and C down, the difference of the pressure being also $h\rho g$. For a given difference of pressures, $h\rho g$ would be constant and hence lighter is the liquid used, the larger would be the difference of levels. Thus a lighter liquid is more convenient to measure a small difference of pressures.

When the pressure to be measured is *high*, say up to 5 or 10 atmospheres, the limb C is closed at the top and the part AC contains some dry air. As the pressure within the receiver is increased the liquid level in D is depressed and that in C is forced up, so that air enclosed between AC is gradually compressed. When the temperature remains fairly uniform and assuming that the bore of the part AC is uniform, we can apply Boyle's law to the resulting pressure. If the length of the air column in AC is l, when the two levels are same under atmospheric pressure P and if the length of the column conracts to l_2 under a pressure P_2 then from the Boyle's law

$$P_2 l_2 = P_1 l_1, \text{ whence } P_2 = P_1 \frac{l_1}{l_2} \qquad \ldots(156,1)$$

If, at this stage, the difference of levels of the liquid be h, then the gas pressure within the receiver $P_2 + h\rho g$. For example, suppose that mercury of density 13·59 gm./c.c. is used as the gauge liquid and l_1 is 30 cms. If on connecting the receiver l_2 becomes 8 cms., then the pressure P_2 within the closed end in $30 \div 8 = 3·75$ atmospheres, If the barometer shows the atmospheric pressure to be 75·14 cms. of mercury, then $P_2 = 3·75 \times 75·14$ cms. $= 281·775$ cms. of mercury. If now the difference of the levels of the mercury column be 43·12 cms, then the gas pressure within the receiver is $281·771 + 43·12 = 324·895$ cms of mercury.

McLeod Gauge—This pressure gauge was designed by McLeod in 1874 and is used extensively in measuring very low pressures which are far beyond the scope of a Siphon gauge. The apparatus consists of a closed top calibrated glass capillary tube B of about ·5 mm. bore and of length about 50 cms. fixed at the top of a cylindrical glass bulb of capacity 200 c.c. to 500 c.c. (Fig. 231). The bulb at its lower end is connected by a uniform glass tube extending up to E. Another capillary glass tube with open end at D is connected just below the cylindrical glass bulb and is about a metre in length. The two capillaries run *vertically* alongside a metre scale (not shown in the diagram). A rubber tubing is connected at E and its other end is in communication with a mercury reservoir F. The reservoir can be placed at any height by suitable clamping arrangement.

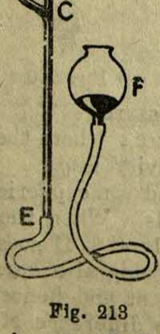


Fig. 213

The end D is connected to the receiver whose pressure is to be measured and so long as the receiver is not *attaining a uniformly low pressure*, the reservoir F is lowered sufficiently, so that mercury level in the tube CE is below the poinit C. At this stage there is a free communication of pressure of the receiver to the bulb and the capillaries. Now the reservoir is slowly raised and when the level of mercury is above the point C the reservior is cut off from the bulb. The reservoir is then raised more and more till mercury fills the tube and reaches a point B in the closed capillary. The mercury in D would remain at a higher level simply because after the bulb is cut off from the receiver by the mercury column, any compression of the residual gas in the closed tube obeys Boyle's law,—pressure inside increasing as volume is compressed. But mercury within the other capillary rises against a constant pressure of the source.

If V represents the volume of the bulb and the closed capillary, v_0 the volume of 1 mm. length of the capillary and a the length of the enclosed air column and h the difference of levels in mm. of mercury within two capillaries, ρ the receiver pressure in mm. of mercury, then remembering that at the cut off point volume of the gas is V and the pressure is ρ and applying Boyle's law we have,

$$p\mathrm{V} = av_0(\rho + h) \quad \text{or,} \quad \frac{\rho + h}{\rho} = \frac{\mathrm{V}}{av_0} \quad \text{or,} \quad 1 + \frac{h}{\rho} = \frac{\mathrm{V}}{av_0}$$

$$\text{whence} \quad \rho = h\frac{av_0}{\mathrm{V} - av_0} = \frac{hav_0}{\mathrm{V}} \text{ very approximately} \quad \ldots \quad (156, 2)$$

Suppose that $h = 1$ mm., $av_0 = 5$ cu. mm. and $\mathrm{V} = 500$ c.c. $= 5 \times 10^5$ cu. mm.; then substituting in the equation, the pressure recorded is 10^{-5} mm. of mercury. Pfund has constructed a most sensitive form of a Mcleod gauge capable of measuring pressure of 10^{-7} mm. of mercury. Very lately the gauze has been further modified to read a pressure of 10^{-9} mm.

Limitation of so-called Vacuum– It is not very definitely known what is exactly the highest possible vacuum reached till the present time. With a very best type of pump and pressure gauge the lowest pressure recorded so far is about 10^{-9} mm. with a flickering value of 10^{-10} mm. of mercury. For a steady and continuous working of a pump, a receiver shows a constant pressure of 10^{-8} mm. So it is safe to say that a pressure of 10^{-8} mm. is always under our control. We may now calculate how many molecules of the gas are still present per unit volume of the receiver at such a low pressure. From Avogadro's law of gaseous volume and from other evidences, we now know that at normal temperature and pressure 1 cubic cm. of a gas contains 3×10^{19} molecules. From this if we calculate the number of molecules in unit volume still present at pressure of 10^{-8} mm. the number comes out to be nearly 390 million. Thus perfect vacuum as speculated by early philosophers is still remote.

EXERCISES ON CHAPTER XII

	Reference
1. Why is it not possible to raise water head more than 34 ft by means of a suction pump? State reasons and give calculations. (U. P. B.—1961; Dac. U.—1952)	Art. 143
2. The spout of common pump is 7 ft. above the level of water in a well The diameter of the piston is 4 inches. What is the force on the piston rod when the pump is working? Weight of 1 cu. ft. of water is 62·5 lbs. *Ans.* 38·1 lbs.-wt.	Art. 143

Reference

Art. 145 3. Draw the sectional diagram of a force pump capable of delivering a continuous supply of water. Explain how it works.
(*Cf.* Pat. U.—1965 ; Utkal U.—1952)

Art. 146 4. A siphon is used to empty a cylindrical vessel filled with mercury. The shorter limb of the siphon reaches the bottom of vessel which is 45 inches deep, but it is found that the mercury ceases to run before the vessel is empty. Explain this.
(Dac. U.—1960)

Art. 146 5. Explain the principle and action of a siphon.
(Utkal U.—1952 ; Pat. U.—1963 ; C. U.—1957 ; U. P. B.—1946 ; Del. H.S.—1960)

Art. 146 6. Describe a filter pump and explain its mode of action.
(U. P. B.—1956 ; Raj. U.—1959)

Art. 149 7. Describe an air-pump and explain its action.
(Vis. U.—1955 ; *Cf.* Raj U.—1962 ; Gau. U.—1958 ; C. U.—1957 ; Pat. U.—1952 ; U. P. B.—1952 ; *Cf.* E. P. U.—1963)

Art. 143 8. Describe with the help of a clear diagram a pump for lifting water from a well about 15 ft. deep. What trouble will be experienced if the depth of the well exceeds 30 ft. (U. P. B.—1955)

Art. 149 9. If the barrel of air-pump is one-third the size of the receiver, what fractional part of the original air will be left after 5 strokes? What will a barometer within the receiver read, the outside pressure being 76 cms.? (Gau. U.—1963)
Ans. $\frac{243}{1024}$; 18 cms. (nearly)

Art. 149 10. If the pressure in a pump were reduced to $\frac{1}{3}$ of the atmospheric pressure in 4 strokes, to what extent it would be reduced in 6 strokes?
Ans. 0·192 of the initial pressure.

Art. 149 11. The volume of the barrel of an air-pump is 1/5th that of receiver. Find after how many strokes the density of air in the receiver will be reduced in the proportion 125 : 216. (Vis. U.—1955)
Ans. 3.

Art. 149 12. The contents of the receiver of an exhausting air-pump is 6 times that of a barrel. Find the pressure of the air in the receiver at the end of the eight strokes of the piston, when the atmospheric pressure is 15 lb. per square inch. (Utkal U.—1951)
Ans. 4·37 lb./sq. in.

Art. 149 13. Describe a double-barrelled air-pump and explain its action.
(C. U.—1948 ; '53)

Arts. 149 & 154 14. Compare the pressures in the receiver of condensing and exhausting air-pump after the same number of strokes, and accounting for the fundamental difference in form of the two expressions,
(*Cf.* C. U.—1958 ; Pat. U.—1952)

Art. 152 15. Explain the action of a rotary oil pump. (E. P. U.—1952)

Art. 154 16. Describe in detail, with a diagram, a condensing pump and its mode of action. (Utkal U.—1951 ; C. U.—1958)

Art. 154 17. The volume of the receiver in a condensing pump is 25 times that of the barrel. Find after how many strokes the pressure inside the receiver will be increased from one to three atmospheres.
Ans. 50.

Arts. 154 & 155 18. What do you mean by a compression pump? Give two common examples. Describe with a diagram, the working of an ordinary bicycle pump and the action of the value in the bicycle tube. (Gau. U.—1952)

Art. 155 19. Write a note on Diving Bell? (Utkal U.—1952, '54)

Arts. 152 & 156 20. Explain how would you produce and measure a high vacuum. Describe some uses of high vacuum. (And. U.—1951)

CHAPTER XIII

APPLICATIONS OF VECTORS IN MECHANICS

1. Vector and its Representation.—We have made a preliminary reference of a **vector quantity** and its fundamental difference from a scalar quantity in Art. 21. We shall make a more detailed study of a vector quantity in this Chapter. A vector requires for its specification, (i) a *unit* of some kind, (ii) a number or a numeric giving the magnitude of this quantity in terms of this unit, which we often call the *size of the vector* and (iii) an indication of a *direction.* In this sense quantities like displacement, velocity, force etc. occuring frequently in Mechanics are vectors. Scalar quantities have magnitudes only such as a mass, distance, time, etc.

Mathematical Representation.—Since a vector is a directed quantity with some *physical* unit, it may be represented by a straight line of some specific length such as AB representing its magnitude with an arrow-head over it in the form $\overrightarrow{AB}$ to signify its direction (Fig. 1). If AB be of length *M*, then $\vec{M}$ may repersent the vector along the specific direction. In some standard Books a Clarendon type is used to represent a vector which includes a magnitude and direction with proper units. The magnitude **M** of the vector is called the size or **modulus** of the vector. In fig. 1, if the moduli of the vectors AB, CD and PQ be represented by *M*, *N* and *R*, then these vectors can be represented by any one of the forms $\overrightarrow{AB}=\vec{M}=\mathbf{M}$, $\overrightarrow{CD}=\vec{N}=\mathbf{N}$ and $\overrightarrow{PQ}=\vec{R}=\mathbf{R}$. In this Book, the Clarendon type method of representing a vector will be generally adopted. Remember that although a Clarendon type vecor is not given an arrow-head, it represents simulaneously the magnitude (size), the unit and the direction when placed over a straight line with an arrow-head. In a hand-written script, as it is difficult to write a Clarendon type, the *students are advised* to use either an italics type such as $\vec{M}$ or $\overrightarrow{AB}$ for a vector.

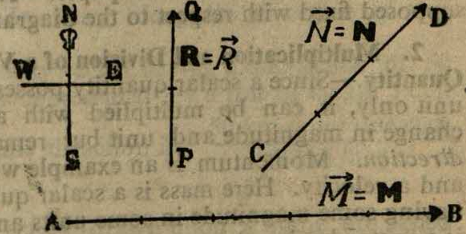


Fig. 1

Graphical Representation.—A straight line with an arrow-head at any end usefully represents a vector; the *length* of the line is taken to be proportional to the magnitude or *size* of the vector and the arrow-head is taken to represent the *direction.* The *unit of the quantity* involved in the vector is specified to represent its *nature* or *class.* Suppose that we have to represent graphically a velocity vector of 50 kilometres per hour towards east. Here kilometres per hour represent the unit, 50 is the magnitude and east is the direction. For specification of cardinal directions a compass with east-west and north-south directions is provided on the extreme left of fig. 1. Draw a straight line AB of a suitable length, say 5 cm., east-west and call this length proportional to 50 kilometres and put an arrow-head at the terminal B (fig. 1). Then the scale to which the vector is drawn is 1 cm. equivalent to 10 kilometres and since it is a velocity vector the unit of representation should be 1 cm.≡10 km./hour. On the same scale the vector $\overrightarrow{CD}$ of length 3 cm. will represent a velocity of 30 km./hour due north-east. For a similar reason $\overrightarrow{PQ}=\overrightarrow{R}=\mathbf{R}$ represeets a velocity 20 km./hour due north. It is a general rule that vectors belonging to the *same class* and represented in the *same diagram* must have a *uniform scale of drawing.*

Again the 3 specified directed lines (fig. 1) may represent vector of *three* different classes, such as **M** representing displacement, **N** a velocity and **R** a force. In such a case the unit and scale of each vector *must be mentioned* for understanding.

So we conclude this article by stating that vectors drawn on a plane surface or in three dimensional space must have *definite lengths* and arrow-head at their terminals. The cardinal directions east-west etc. may not be drawn on the paper. The vector directions may be supposed fixed with respect to the diagram.

2. Multiplication and Division of a Vector by a Numeral or Scalar Quantity.—Since a scalar quantity possesses a magnitude with some unit only, it can be multiplied with a vector which undergoes a change in magnitude and unit but remains a vector having the *same direction.* Momentum is an example which is the product of a mass and a velocity. Here mass is a scalar quantity but velocity is a vector having some magnitude in some units and direction. So momentum is a vector having the same direction as the velocity. In this case the momentum vector changes its class, the direction remaining same. A mass of 10 gm. (scalar) moving with a velocity of 20 cm./sec. due east (vector) possesses a momentum of 200 gm. cm./sec. due east (vector). Similar example is found in product of a mass and acceleration vector which produces a force vector of a different magnitude but *having the same direction.* Reversion from momentum to velocity vector requires a division of a vector by a scalar quantity.

When a vector is multiplied or divided simply by a number, it retains its class but its magnitude changes while the direction remains fixed. Suppose a is a number and **B** is a vector, then aB is a vector of the same class but it is a time larger than B. Division of **B** by a is the same as multiplication of B by $\frac{1}{a}$. If a is greater than unity, $\frac{\mathbf{B}}{a}$ must be less in magnitude than B, its direction remaining the same. Fo rexample, if we have a force vector of 100 dynes impressed due north, then this vector multiplied by 5 will give 500 dynes due north, or when it is divided by 10 will give 10 dynes impressed due north. This sort of division of a vector by its magnitude has given us an idea of unit of a vector.

2. Unit Vector.—Since every vector, irrespective of its class must have a definite magnitude, unit and direction, there must be a relation between the vector and its magnitude. In Fig. 2, vectors A, E and U are drawn to suitable scales. Let *A*, *E* and *U* be the corresponding magnitudes and units of these vectors. These italic types are neccessarsly scalar values of the vector quantities. Then the ratios $\frac{\mathbf{A}}{A}$, $\frac{\mathbf{E}}{E}$ and $\frac{\mathbf{U}}{U}$ are called the **unit vectors** along the directions of respective vectors.

Call $\frac{\mathbf{A}}{A}=\mathbf{a}$ or $\vec{a}$; $\frac{\mathbf{E}}{E}=\mathbf{e}$ or $\vec{e}$

and $\frac{\mathbf{U}}{U}=\mathbf{u}$ or $\vec{u}$. Then **a**, **e** and **u** are the unit vectors of the vectors A, E and U aloag their directions. So a unit vector of any vector is defined to be a *direction finder unity along the direction of that vector.* In fig. 1, $\overrightarrow{AB}$ is a velocity vector having a magnitude of 50 km./hour due east and it is represented by a line of length 5 cm.

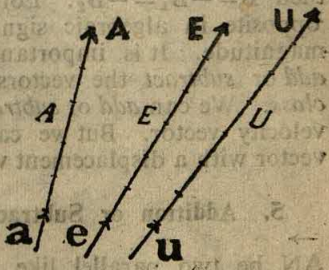


Fig. 2

Hence the unit vector along this direction is $\frac{\text{50 km./hour due east}}{\text{50 km./hour}}$

=unit due east.

4. Co-planar Vectors.—Vectors drawn upon a plane are called co-planar vectors. These vectors may belong to same or different classes, for example, some of them may be displacements, some may be velocities etc. In Fig. 3 we have a number of coplanar *parallel* vectors represented *by* **B**, **B′**, $\mathbf{B}_1$ $\mathbf{B}_2$, **D** etc. Vectors may be divided into two groups: Some vectors are *fixed* or *localised.* A vector whose point of application is fixed *with respect to a point,* is called a localised vector. Suppose that the point A of the vector $\overrightarrow{AB}$ is fixed to a body (fig. 1). It is then a localised vector. Examples of localised force vectors are obtained in moments and couples acting on a body.

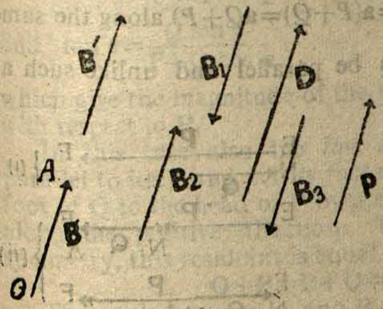


Fig. 3

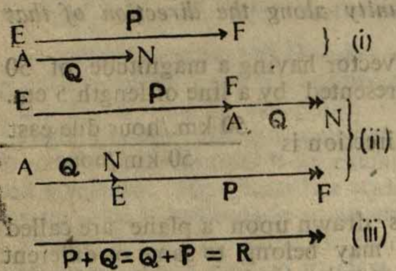
It is found that as much as a motion of translation of a rigid body is concerned, the effect does not alter if velocity vector is moved *parallel to itself.* Again if the body is imagined to be absolutely rigid, it matters little if the point of application of the vector is *shifted to any other point along its line of action.*

Parallel arrows of equal length drawn to the same scale and having the same direction are *equal like vectors.* In Fig. 3 $\mathbf{B}=\mathbf{B}'=\mathbf{B}_2$ are all equal like vectors having equal unit vector. Reversing the arrow head would change the algebraic sign of the vector; then $\mathbf{B}=-\mathbf{B}_1=-\mathbf{B}_2$. For **B** and $\mathbf{B}_1$ the unit vectors are equal but opposite in algebraic sign. **D** represents a like vector of large magnitude. It is important to remember that in order to be able to *add* or *subtract* the vectors, *such vectors should belong to the same class.* We can *add* or *subtract* one velocity vector to or from another velocity vector. But we cannot get the summation of one velocity vector with a displacement vector.

5. Addition or Subtraction of Parallel Vectors.—Let $\overrightarrow{EF}$ and $\overrightarrow{AN}$ be two parallel like vectors of the same class represented by **P** and **Q** (fig. 4i). To find their resultant, bring **Q** parallel to itself and put it at the *head* of **P** (Fig. 4 ii) or alternatively bring P parallel to itself and put it at the *head* of **Q**. The resultant vector is either $\mathbf{P}+\mathbf{Q}$ or $\mathbf{Q}+\mathbf{P}$ and may be put equal to **R** (Fig. 4 iii).

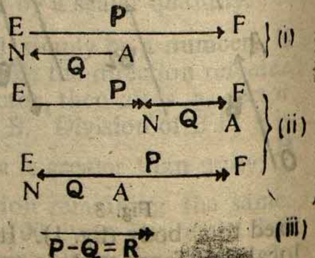
$$\therefore\ \mathbf{P}+\mathbf{Q}=\mathbf{Q}+\mathbf{P}=\mathbf{R} \quad \ldots (4,1)$$

This is like ordinary law of addition of Algebra, Since **P** and **Q** are parallel, their unit vector, say **a** is same. Let the moduli of these two vectors be P and Q. Then $P+Q=\mathbf{a}\ P+\mathbf{a}Q=\mathbf{a}(P+Q)=\mathbf{a}Q+P)$ along the same direction.



Fig, 4

If the two vectors **P** and **Q** be parallel and unlike such as shown in Fig 5i, then to find their resultant, move **Q** parallel to itself and *make its foot touch the head of* **P**. So the vector difference **P—Q** $=\overrightarrow{EN}$ (Fig. 5ii). Alternatively move **P** parallel to itself and make its foot touch the head of **Q**, so that $\mathbf{P}-\mathbf{Q}=\overrightarrow{AF}$ Since both $\overrightarrow{EN}$ and $\overrightarrow{AF}$ are equal in length and parallel we get $\mathbf{P}-\mathbf{Q}=\mathbf{R}$...(5,iii).



Eig. 5

Since **Q** of Fig. 4 and **Q** of Fig. 5 are *oppositely directly* and are equal in magnitude, one is $+\vec{\mathbf{Q}}$ and other is $-\vec{\mathbf{Q}}$ since their unit vectors are equal but opposite in sign. In Fig. 5, we have followed the same general rule of putting the foot of one vector at the head of the other. So we can state equ. (4,2) in the form,

$$\mathbf{P}+-\mathbf{Q}=\mathbf{P}-\mathbf{Q}=\mathbf{R} \quad \ldots \quad \ldots \quad \ldots \quad (4,3)$$

Here is a case Algebraic difference of the magnitudes of the two vectors and the difference acting along the direction of the larger vector.

6. Addition of Two Vectors at an Angle.—Let **P** and **Q** be two vectors at an angle with respect to each other which means that *unit vectors* of **P** and **Q** are different in this case. The angle between two vectors is measured with respect to the positive direction of each (Fig. 6). To add them vectorially shift **Q** parallel to itself and *set* its *foot on the head* of **P** (Fig. 7). Join a straight line from the foot of **P** to the head of **Q** and give an arrow head towards its positive direction. This is the resultant vector **R** (refer to Art 25, pp. 34).

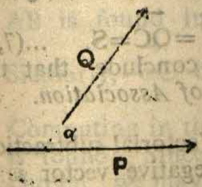


Fig. 6

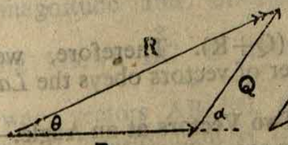


Fig. 7

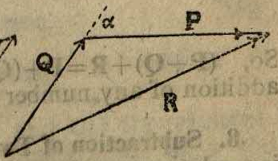


Fig. 8

$$\therefore \quad \mathbf{P}+\mathbf{Q}=\mathbf{R} \quad \ldots \quad \ldots \quad \ldots \quad (6,1)$$

In this case ordinary Algebraic addition will be incorrect, since for ordinary geometry, the sum of two sidcs of a triangle is greater than the third side. If, however, the moduli of the three vectors are represented by P, Q and R, it can be shown by the parallelogram law (Art 24) that

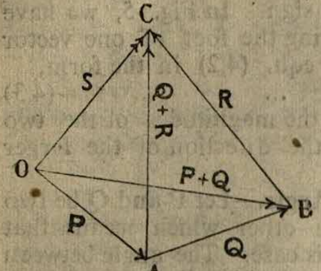
$$\left.\begin{array}{l} R^2=P^2+Q^2+2PQ \operatorname{eos} a \\ \text{and} \quad \tan\theta=\dfrac{Q\sin a}{P+Q\cos a} \end{array}\right\} \quad \ldots \quad (6,2)$$

which give the magnitude of the resultant vecter **R** and its direction θ with respect to **P**.

In this case also the *law of assoeiation* holds good. Shift **P** parallel to itself and *sct its foot on the head* of **Q** (Fig. 8). Join the foot of **Q** to the head of **P** by a straight line and give and arrow-head along the positive direction. This is the resultant vector **R**. By symmetry, this resultant is equal and parallel to **R** of Fig. 7.

$$\therefore \quad \mathbf{Q}+\mathbf{P}=\mathbf{P}+\mathbf{Q}=\mathbf{R} \quad \ldots (6,3)$$

The angle between **P** and **R** in this case is also α, and similar equations between their magnitude and direction as those of equ. (5,2) will hold good. Equation (5, 2) can also be proved from the unit vectors, say **a** and b of P and **Q** from the scalar multiplication of vectors (vide Art 10).



Fig, 9

7. Addition of Three or more Vectors.—The process of addition of vectors by triangular method may be employed for any number of vectors having different directions. In Fig. 9 we have three vectors **P**, **Q** and **R**. The fourth side of the quadrilateral will be the resultant vector **S** with the arrowhead in opposite direction as shown. To prove that join OB with an arrow head. Then $\mathbf{P}+\mathbf{Q}=\overrightarrow{OB}$. Now, $\overrightarrow{OB}+\mathbf{R}=$

$$\mathbf{P}+\mathbf{Q}+\mathbf{R}=\overrightarrow{OC}=\mathbf{S}. \quad ...(7,1)$$

Alternatively, we can join AC, then $\overrightarrow{AC}=\mathbf{Q}+\mathbf{R}$ and $\mathbf{P}+\overrightarrow{AC}=\mathbf{P}+(\mathbf{Q}+\mathbf{R})$

$$=\overrightarrow{OC}=\mathbf{S}. \quad ...(7,2)$$

So, $(\mathbf{P}+\mathbf{Q})+\mathbf{R}=\mathbf{P}+(\mathbf{Q}+\mathbf{R})$. Therefore, we conclude that the addition of any number of vectors obeys the *Law of Association.*

8. Subtraction of Two Vectors at an Angle.—Vectorial subtraction is a relative term in the sense that addition of a negative vector is a subtraction. In Fig. 10, when the vector $\overrightarrow{AB}=\mathbf{P}$ is added to the vector $\overrightarrow{BC}=\mathbf{Q}$ to get the resultant vector $\overrightarrow{AC}=\mathbf{P}+\mathbf{Q}$; then a vector $\overrightarrow{BD}$ just equal and opposite to $\overrightarrow{BC}$ will be a vector $-\mathbf{Q}$. Adding P to $-\mathbf{Q}$, we get as usual, the resultant vector $\overrightarrow{AD}=\mathbf{P}-\mathbf{Q}$. The difference of P and Q may be *less or greater* in magnitude than the summation of the same two vectors, according to the relative position of the two vectors. In Fig. 11 the summation of P and Q is shown to be less than their difference. This paradox is explained by the fact that when the angle between the vectors (angle α between their positive directions) is obtuse, the effect of one vector is partially neutralised by the other in course of addition as in Fig. 11. So in subtraction the resultant is larger in magnitude.

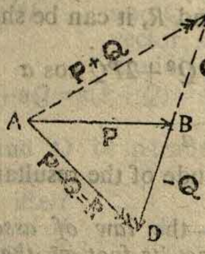


Fig 10

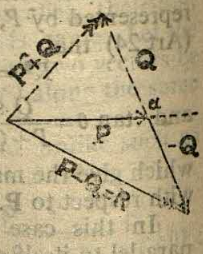


Fig. 11

9. Vector Application in Locations.—There are various methods of locating the positions of an object by vectors lying on the same plane or on different planes.

(i) Point Source : Radius Vector Method.—All that is needed in this method is a point of observation for example an observation station O, (Fig. 12). Distances from O to the ends of each vector are measured by geometric method or by wireless signals in case of rocket flights and the angle between successive vectors is measured. These distances are called radius vectors. Let $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}$ etc. be successive displacements of a moving body. Let $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ be the radius vectors from O. Then $\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB}$. Vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ being known, $\overrightarrow{AB}$ is found in magnitude and direction. Similarly, $\overrightarrow{OC}$ being known $\overrightarrow{BC}$ is found.

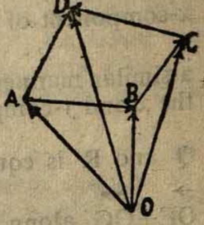


Fig. 12

Computing in this way, vectors $\overrightarrow{AB}, \overrightarrow{BC}$, and $\overrightarrow{CD}$ the resultant $\overrightarrow{AD}$ is found. Since such a measusement is independent of a station chosen, we may say that the vectorial measurement is *independent of any system of co-ordinates*. The method of measursement depends upon the solution of triangles. Illustraiions of applications of such method are found in Echo-depth sounding, Location of moving air craft by Radar, location of tumors on a human body by Sonography etc.

(ii) Rectangular Co-ordinate Method.—In the previous method, we have to find the resultant vector by solution of triangles either directly or indirectly. When the number of coplanar vectors is more than four or five, it is tiresome to find the resultant in magnitude and direction by vector triangles. For vectors on a plane surface, it is convenient to get the vector sum by resolving all the coplanar vectors along two rectangular axes on the plane and then combine these tnto a single vector.

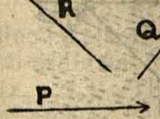


Fig. 13

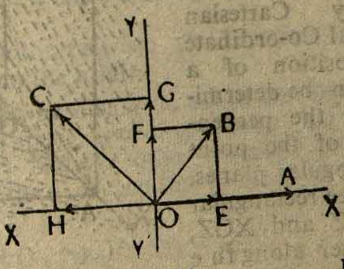


Fig. 14

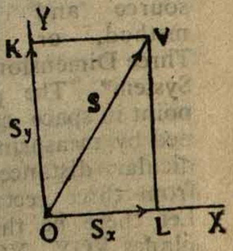


Fig. 15

Let **P**, **Q** and **R** be three co-planar vectors of the same class (Fig. 13). Draw a pair of rectangular axes along any arbitrary direction on the plane designated by X′OX and Y,OY, as is usually done on a graph paper (Fig. 14). For convenience let P coincide with OX and be represented by $\overrightarrow{OA}$.

Slide **Q** parallel to itself until it coincides with $\overrightarrow{OB}$. From B drop a perpendicular BF on OY and BE upon OX. Then $\overrightarrow{OE}$ is the x-component of $\overrightarrow{Q}$ and $\overrightarrow{OF}$ is the y-component of the vector $\overrightarrow{\mathbf{Q}}$. In a similar manner bring R to the position $\overrightarrow{OC}$ and let $\overrightarrow{CH}$ and $\overrightarrow{CG}$ be the x and y-components of this vector. Then the vector sum of P, Q and R is equivalent to $\overrightarrow{OA}+\overrightarrow{OE}-\overrightarrow{OH}$ along x-direction and $\overrightarrow{OF}+\overrightarrow{OG}$ along y-direction. Let their sums be $\overrightarrow{OL}$ and $\overrightarrow{OK}$ to be represented by S_x and S_y. (Fig. 15)

Then $\overrightarrow{OV}=\overrightarrow{OL}+\overrightarrow{OK}=\overrightarrow{OL}+\overrightarrow{LV}$.

If the magnitude of the resultant be S.

Then $S=\sqrt{S_x^2+S_y^2}$

and the inclination θ of the resultant is given by $\left.\tan\theta=\frac{S_y}{S_x}\right\}$ (9,2)

It is customary to write the sum of the components along OX as Σx and that along OY as Σy. So the resultant in magnitude is given by $\sqrt{\Sigma x^2+\Sigma y^2}$ and the direction $\theta=\tan^{-1}\frac{\Sigma y}{\Sigma x}$.

10. Three Dimensional Co-ordinates—When vectors do not lie on a plane their vector sum can be found either by point source and radius vector method, or by Cartesian Three Dimensional Co-ordinate System*. The position of a point in space can be determined by measuring the perpendicular distance of the point from three rectangular planes. Let there be three rectangular planes XOY, YOZ and XOZ, meeting each other along the

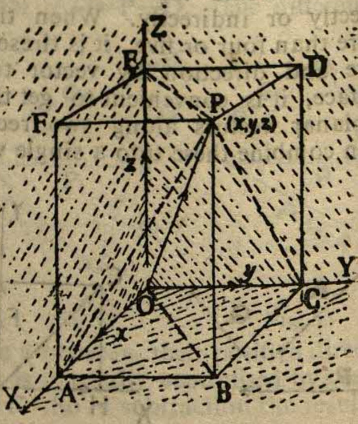


Fig. 16

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[*] There is also another method of projecting the vectors on a plane and finding their resultant on that plane. But this method is not dealt with in these preliminary discourses on the vector.

lines OX, OY and OZ (Fig.16). These lines of section arecalled *three dimensonal co-ordinate axes*. If the perpendicular distances of the point P from the planes YOZ, XOZ and XOY be PD, PF and PB respectively, then these three distances will *uniquely* determine the position of P with reference to the planes. The distance DP of the point P from YOZ plane is denoted by x, distance FP from XOZ plane is denoted by y and distance BP from XOY plane by z. These are the co-ordinate values of P and is denoted by P(xyz). By geometry it can be proved that $DP=OA=x$, $FP=OC=y$ and $BP=OE=z$ where three perpendiculars PA, PC, and PE are drawn on three co-ordinate axes. The point O is the *origin* of the axes and OP is the distance of P from origin. Thus the position of P is uniquely defined by writing P(x, y, z).

The distance of P from the origin O is given by,

$OP^2=PB^2+OB^3$ (since PBO is a triangle right angled at B)

$=PB^2+AB^2+OA^2$ (Since BAO is a right angled triangle at A)

$$=z^2+y^2+x^2 \qquad (10,1)$$

If OP is denoted by R, then $R=\sqrt{x^2+y^2+z^2}$

To get the direction of OP with reference to *axes of co-ordinates*, we have to know the *angles made by the line OP with the axes*. If $\angle POA=\alpha$, $\angle POC=\beta$ and $\angle POE=\gamma$,

$$\text{Then } \cos\alpha=\frac{x}{R},\ \cos\beta=\frac{y}{R} \text{ and } \cos\gamma=\frac{z}{R} \qquad (10,2)$$

These are respectively called the *direction cosines* of R with respect to the axes and are designated by l, m and n.

Now, $R^2=x^2+y^2+z^2$ ...(10,3)

or, $R=\frac{x}{R}x+\frac{y}{R}y+\frac{z}{R}z=lx+my+nz$

Thus $OP=R$ is known in terms of co-ordihates and directions consines.

Further from equ. (10,3).

$$1=\frac{x^2}{R^2}+\frac{y^2}{R^2}+\frac{z^2}{R^2}=\cos^2\alpha+\cos^2\beta+\cos^2\gamma=l^2+m^2+n^2 \quad ...(10,4)$$

which gives a relation between the direction consines of a radius vector OP drawn from the origin.

Length of a line whose base is not at the Origin—Let a straight line be placed on the space co-ordinates such that one end point has co-ordinate $(x_1y_1z_1)$ and the other terminal point has co-ordinate $(x_2y_2z_2)$. The length r of the line in terms of the co-ordinates is given by,

$$r=\sqrt{[(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2]} \quad ...\ (10,6)$$

The direction cosines of this line are given by,

$$\frac{x_1-x_2}{r}=\cos\alpha\ ;\ \frac{y_1-y_2}{r}=\cos\beta \text{ and } \frac{z_1-z_2}{r}=\cos\gamma \quad ...\ (10,7)$$

11. Orthogonal System of Vectors : Unit Co-ordinate Vectors :— We have already seen that any number of vectors can be added and their resultant can be found by the polygon method. Even if these vectors do not lie on the same plane, we can find their resultant by placing the successive vectors *head to tail* maintaining proper directions. This will give an unclosed figure in three dimensions. Finally close the figure by *joining a line from the foot of first vector to the tip of the last vector* and give an arrowhead on this line directed away from origin. This will be the resultant vector.

In a reverse way any vector can be resolved into any numbea of components in three dimensional space, so that single vector may represent the resultant of all those component vectors. It is very conventent to compose or resolve vectors along Cartesian Rectangular axes XYZ, as the mathematical transformations become easy to manupulate.

Let $\overrightarrow{OP}$ be a vector of any class to be specified by **H**. At the origin O of the vector let us draw three rectangular axes OX, OY and OZ (fig. 17). From P draw perpendiculars on the three axes at points A, C and E respectively. From the precediug theorem OA $=H_x$, OC$=H_y$ and OE$=$BP$=H_z$ are the projections of the line OP along three rectangular axes. Taking the co-ordinates of P to be (x,y,z) and taking $H_x=x$, $H_y=y$ and $H_z=z$, the relation between such quantities may be obtained from equa. (10,2).

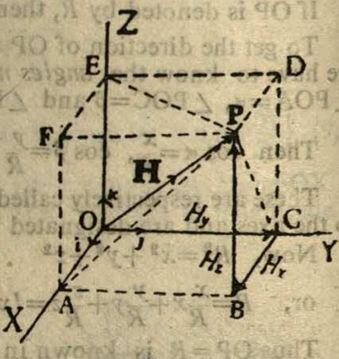

Fig. 17

Now let us observe this diagram from the view-point of vectors.

Let $\overrightarrow{OC}$ and $\overrightarrow{OE}$ be changed into vector by giving arrow-heads along their incremental directions. Then from the polygon theorem,

$$\mathbf{H}=\overrightarrow{OP}=\overrightarrow{OC}+\overrightarrow{CB}+\overrightarrow{BP}=\overrightarrow{OC}+\overrightarrow{OA}+\overrightarrow{OE} \qquad ...(11,1)$$

These component vectors are *mutually perpendicular* and hence they are called **orthogonal vectors.** But the magnitudes of $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OE}$ are H_x, H_y and H_z respectively. In the original co-ordinate system, *the units of measurements along the three axes must be equal* although they are directed in mutually perpendicular position.

Let $\overrightarrow{OA}=\mathbf{i}\,H_x$, $\overrightarrow{OC}=\mathbf{j}\,H_y$ and $\overrightarrow{OE}=\mathbf{k}\,H_z$, where **i**, **j** and **k** are *vectors of unit length* along three mutually perpendicular directions. These are called *orthogonal unit vectors.* These vectors have the property that multiplied by any scalar quantity or a number (such as H_x, H_y and H_z), they will convert the product into vectors *along their respective directions.*

Thus $\mathbf{H}=\overrightarrow{\mathrm{OP}}=\mathbf{i}\,H_x+\mathbf{j}\,H_y+\mathbf{k}\,H_z$ ... (11,2)

Also we know from preceding Art that the magnitude of the vector **H** will bear the following relations to the magnitudes of the component vectors

$$H^2=H_x{}^2+H_y{}^2+H_z{}^2$$

and $\left.\cos\alpha=\dfrac{H_x}{H}\,;\ \cos\beta=\dfrac{H_y}{H}\ \text{and}\ \cos\gamma=\dfrac{H_z}{H}\right\}$ ... (11,3)

Example.—A force of 100 gm. wt. is drawn as a vector of length 20 cm, inclined at at an angle of 30° to the positive direction of OX and 90° to the positive direction of OY of rectangular system of co-ordinates. Find the unit vectors along the radius vector and along the axes.

Ans.—If $\overrightarrow{OP}$ be the direction of the vector, then $\overrightarrow{OP}=100$ gm. wt. and $OP=20$ cm.

Then $\dfrac{\overrightarrow{OP}}{OP}=\dfrac{100\text{ gm. wt.}}{20\text{ cm.}}=5$ gm, wt. per cm, is the magnitude of unit vector along $\overrightarrow{OP}$. Its direction is given by direction-cosines $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ where $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$. Now $\cos 30°=\dfrac{\sqrt{3}}{2}$; so $\cos^2 30°=\dfrac{3}{4}=0{\cdot}75$ $\cos 90°=0$; so $\cos^2 90°=0$, $\therefore\ \cos^2\gamma=1-0{\cdot}75=0{\cdot}25$, whence $\cos\gamma=0{\cdot}5$

Hence $\gamma=60°$. So $\overrightarrow{OP}$ is inclined to OX, OY and OZ by 30°, 90° and 60° respectively. These give the direction of the vector.

Again if (x, y, z) represent the terminal point of OP of length 20 cm. then its projection on X-axis is given by 20 cos 30° cm. and 100 gm. wt. has the component 100 gm. wt. × cos 30°.

$$\therefore\ \mathbf{i}=\frac{100\text{ gm. wt.}\times\cos 30°}{20\text{ cm.}\times\cos 30°}=5\text{ gm. wt. per cm. along }OX.$$

From a similar argument $\mathbf{j}=5$ gm. wt. per cm. along OY and $\mathbf{k}=5$ gm. wt./cm. along OZ. Thus $\mathbf{i}=\mathbf{j}=\mathbf{k}$ in magnitude but they differ orthogonally in directions.

12. Scalar or Dot Product of Vectors—We have seen in Chap. V, Art 77 that the magnitude of the work is given by the product of a *force* and the *displacement* of the point of application of the force. In this case force and displacement are both vectors but their product gives the work done which is a scalar. Such a *product of two vectors, giving a scalar result is called the dot product of vectors.* In fig. 18, let **F** be the force vector and **S** be the displacement vector, then their dot product is expressed as **S. F** whatever be the angle between them. If we consider the vector **S** first and start to measure towards **F** in the anti-clockwise direction, we call this angle *positive.* If farther the sizes or moduli of the vectors be OC and OB to be represented by S and F.

Then $\mathbf{S}.\,\mathbf{F}=S\,F\cos\theta$ (in scalar values) ... (12,1)

If again we get dot product of the vectors in the form **F. S**, then we come from **F** to **S** in the anti-elockwise direction, which makes the angle θ negative.

Then $\mathbf{F}.\ \mathbf{S}=F_S\ \cos\ (-\theta)=$ $FS\cos\theta$ (in scalar values)

Since the two values are equal, we conclude that **S. F=F. S. Therefore** the scalar or dot product of two vectors is *commutative obeying* ordinary laws of Algebra. But the three following special cases arise :

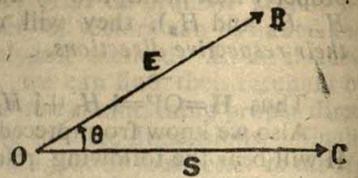

Fig. 18

(i) When either of the vectors is zero, the scalar product gives zero value. As an illustration, when either the force or displacement is zero the work done is nil. This follows the ordinary rule that a quantity multiplied by zero gives zero value.

(ii) When the vectors are at *right angles* to each other, then $\mathbf{F}.\ \mathbf{S}.=FS\cos 90^\circ=0$. For this case Vector Algebra differs from Scalar Algebra.

(iii) When the vectors are parallel $\mathbf{F}.\ \mathbf{S}.=FS$

Therefore the magnitude of the dot product depends not only upon the *individual magnitudes* of the vectors but it also depends upon the *cosine of the angle* between two vectors.

13. Self-Product of Vectors—When the two vectors are parallel then $\mathbf{F}.\ \mathbf{S}.=FS$. If the vectors *belong to the same class* and are equal in magnitude, that is when **S** is replaced by **F**, then $\mathbf{F}.\ \mathbf{F}=\mathbf{F}^2=F^2$. This is a case of a scalar product of a vector with the same vector. This is called the *self-product of a vector.*

Applying the law of dot product to unit vectors, **i, j** and **k** along rectangular co-ordinate axes, we have,

$\mathbf{i.i}=\mathbf{i}^2=\mathbf{j.j}=\mathbf{j}^2=\mathbf{k.k}=\mathbf{k}^2=1$ ... (13,1)

Since each pair is parallel and equal.

and $\mathbf{i.j}=\mathbf{j.k}=\mathbf{k.i}=0$ Since they are perpendiculars ... (13,2)

We can now investigate the nature of Vector products of equ. (8,2) in which

$\mathbf{H}=\mathbf{i}H_x+\mathbf{j}H_y+\mathbf{k}H_z$ Squaring each side we get,

$$\mathbf{H.H}=\mathbf{H}^2=H^2=[\mathbf{i}H_x+\mathbf{j}H_y+\mathbf{k}H_z]^2$$

$$=\mathbf{i\,i}\,H_x{}^2+\mathbf{j.j}\,H_y{}^2+\mathbf{k.k}\,H_z{}^2+\mathbf{i.j}2H_xH_y+\mathbf{j.k}2H_yH_z+\mathbf{i.k}2H_xH_z$$

$$=H_x{}^2+H_y{}^2+H_z{}^2 \qquad (13,3)$$

by virtue of equs. (13,1) and (13,2)

Vectorial Proof of the Parallelogram Law :—In Art 6, we found the value and direction of the resultant vector **R** of **P** ond **Q** by algebraic process. It can also be proved vectorially in the following manner.

Let **p, q** and **r** respectively be unit vectors along P, Q, R. Then $\mathbf{P}=\mathbf{p}P$, $\mathbf{Q}=\mathbf{q}Q$ and $\mathbf{R}=\mathbf{r}R$.

Since $\mathbf{R}=\mathbf{P}+\mathbf{Q}$ or, $\mathbf{r}R=\mathbf{p}P+\mathbf{q}Q$

$\therefore\ \mathbf{r}^2\mathbf{R}^2=(\mathbf{p}P+\mathbf{q}Q)={}^2\mathbf{p}^2P^2+\mathbf{q}^2Q^2+2\mathbf{pq}PQ$

or, $R^2=P^2+Q^2+2PQ\ \mathbf{p.q}$.

Since self product of any unit vector is 1. and $\mathbf{pQ}=\cos\sphericalangle$

$\therefore \quad R^2=P^2+Q^2+2PQ \cos \alpha$

and tan θ is to be found from geometry.

14.. Vector Product of Vectors—In Art 44, we have dealt with the effect of a moment of a force on a body. The effect of such forces can be handled with vectors. We already know that the moment of a force is obtained by the product of the force and an arm which is the perpendicular from the point of rotation on the line of force. In treatment with vectors the force is already defined to be a vector but the arm of the fôrce is treated as a radius vector which may or may not be perpendicular to the force vector. So this is also a case of product of two vectors known as the *cross product* of vectors.

Let us first define the vector product of two vectors in the following way. The vector product of two vectors **A** and **B** is defined as a third vector A×B having a magnitude equal to the product of the magnitudes of the vectors and the sine of the angle between their directions. In fig. 19 θ is the angle between the positive directions of **A** and **B**. If the vector **A** is turned towards **B** through a given *angle* the necessary rotation must be *anticlockwise when* sighting along OT, which is the positive normal to the plane containing AOB. If the resultant torque be **T**, this is also a vector. If the unit normal in this direction be **n**, the value of torque vector $\mathbf{T}=\mathbf{n}T$, where T is the magnitude of the torque. A right handed screw moved from **A** to **B** in the anticlockwise direction will move along the positive normal. So we ean write ... (14, 1)

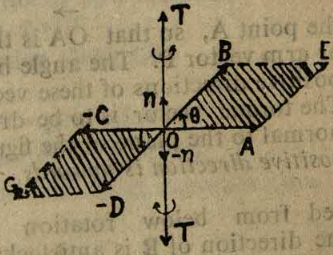

Fig. 19

$$\mathbf{T}=\mathbf{n}T=\mathbf{A}\times\mathbf{B}=AB \sin \theta$$

Now, AB sin θ is equal to the area of the quadrilateral OAEB. Hence the vector product of two vectors sweeps out an area known as a *vector area* whose magnitude is the area of the figure and whose direction is *perpendicular to the area.* In dealing with moment, we call $\overrightarrow{OA}$ as the arm vector and $\overrightarrow{OB}$ as the force vector. If $\overrightarrow{OB}$ is shifted to $\overrightarrow{AE}$ then we have the moment of the force $\overrightarrow{AE}$ about the axis of rotation TOT. The quantity OA sin θ is the value of the perpendicular dropped from O on the force AE.

If however we take $\overrightarrow{OB}$ as the arm vector and $\overrightarrow{OA}$ as the force vector, then the effect of the moment is a clockwise motion when sighted along the positive normal. Since $\overrightarrow{OA}$ can be shifted to $\overrightarrow{BE}$. In this case, we obtain the motion of a right handed screw when rotated from **B** to **A** in the clockwise direction. The motion of the screwhead is downwards which is taken as a negative direction of vector torque **T**.

Then $\mathbf{B}\times\mathbf{A}=-\mathbf{n}T=-\mathbf{T}=AB\sin(-\theta)=-AB\sin\theta$ ... (14,2) Suppose that opposite to **A** there is a vector $-\mathbf{C}$ and opposite to **B** there is a vector $-\mathbf{D}$. What is the value of the cross product or vector product of these two vectors ? If $-\mathbf{C}$ rotates towards $-\mathbf{D}$, it is an anticlockwise (positive) rotation and so the torque vector is positive. If again $-\mathbf{C}\times-\mathbf{D}=\mathbf{A}\times\mathbf{B}$ in magnitude, the combination represents a couple whose vectorial moment

$$=-\mathbf{C}\times-\mathbf{D}+\mathbf{A}\times\mathbf{B}=\mathbf{C}\times\mathbf{D}+\mathbf{A}\times\mathbf{B}=2AB\sin\theta \qquad ...(14,3)$$

Equation (14,3) shows that the torque of individual arm-force product has the same sign and is equal in magnitude, it is possible to reverse the algebraic signs of **C** and **D** and their product may be added to the vector product of **A** and **B**.

Fig 20 gives a physical picture of vector product in Mechanies. A body is capable of rotation along an axis passing through the point O. A force vector **R** is acting at the point A, so that $\overrightarrow{OA}$ is the position or arm vector **P**. The angle between the positive directions of these vectors is α. The torque vector is to be drawn at A normal to the plane of the figure and its *positive direction is upwards*. So observed from below rotation of $\overrightarrow{\mathbf{P}}$ in the direction of **R** is anticlockwise and a right hand screw method along that direction will go downwards along the negative direction of torque. Therefore, $\mathbf{P}\times\mathbf{R}=-\mathbf{n}T$. From a similar argument we can show that $\mathbf{Q}\times\mathbf{S}$ has a *positive value* and can be written at $\mathbf{Q}\times\mathbf{S}=\mathbf{n}T_1$.

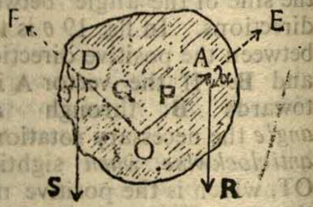

Fig. 20

The vector products occur frequently in Electro-Magnetism and will be dealt with in their proper places.

15. Application of Cross-product of Vectors—From the preceding Article, we learn that the magnitude of the cross or vector product of two vectors depends upon the magnitudes of the indiviual vectors and smaller angle between the positive directions of the vectors. The algebraic sign depends upon the direction of rotation along which one vector may be turned so as to *approach* the otber vector. If one vector is made to approach the other along the direction of a right handed screw, the motion of the screw will indicate the positive direction of the resultant vector.

Following this rule, if we have two vecctos **A** and **B** on the XOY plane in a right handed system of rectangular Cartesian Co-ordinates and if the smaller angle between their *positive directions* be θ.

Then $\mathbf{A}\times\mathbf{B}=AB\sin\theta$, provided we are able to make **A** approach **B** towards a rotation from OX to OY. The resultant of this cross product is along OZ. But in this case if **B** made to approach **A**, the rotation is from OY to OX in a reverse direction (counter clockwise if seen along OZ) and the resultant will be along $-$OZ. This is the surest way of rockoning the algebraic sign of the resultant.

Thus $\mathbf{A}\times\mathbf{B}=AB\sin\theta=\mathbf{n}T$, say

and $\mathbf{B}\times\mathbf{A}=-AB\sin\theta=-\mathbf{n}T$, say

$\therefore\ \mathbf{A}\times\mathbf{B}=-\mathbf{B}\times\mathbf{A}$

If $\theta=0^\circ$, $\sin\theta=0$, hence the cross-product vanishes. In case of a mechanical moment we explain this condition as the arm vector and the force vector being along the same line. This amounts to the case where the force acting on a rigid body passes through its axis of rotation and hence there is no rotatory effect by the force upon the body resulting in the torque being zero.

If $\theta=90^\circ$, $\sin\theta=1$, hence the cross product is maximum. This is the case when the arm vector is *perpendicular* to the force vector. The effect of the torque is maximum.

Cross-product of Rectangular Unit Vectors—The unit vector **i, j** and **k** along OX, OY and OZ respectively are of unit magnitudes and are at right angles to each other for which $\theta=90^\circ$

Henca $\mathbf{i}\times\mathbf{j}=ij\sin 90^\circ=ij$ having direction along OZ

$$\left.\begin{array}{l}\therefore\ \mathbf{i}\times\mathbf{j}=-\mathbf{j}\times\mathbf{i}=\mathbf{k}\\ \text{For a similar reasoning } \mathbf{j}\times\mathbf{k}=jk\sin 90^\circ=jk=-\mathbf{k}\times\mathbf{j}=\mathbf{i}\end{array}\right\}$$

The strict cyclic order in such results should be observed.

Again since cross-product of parallel or conicident vectors is zero, since $\theta=0^\circ$,

$\mathbf{i}\times\mathbf{i}=0=\mathbf{j}\times\mathbf{j}=0=\mathbf{k}\times\mathbf{k}$.

Examples :—

1. $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CD}$ are the successive displacements of a rocket in space as detected from an ovservation station O on earih. The distances AO, BO, CO and DO as measured by wireless signals are respectively 50, 60, 120 and 80 kms. Further angles AOB, BOC, COD and DOA are found to be 30°, 45°, 60° and 30° respectively. Find the individual and resultant displacements.

(Refer to Fig. 12 Art 8 of this Chapter).

Ans. From $\triangle AOB$, $AB^2=OA^2+OB^2-2OA,\ OB\cos\angle AOB$

$$=50^2+60^2-2\times50\times60\times\cos 30^\circ=904.\quad\text{whence } \overrightarrow{AB}=30{\cdot}6 \text{ kms.}$$

Again from $\triangle BOC$, $BC^2=OB^2+OC^2-2.\ OB.OC\cos\angle BOC$

$$=60^2+120^2-2\times60\times120\times\cos 45^\circ=7818 \text{ whence } \overrightarrow{BC}=88{\cdot}4 \text{ kms.}$$

Also from $\triangle COD$, $CD^2=OC^2+OD^2-2.OC.OD\cos\angle COD$

$$=50^2+80^2-2\times50\times80\times\cos 30^\circ=1972 \text{ whence } \overrightarrow{AD}=44{\cdot}4 \text{ kms.}$$

$=$ Resultant.

2. Three coplanar vectors $\overrightarrow{P}$, $\overrightarrow{Q}$, and $\overrightarrow{R}$, on a vertical plane having azimuthal angles of 45°, 30° and 120° have a resultant $\overrightarrow{S}$ whose projection on a horizontal plane is $\overrightarrow{30}$ cms. Find the magnitude of the resultant.

Ans. Ref to Fig. 14 Art. 7.

Let $\alpha=45°$, $\beta=30°$ and $\gamma=120°$, Then if δ be the azimuthal angle of the resultant, $\delta=\gamma+\beta-\frac{\pi}{2}=150°-90°=60°$

Then the resultant$=\frac{30}{\cos 60°}=\overleftarrow{60}$ cm.

3. Show that the vectors $\vec{P}=2i-j+k$, $\vec{Q}=i-3j-5k$ and $\vec{R}=3i-4j-4k$ form the sides of a right angled triangle. [C. U.—1961]

Ans. We know that the square of the magnitude of a vecior is equal to the sum of the squires of the co-efficients of i, j and k which compose that vector.

$\therefore$ Length of the vector $\vec{P}=\sqrt{4+1+1}=\sqrt{6}$

Length of the vector $\vec{Q}=\sqrt{1+9+25}=\sqrt{35}$

Length of the vector $R=\sqrt{9+16+16}=\sqrt{41}$

Since $P^2+Q^2=41=R^2$; P, Q, R form a right angled triangle of sides P and Q, and its hypotenuse is R.

EXERCISES XIII

1. Distinguish between a scalar and a vector quantity. How is a vector mathematically and graphically represented ? Is it necessary that vectors drawn on the same diagram should have the same scale,—Explain with diagram.

2. Explain the principle of multiplication and division of a vector by simple numbers. How does the sense of a unit vector arise from this principle ?

3. Show by a diagram that the vector addition obeys the associative law.

4. Explain different methods of identifying vectors and getting their resultant.

5. Explain with a neat diagram of expressing the position of a point in three dimensional orthogonal co-ordinates. How can the length of a straight line starting from the origin be expressed in terms of the co-ordinates of its other end point ?

6. What are the direction cosines l, m and n of a straight line of length H in a rectangular co-ordinate system ? If the line starts from the origin, how can you represent the line in termsof tis direction cosines ?

7. Define orthogonal system of unit vectors. Prove that they are equal in magnitude. Express a position vector in terms of orthogonal unit vectors.

8. What is a scaler product of vectors ? Give an example of a scalar product. Define self product of vectors. Show the relations if i, j and k in self product.

9. Define vector product of vectors. Find a relation of the resultant of vector products and individual vectors by drawing a neat diagram.

10. Show that the effect of the moment of a force acting on a rigid body can be represented by the cross-product of two vectors. Express the relations between unit vectors in cross-products.

HEAT

CHAPTER I

THERMOMETRY

1. **Heat.**— From our earliest days we are familiar with the sensations of heat and cold. If we stand in the sun or before fire, we feel hot, whereas if we hold a piece of ice in hand, we feel cold. The *physical cause*, due to which we have such feelings of hotness or coldness, is termed **heat.**

From pre-historic times it was known to man that rubbing of two splinter woods created fire ; and it was fire which made things burn and produce heat. For example, heat is generted when we rub our hands together or when a piece of iron is hammered on an anvil. Both Indian and Greek philosophers of early days considered 'fire' (Tejas in Vedic term) as one of the five elements ever present in nature. But experiments to show that heat can be obtained from other energies or that heat can be transformed into other forms of energy were done in eighteenth and nineteenth centuries. These experiments showed that *Heat is a form of Energy and it can be measured in terms of some units*

2. **Temperature**— When heat is applied to a body which is not in a state of melting or vaporising, the body grows hotter. Conversely, when heat is somehow taken away from the body, it becomes colder. Generally speaking, *temperature is the measure of hotness of a body*. If on touching two bodies A and B, A appears to be *hotter* than B, then A is said to be at a *higher* temperature than B. Thus in material bodies temperature is that manifestation of heat which we can compare by our sense of perception.

Accurately defined, *temperature is the thermal condition of a body which determines whether the body will communicate heat to, or receive heat from, another body, when both the bodies are put into thermal communication.* If a hot metal ball be dropped into a mass of cold water, the ball after some time appears colder than before, while the water appears warmer. Here heat has left the hot ball and has passed into the cold water. The passing of heat from one body to another, therefore, depends on their difference of temperatures.

Distinction between Heat and Temperature—Although the terms heat and temperature appear so much alike, they do not convey the same meaning : (1) Temperature indicates the thermal *condition* of a body, whereas the quantity of heat possessed by it is a form of *energy*. (2) Heat generally increases the temperature of a body. But if a body melts or boils, it does so at a definite temperature known as the melting point or boiling point. The application cf heat at these two states is utilised in melting or boiling out more of the substance without any change of temperature. (See Chapter VI) (3) Two bodies may be at the same temperature, although the quantities of heat possessed by them may be different. If, out of a large mass of boiling water, a small portion is taken off, the temperature of the two masses is the same, although the quantity of heat in the former is far greater than that in the latter. (4) The flow of heat from one body to another depends only on their temperatures and *not* on the quantity of heat possessed by either. The amount of heat in a red-hot iron ball may be far less than that in a bucket of warm water ; but if the ball be put into the water, the former will communicate some amount of heat to the latter. (5) Temperature is analogous to the surface level of a mass of liquid. If the level of water in a vessel be higher than that in another, water will flow from the former to the latter, when communication is made between the two vessels ; this takes place irrespective of the quantity of liquid in each. The flow ceases as soon as the level of water in both the vessels becomes the same. Similarly, heat will pass from a body at a higher temperature to one at a lower temperature, irrespective of the quantity of heat possessed by each and there will be no further transference when the temperatures of the two bodies become the same.

3. Effects of a Change of Temperature—When a body is heated or cooled so that its temperature changes, the following among other effects may by observed :—

(i) *Change of Volume*—It is found that, in general, bodies increase in volume when heated, and contract when cooled.

(ii) *Change of State*—On being heated, a solid at a certain temperature generally passes into the liquid state and similarly a liquid at another fixed temperature passes into the gaseous state. Thus when ice is heated, it gradually melts into the water and when the water is heated, it passes into steam. Conversely, on being cooled, steam changes into water and water being sufficiently cooled freezes into ice.

(iii) *Change of Physical Properties*—A change of temperature modifies almost all the physical properties of matter. Thus the elasticity of solid, the surface tension and the solvent power of a liquid, the power of metals to conduct heat and electricity, etc., are affected by changes of temperature. Zinc is hard and brittle at ordinary temperature but soft and flexible when heated ; iron is softened or tempered by alternate heating and cooling.

(iv) *Chemical and Electrical Effects*—Many substances (such as lime) are descomposed when sufficiently heated. Again, some chemical union is brought about by application of heat. The electrical resistance of a conductor changes with its change of temperature by heat. If by joining wires of two dissimilar metals a closed circuit is made and if any junction be heated, an electric current flows through the circuit.

4. **Measurement of Temperature**—All our ideas regarding difference of temperatures are primarily derived from the *sense of touch.* In such cases the temperature of the hand is, therefore, the criterion. But this method of estimating the relative hotness or coldness of a body cannot be taken to be accurate. The following are the reasons therefore : (1) Our sensations are not sufficiently delicate (2) Judgment as to the temperature of a body is sometimes incorrect. For instance, if the right hand is placed in hot water and the left hand in cold water and then both be immersed into a mass of lukewarm water, the water appears *cold* to the right hand but *not* to the left. Again when touched by hand, different bodies at the same temperature may give different sensations of hotness or coldness Thus if a piece of wood and a piece of a polished iron lying side by side in air be touched, the latter will appear colder, but as a matter of fact both are at the same temperature. The instrument, that is devised for comparing and measuring different temperatures, is known as **thermometer.**

We take recourse to one or the other physical property of a substance which should change uniformly with temperature and which at the same time must be capable of easy and accurate measurement. It is an observed fact that when a substance is heated, its volume increases continuously with the rise of temperature. The change of the volume of a suitable substance may, therefore, be taken to indicate the corresponding change in temperature. Mercury is most suitable as a thermometric substance,

5. **Early History of Thermometers.**—Of all the changes of physical properties of a substance with temperature, the change of volume with temperature attracted the attention of early scientists. Galileo Galilie (1564-1642) is said to have first constructed a form of thermometer round about the year 1604. The instrument was really a combination of a thermometer and a baroscope and so it was named a *thermoscope* (Fig. 1). The apparatus consists of a glass bulb *B* with a narrow uniform pipe vertically placed over a pot *A* having a spout. Some air is enclosed within the bulb extending to some level. Some coloured liquid fills a part of the pot and rises up to a certain height. An arbitrary scale is attached to the pipe. If now temperature rises, the air within the bulb expands forcing the level down. On the other hand when temperature falls, air contracts and the liquid level rises up. So observation of the liquid level is an estimate of hotness or coldness. No fixed points could be devised in those days. Since the spout is open

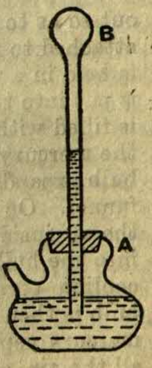

Fig. 1

to atmosphere, a change of atmospheric pressure will also cause the level to rise up or fall down. Hence is the name thermoscope. Robert Boyle (1627-1691) carried such a thermometer in England for the investigation of the famous Boyles law of pressure-volume relation at a constant temperature, which he successfully carried out in 1660. The use of alcohol as a thermometric substance in a sealed tube began in Italy by a group of scientists between the years 1640 to 1650. But it is important to note that use of mercury as a thermometric substance and fixed points of a thermometer all came in during eighteenth century.

6. Mercury Thermometer and its Construction—The most common form of thermometer is a mercury in-glass thermometer. It consists of a small cylindrical bulb (Fig. 2) provided with a stem of very narrow and uniform bore. The other end of the stem is closed and there is mercury inside. The volume of the enclosed mercury changes with the change of temperature. For a definite temperature, the volume occupied by mercury is constant, as shown by a definite position of the mercury column along the stem. The stem is graduated in accordance with some standard scale, so that each division represents an equal change in temperature.

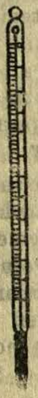

Fig. 2.

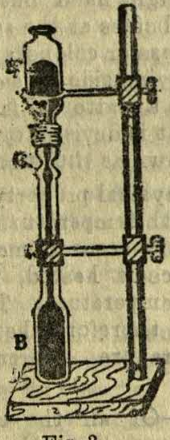

Fig. 3.

Construction of a Thermometer—A piece of thick-walled *capillary glass* tube of *uniform* bore is taken. It is thorougly cleaned and dried, A bulb B is blown at one end (Fig. 3.) At C, a little below the open end, the tube is heated and drawn out so as to form a narrow neck. A small cylindrical funnel E is then attached to the open end by a short piece of rubber tubing. The tube is held in a vertical position and a quantity of pure and dry mercury is put into the funnel. As the bore is very narrow and as the tube is filled with air, mercury does not run down the tube. To introduce the mercury into the bulb, the latter is gently heated ; the air in the bulb expands and a portion of it bubbles out through mercury in the funnel. On being now removed from the flame the bulb cools down ; the air inside contracts and sucks in some quantity of the mercury into the bulb. By repeating this process of alternate heating and cooling several times, the bulb is almost filled with mercury. The bulb is then strongly heated until the mercury boils and gives out mercury vapour. As the vapour continues to escape, it drives away all the air and moisture enclosed. On cooling, mercury completely fills the bulb and the stem. While the bulb is still warm, the excess of mercury in the funnel is taken out.

Next the bulb with the stem is subjected to a temperature a little above the boiling point of water. Some amount of mercury expands into the funnel and is removed. When the bulb slowly cools, mercury contracts and as it just passes the constriction C, the tube is sealed at the point by means of a strong pointed flame. On coolin, mercury contracts and fills up the bulb and a part of the stem.gThe closed tube is then allowed to cool down for a considerable period before the instrument is *graduated.* In *selecting* a scale for the measurement of temperature with a mercury thermometer, it is necessary to mark on the stem the positions of the mercury thread corresponding to *two definite* temperatures. These are called fixed points of a thermometer. The lower fixed point is taken to correspond to the temperature at which pure ice melts under normal atmospheric pressure ; this is known as the *ice-point.* The upper fixed point is taken to correspond to the temperature at which pure water boils under standard atmospheric pressure of 76 cm. of mercury. This is termed the *steam-point.*

Determination of Fixed Points—(a) *The Lower fixed point or the* **ice-point**—In order to determine the ice-point, the bulb of the thermometer, constructed as above, is placed within small lumps of pure melting ice kept in a funnel (Fig, 4) The level of the mercury column gradually sinks. Finally at a particular position it becomes stationary. After some time the fixed position is marked by a fine scratch made on the stem with a file. This gives the lower fixed point. It is also termed the *freezing point.* If the bulb of a centigrade thermometer is placed in melting ice in the above way and if the mercury level stands at the zero mark, its freezing point has been correctly marked.

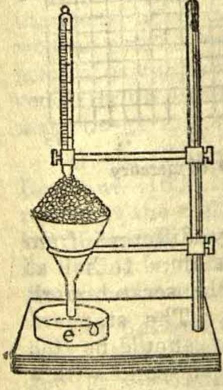

Fig. 4

(*b*) *The Upper fixed point* or the **steam-point**—To determine the upper fixed point, the thermometer is placed into steam of boiling water in a specially constructed vessel, called a *hypsometer.* It consists of a copper vessel A (Fig 5) with a vertical tube B which is surrounded by the jacket C. The jacket has two openings, one at the top and the other D at the side. The vessel A is partly filled with water and the upper opening is closed by a cork havinga hole, through which a thermometer T is inserted into B so as to remain well above the water level in A. As water begins to boil, steam rises through B, passes into the jacket C and finally escapes through the pipe D. The steam in the jacket C protects circulating steam in B from being cooled by external air. Attached to the side of the vertical tube opposite to D, a manometer M indicates the pressure of the steam. The bulb and the stem being thus surrounded by steam, the level of mercury gradually rises and after some time attains a stationary position. The position of the mercury level is just visible above the cork. When after a few minutes mercury has reached a steady temperature throughout, the position of the mercury level is marked by a scratch. This mark indicates the temperature, at which water boils under the atmospheric pressure which is noted from the barometer,

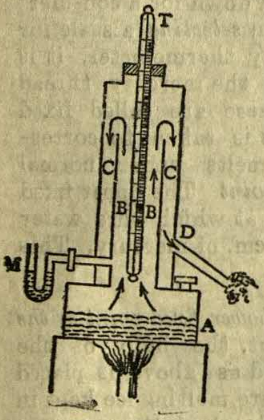

Fig. 5

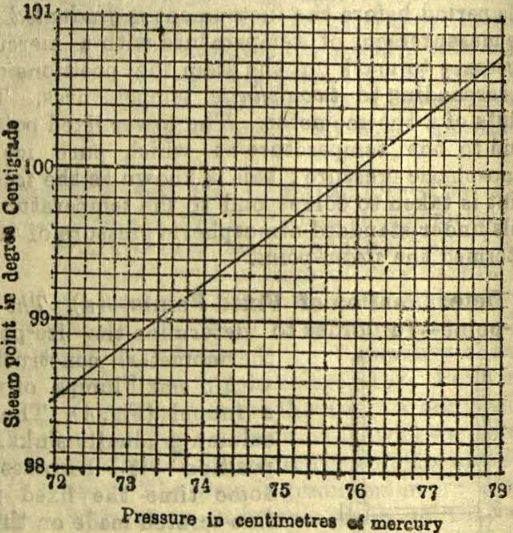

Fig. 6

As the observed atmospheric pressure is generally different from the standard pressure of 76 cm. and as pure water is found to boil at different temperatures under different pressures, the scratch mark so made does not tally with the standard steam-point. The standard steam-point according to a centigrade thermometer should be 100. The graph shows the variation of the observed steam-point with a change of pressure (Fig. 6). It is found that near the normal atmospheric pressure an alteration of pressure by about 28 mm. changes the boiling point of water by 1 °C. The correction is to be added or subtracted according as the observed pressure is below or above the normal atmospheric pressure,

Suppose that the scratch mark of the observed steam-point is made when the barometric pressure is found to be 75 cm. From the graph the corresponding steam temperature is found to be 99·65°C. The length on the stem from the lower to the upper scratch marks is accurately read with a vernier microscope. Let it be l cm. If L cm. be the true length between the lower scratch mark and the standard steam-point, then

$$\frac{L}{l}=\frac{100}{99{\cdot}65}, \text{ whence } L=\frac{100\,l}{99{\cdot}65}.$$

Thus measuring a distance L cm. from the lower scratch mark, we put there the standard mark 100 for the upper fixed point.

After the two fixed points have been marked on the stem, the space between them is divided into a number of equal parts, according to a standard scale of temperature. Each such division is called a degree. The graduations are continued some way both above and below the upper and the lower fixed points. To mark the graduations on the stem, the latter is coated with a thin layer of wax and equidistant lines are drawn with a fine pointer on the layer. By exposing the thermometer to the vapour of hydroflouric acid, the marks are etched permanently on the stem.

7. Scales of Temperature—There are three scales of temperature—**(1) Centigrade or Celcius, (2) Fahrenheit and (3) Reaumur.**

(1) The *Centigrade scale* was designed by Elvius of Sweden in 1710 and later on by Christin in 1743. Some people associate also the name of Anders Celsius (1701-1741) of Sweden. The melting point of ice is zero and the boiling point of water under normal atmospheric pressure is marked 100.

(2) The *Fahrenheit scale* was devised by German scientist Gabriel Fahrenheit (1686-1736) in 1717. Its zero was specified as the temperature of a given mixture of snow and common salt ; the melting point of pure ice was taken as 32° on the scale and the boiling point of water under normal atmospheric pressure as 212°. The scale is generally used in Great Britain and the United States. Physicians make use of this scale.

(3) In 1730 or round about that year French scientist Antaine Reaumur (1683-1757) designed a scale which goes by his name. Its zero is at the melting point of ice, Under normal atmospheric pressure the boiling point of water corresponds to 80° on this scale. The scale is used for domestic purposes in Russia and in some parts of the European continent.

Comparison of Three Scales of Temperature—The space between the two fixed points contains 100 divisions of the Centigrade, 180 of the Fahrenheit and 80 of the Reaumur Scale. The space is sometimes called the **fundamental interval of a thermometer scale.**

The positions of the two fixed points on the three scales are shown below :

Scale	*Ice-point*	*Steam-point*	*Number of equal divisions between fixed points*
Centigrade (C)	0°	100°	100
Fahrenheit (F)	32°	212°	180
Reanmnr (R)	0°	80°	80

$\therefore$ 100 Centigrade divisions = 180 Fahrenheit divisions = 80 Reaumur divisions.

or, $1°\text{C} = \frac{9}{5}$ of $1°\text{F} = \frac{4}{5}$ of $1°\text{R}$.

Take a thermometer whose lower and upper fixed points are A and B respectively (Fig. 7) By its side place the three scales. Put the thermometer in a suitable bath and let P be the position of the top of the mercury column in AB. Corresponding to the position P note the readings on the three scales, by x°C, y°F, and z°R. It is easy to see that on the three scales the distance AP represents x, $(y-32)$ and z degrees and that AB represents 100, $(212-32)$ and 80 respectively. No matter what scale is used, as AP bears to AB the same ratio.

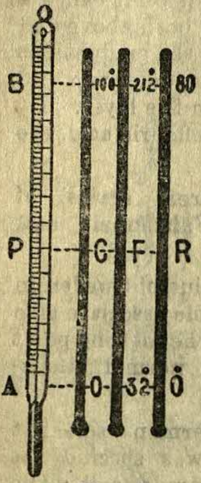

Fig. 7

$$\frac{AP}{AB}=\frac{x^\circ C}{100}=\frac{(y-32)^\circ F}{180}=\frac{z^\circ R}{80}$$

$$i.\,e.\ \frac{x^\circ C}{5}=\frac{(y-32)^\circ F}{9}=\frac{z^\circ R}{4} \qquad (\ldots 6,\ 1)$$

Examples :—

1, The normal temperature of a healthy person is found to be $98^\circ{\cdot}4$F. What would be the corresponding reading on the Centigrade and the Raeumur ?
[*Cf.* V. U.—1952]

Ans. Symbols carrying the usual significance, we find $\frac{x}{5}=\frac{98{\cdot}4-32}{9}=\frac{R}{4}$,

whence $x=39{\cdot}9^\circ$C and $y=29{\cdot}5^\circ$R

2. The fundamental interval of a thermometer A is arbitrarily divided into 60 equal parts and that of another thermometer B into 120 equal parts. If the freezing point of A is marked 60° and that of B marked 0°, what is the temperature by A when it is 100° by B ? [Pat. U.—1954]

Ans. The interval between the ice-point and the steam-point is divided into 60 parts for A and 120 parts for B. Therefore each degree graduation of A is equivalent to two degree graduations of B.

∴ 100 degree graduations of B=50 degree graduations of A.

Since A starts from 60°, the corresponding reading of A would be $(60+50)^\circ=110^\circ$.

3. What is the temperature which when read on a Centigrade and a Reaumur thermometer differs by 2° ?

Ans. Let the reading on the Centigrade scale be x. Then on the Reaumur scale. it is either $x+2$ or $x-2$.

Since $\frac{C}{5}=\frac{R}{4}$, we gather that either $\frac{x}{5}=\frac{x+2}{4}$ or else $\frac{x}{5}=\frac{x-2}{4}$,

whence $x=-10^\circ$C or. 10°C, and $R=-8^\circ$ or 8° respectively.

4. When the temperature is 0°C. a mercury thermometer reads $\cdot 5^\circ$C, while at 100°C, it reads $100{\cdot}8^\circ$C. Find the true temperature when the thermometer reads 20°C, assuming that the bore is cylindrical and that the divisions are of uniform length.

Ans. Since at the true zero-point the thermometer reads $0{\cdot}5^\circ$C and at 100°C it reads $100{\cdot}8^\circ$C. there are $(100{\cdot}8-0{\cdot}5)$ or, $100{\cdot}3$ divisions between the two fixed points.

Each division of the thermometer corresponds to 100/100·3 of a correct centigrade thermometer the reading 20° of this thermometer is, in reality, (20 − 0·5) or, 19·5 division above the freezing point.

The corresponding true temperature is $\frac{19{\cdot}5 \times 100}{100{\cdot}3} = 19{\cdot}44°C$.

5. A faulty thermometer raads 5°C in melting ice and 99°C in dry steam at normal pressure. Find the correct temperature in Fahrenheit when the thermometer reads 52°C. [Pat. U. 1951]

Ans. Obviously, (99—5)°, =94° of the thermometer correspond to 100° of a standard thermometer.

∴ 1° of this thermometer $= \frac{100}{94}$ of the standard centigrade thermometer.

When this thermometer reads 52°C, its corresponds, in fact, to 52—5 or 47 scale divisions above the ice-point.

This comes out to be $\frac{47 \times 100}{94} = 50°C$ of a standard centigrade thermomter.

Again, since $\frac{C}{5} = \frac{F-32}{9}$, we, get $\frac{F-32}{9} = \frac{50}{9}$, whence the desired temperature is 122°F·

7 Advantages of Mercury as a Thermometric Substance—

(1) Mercury boils at 357°C and freezes at—39°C ; so it remains in the liquid state over a *long range of temperatures.* Ordinarily a mercury thermometer may be conveniently used for an approximate range of temperature from—30°C to abaut 300°C.

A mercury thermomter may also be used for measuring high temperature by filling the space above the mercury with nitrogen or carbon dioxide at a high pressure so as tc prevent the boiling of mercury. A temperature up to 600°C can thus be measured by using glass with high melting point and the range can even be increased to 750°C by taking a tube of silica.

(2) It is a good *conductor*; that is, it quickly transmits heat throughout its body and so all mercury in the thermometer bulb readily attains the temperature of the substance in which it is placed.

(3) Its expansibility is fairly large with a rise of temperature.

(4) Its expansion is *fairly* uniform, so that at all parts of the scale for an equal rise of temperature its volume registers an equal rise.

(5) It has a *low thermal capacity*. This means that for a given rise of temperature it absorbs a small quantity of heat.

(6) As at ordinary temperature the vapour pressure of mercury is very low, the space above mercury contains very little mercury vepour.

(7) It can be readily obtained in a pure state· It is a shining opaque liquid and so it can be easily observed within a glass tube. It does not wet glass and does not, therefore, stick to the sides of the glass tube.

8. Alcohol Thermometer—Alcohol is sometimes used as a thermometric substance. The liquid is coloured with some dye, as otherwise the position of the top of the liquid column cannot be readily observed. Its advantages and disadvantage as the thermometric substance in comparison to mercury are given below :

Advantages :

(i) Alcohol freezes at—130°C. while mercury freezes at—39°C, so for measurement of *low temperature*, an alcohol thermometer is preferable Consequently, alcohol may be used in thermometers down to about − 120°C. (Liquid pentane may, however, be used down to − 200°C.)

(*ii*) For a given rise of temperature, alcohol expands much more than mercury ; so an alcohol thermometer is *more sensitive* than a mercury thermometer to temperature variations.

(*iii*) For a given volume, so as to be raised through the same range of temperature, alcohol requires much less quantity of heat than mercury,

(*ix*) Alcohol is a light liquid and wets glass ; so when temperature rises, the thread of alcohol can move smoothly in a tube of very fine bore, while mercury tends to move in a jerky manner,

Disadvantages :

(i) Alcohol boils at 78°C, while mercury boils at 357°C So for accurately measuring temperatures higher than about 60°C an alcohol thermometer cannot be used.

(ii) Conductivity of alcohol is less than that of mercury.

(iii) The rate of expansion of alcohol is not uniform, but increases with rise of temperature. An alcohol thermometer is, therefore, graduated by comparison with a mercury thermometer, both being placed in the same bath.

(iv) It is highly volatile and so its vapour readily collects into the space above the liquid and exerts an appreciable pressure.

(v) As alcohol wets glass, a film of the liquid sticks to the walls of the tube when the temperature falls.

9. **Maximum and Minimum Thermometers**—These thermometers are so constructed as to be able to record *automatically* the highest or the lowest temperature attained during a given period of time. They are used for meteorological purposes.

Rutherford's Maximum Thermometer—This is a mercury thermometer with the bulb B and a small steel index P shaped like a double-headed pin resting within the stem (Fig. 8). The thermometer is mounted horizontally in a wooden or metallic frame so that at any position the index can remain statio- nary.

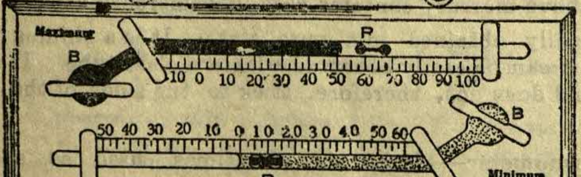

Fig. 8—Maximum & Minimum Thermometers

In setting the thermometer, the steel index is moved from outside by a strong magnet, till one end is brought just in contact with the surface of the mercury. As the temperature rises, mercury expands and pushes the index forward. With a fall of temperature the mercury contracts ; but the index is left at the position to which it had been pushed. The higher is the rise of temperature, the further the distance the index is pushed through. The end of the index towards the mercury surface records the maximum temperature attained during the period under observation.

In some form of Maximum Thermometers the stem is placed in a horizontal position and the bulb is kept at a slightly lower level. At the junction of the bulb and the stem there is a constriction. With rise of temperature mercury is forced into the stem ; but when temperature falls, the mercury column within the stem is separated from the mercury within the bulb, thereby recording automatically the maximum temperature. Use of a separate index is not required in this type of thermometer.

Rutherford's Minimum Thermometer—In this thermometer alcohol is used in place of mercury. There is a small index P made of enamel or coloured glass and shaped like a double-headed pin resting within the liquid in the stem (Fig 8). In a suitable frame the thermometer is fixed horizontally. The instrument is set by tilting and the index is moved till one of its ends is just in contact with the liquid surface, the other end being within the liquid. As the temperature falls, the liquid contracts and due to surface tension the concave surface of alcohol drags the index along with it. But when the temperature rises, alcohol expands and leaves the index at the position attained. The end of the index towards the meniscus records the minimum temperature reached during the period under observation.

Six's Thermometer – In this case the maximum and the minimum thermometers are combined into a single instrument. It essentially consists of a U-tube (Fig. 9) with bulbs at the two ends. The bulb B and a part of the stem are filled with alcohol. The portion P_1 P_2 is filled with mercury. The portion of the tube above P_1 and a portion of the bulb *b* are filled with alcohol. The remaning part of it contains alcohol vapour only. Room for expansion is left thereby.

Shaped like dumb-bells, two small steel indices P_1 and P_2 float on the ends of the mercury column. Each index is provided with a small side-spring which presses against the wall of the tube and keeps the index in position. The instrument is set by bringing the indices just in contact with the respective mercury surface by means of a strong magnet working from outside. Running by the sides of the limbs of the U-tube, there are two graduated scales, the one of the 'maximum' side being graduated from below upwards, and the other of the 'minimum' from above down wards.

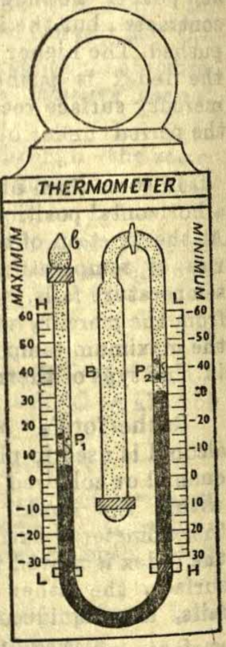

Fig. 9,

Temperature rises and the alcohol in the larger bulb B expands and pushes down the mercury column with which it is in contact. Pushing forward the index P_1 and driving the alcohol in the other bulb *b*, the other end of the mercury column rises. When the temperature falls, the alcohol in the large bulb contracts and the mercury level on this side rises and raises the index P_2, leaving it at the highest position reached. The position of the lower end of P_1 records the *maximum* temperature during the period under observation, the reading being obtained from the corresponding scale. From the corresponding scale the position of the lower end of P_2 give the *minimum* temperature. After each observation the instrument is readjusted.

10. Clinical Thermometer (*Physician's Thermometer*)—This is a form of sensitive maximum thermometer and is used in recording the temperature of human body. It is a mercury thermometer with a constriction at C in the bore close above the bulb (Fig. 10). It has a short stem which is graduated generally from 95°F, to 110°F, each degree being divided into five equal parts. The normal temperature of a healthy person varies between 96·6°F and 98·4°F When the bulb is placed into thermal communication with the human body, the temperature of the bulb rises and the mercury inside expands. The expansive force drives the mercury past the constriction into the stem. When the thermometer is removed, the temperature falls and the mercury below C contracts ; but the thread above C cannot follow, as the constriction provides an obstruction. The farthest end of this portion indicates the maximum temperature of the body. When the thermometer is to be used again, it is shaken with the bulb downward, so that the

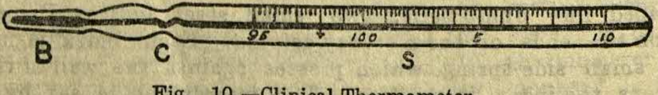

Fig. 10.—Clinical Thermometer

mercury thread in the stem is made to pass through the constriction partly into the bulb. Very recently clinical thermometers are graduated in centigrade scales. (For Dial and Metal Thermometers vide HEAT, Appendix E, at the end of Volume I).

***11. Sources of Errors in Mercury Thermometer.**—The following are some of the more important sources of error in a mercury thermometer and their corrections.

(1) *Change of Zero*—By placing the bulb of an ordinary marcury thermometer in melting ice, it is sometines found that the mercury stands at a slightly *higher* position than the freezing point marked on the stem. This occurs when a sufficient interval is not allowed between the sealing and the graduation of the thermometer tube. The tube contracts slowly and so the volume of the mercury appears to be in excess, showing a rise in the freezing point. The excess reading for ordinary glass is about 0·04°C.

(2) *Temperature of Steam*—It has been mentioned in Art. 5 that in determining the upper fixed point a correction is to be applied to it, if atmospheric pressure at the time be not normal. But if both the fixed points possess certain errors, they may be eliminated by supplying for every such thermometer a correction table between 0°C and 100°C.

(3) *Temperature of the Exposed Column*—When a thermometer is used, a part of the stem always remains outside and is, therefore, at a different temperature from the bulb. For this reason the readings, which are recorded, would be slightly lower; hence it is better to immerse as much of the stem as possible. A correction for this may be applied.

(4) *Internal Pressure*—A vertical column of mercury, 20 cm. long, exerts a pressure on the thermometer bulb of nearly at quarter of atmospheric pressure. Hence when a rather higher temperature is recorded by a mercury thermometer, pressure of this order is exerted on the bulb, the bulb gets distended and the reading is consequently lower. But if the thermometer is kept horizontal, the problem of internal pressure does not crop up.

(5) *Non-uniformity of the bore*—In a thermometer a change of temperature is measured by the change of volume of a definite mass of the liquid in the bulb. The changes in volume produce corresponding movement of the liquid head along the stem. The distance on the stem between the two fixed points is divided into equal intervals or *degrees*, so that each degree represents the same change of temperature. This is true only when the bore is uniform; else an equal expansion of the liquid for an equal rise of temperature, though regular, will not be indicated by an equal length in different parts of the stem. In such a case, the thermometer may be calibrated all over the length of the bore by detaching a thread at mercury and measuring the number of degree divisions it occupies at various parts of the tube.

EXERCISES ON CHAPTER I

Reference

Arts. 2 & 9 1. What do you mean by the temperature of a substance? Describe with a neat diagram a Six's thermometer and discuss the principle on which it works. State some of its uses.

Arts. 6. Determine the temperature which is indicates by the same number on Centigrade and Fahrenheit scales.
Ans.—40°C, or, —40°F. (C. U.—1942,'58)

Art. 6 2. The freezing point on a thermometer is marked 20 and the boiling point 150. What reading would this thermometer give for a temperature. of 45°C? (C. U.—1957)
Ans. 78·5°.

Art. 5 3. How will you construct a mercury thermometer and graduate it so that your reading will agree with that of another thermometer? (All precautions and corrections should be metioned). (Pat. U.—1961; Del. H. S.—1970; U. P. B.—1969? C. U.—1956)

Art. 5 4. What is meant by Fundamental Interval of a thermometer? Describe experiment to determine it accurately. (Pat. U.—1970)

Art. 5 5. Describe fully the method followed to mark the scale on a mercury in glass thermometer. (C. U.—1952 *Cf*- 1956)

Arts. 5 & 6 6. Describe how the fixed points of a mercury thermometer are determined.

A faulty thermometer reads 1° when placed in melting ice and 96° in steam at normal barometric pressure. Find the correct temperature when the thermometer reads 39°, the bore of the thread and graduations being supposed uniform. (Dac. U.—1963)
Ans. 40°C.

Art. 5 7. Describe how you would construct and graduate a liquid-in-glass thermometer to read temperature between 2°C and 10°C. Another sensitive thermometer for this range will be supplied.

Give reasons for choosing a particular liquid as the thermometric substance. Also discuss whether water can be used for this range and explain how you would read the temperature without any ambiguity with water-in-glass thermometer in the above range. (Pat. U.—1962)

Arts. 7 & 8 8. Discuss the relative merits and demerits of mercury and alcohol as thermometric substances. (V. U.—1953)

Art. 9 9. Describe with a neat diagram the working of Six's maximum and minimum thermometer. (Utk. U.—1964)

Art. 9 10. Write a note on maximum-minimum thermometer. (V. U.—1954)

Art. 9 11. Describe with a neat diagram a form of maximum and minimum thermometer. (Pat. U.—1974)

Art. 8 12. Give an account of the various methods for measurement of temperature and classify according to the range of temperature for which they are suitable. Give their relative accuracy of measurement. (R. P. B.—1968)

Art. 10. 13. Describe a clinical thermometer. What makes it possible to read the temperature after removing the thermometer from the body? (*Cf*. V. U,—1953; C. U.—1967)

CHAPTER II

EXPANSION OF SOLIDS

12. Expansion of Solid—In most cases solids expand when heated. This can be shown by an apparatus known as *Gravesand's Ring.* It consists of a metal ring (Fig, 10) clamped to a stand. Suspended by a chain, a ball of the same metal can just pass through the ring, when both are at the same temperature. When heated in a flame, the ball would no longer pass through the ring, showing that it has expanded. On cooling, the ball contracts and slips through the ring.

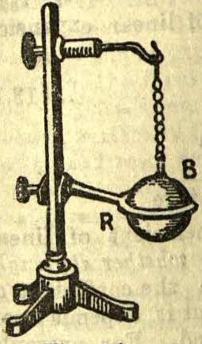

Fig. 11

A short metal rod with flat ends and provided with a wooden handle fits exactly into a metal gauge at ordinary temperature. When heated, the rod no longer fits into the gauge, as it has expanded.

Rivetted together at several points, two similar strips, one of iron and the other of copper, are taken (Fig. 11). When heated, the composite bar slowly gets curved, the copper remaining on the outside. If, on the other hand, the bar is placed for sometime in ice, it bends in the opposite direction with the copper strip on the inside. This shows that copper expands or contracts more than iron. In a similar way it can be shown that different solids expand differently.

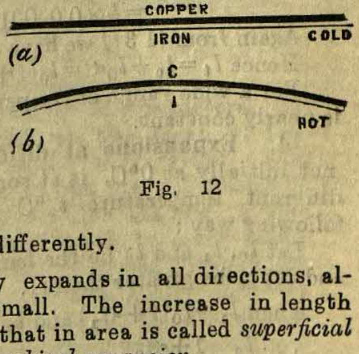

Fig. 12

When a solid is heated, it generally expands in all directions, although the expansion is usually very small. The increase in length of the body is called *linear expansion*, that in area is called *superficial expansion* and that in volume is called *cubical expansion.*

13. Coeficient of Linear Expansion—When a solid is heated, it is found that the increase in length (i) *is proportional to the original length of the body*, (ii) *is proportional to the increase of temperature to which the body is subjected to and* (iii) *depends on the nature of the material of the solid.*

The coefficient of linear expansion of a solid is defined to be *the increase in length* per unit length *of the material for a rise of temperature of* 1°C for ordinary range of temperatures.

The *coefficient of linear expansion of iron is* 0·000012. This statement means that a rod of iron, 1 cm. in length, increases by 0·000012 cm. for rise of temperature of 1°C.

Below we work out the formula for this coefficient.

Let the original length of the rod at 0°C be $= l_0$

„ length of the rod when heated through t°C „ $= l_t$

and „ cofficient of linear expansion of the solid „ $= \alpha$,

Then the increase in length for a rise of t°C $= l_t - l_0$.

∴ the increase „ „ „ of 1°C $= (l_t - l_0)/t$

Hence, by definition, the mean coefficient (α) of linear expansion between 0°C is given by

$$\alpha = \frac{l_t - l_0}{l_0 t}. \qquad \ldots(13.1)$$

In other words, the cefficient of linear expansion

$$= \frac{\text{increase in length}}{\text{original length} \times \text{rise of temperature}}.$$

From (13.1) it is thus apparent that the coefficient of linear expansion of a metarial *would have the same value, whether its lenght is measured in centimetres or in feet.* Therefore the coefficient of expansion is independent of the unit of length. But it depends upon the scale of temperature in which the value is found. For example, if the Fahrenheit scale is used, the coefficient of linear expansion becomes different from that measured in centigrade scale. As 1°F $= \frac{5}{9}$ of 1°C, the cefficient of linear expansion of iron

$$= \tfrac{5}{9} \times 0{\cdot}000012 = 0{\cdot}0000066 \text{ per } {}^\circ\text{F}.$$

Again from (1·31) we have $l_t - l_0 = l_0 \alpha t$.

Hence $l_t = l_0 + l_0 \alpha t = l_0(1 + \alpha t)$. ...(13 2)

For a wide range of temperature, the coefficient of linear expansion is nearly constant.

14. Expansions at different temperature—Suppose a body, not initially at 0°C, is at some other temperature t_1°C. Then at a different temperature t_2°C its length may be obtained in the following way ;

Let l_0, l_1 and l_2 be the lengths at 0°C, t_1°C andt^1°C, respectively and let t_2 be greater than t_1.

Then $l_1 = l_0(1 + \alpha t_1)$ and $l_2 = l_0(1 + \alpha t_2)$.

$$\therefore \quad \frac{l_2}{l_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1} = (1 + \alpha t_2)(1 + \alpha t_1)^{-1}$$

Since α is a very small quantity, square or higher powers of α may be neglected. Expansion of $(1 + \alpha t_1)^{-1}$ binomially up to two terms, we have $(1 + \alpha t_1)^{-1} = 1 - \alpha t_1$, so that

$$\frac{l_2}{l_1} = (1 + \alpha t_2)(1 - \alpha t_1) = 1 + (t_2 - t_1)\alpha - \alpha^2 t_1 t_2.$$

Neglecting terms containing powers of higher than the first, we derive $l_2 = l_1\{1 + (t_2 - t_1)\alpha\}$. ...(14.1)

$$\text{or,} \quad \alpha = \frac{l_2 - l_1}{l_1(t_2 - t_1)}. \qquad \ldots(14.2)$$

This expansion is similar is the one in (13.1) except that original length is taken not at 0°C. Thus if coefficient of linear expansion of a solid be taken to be very small, it is immaterial whether the original length is measured at 0°C or at the room temperature.

15. **Determination of the Coefficient of Linear Expansion of a Solid by Pullinger's Apparatus**—The apparatus consists of a steam jacket J (Fig. 12) held vertically in a suitable frame F. The ends of the jacket are closed by rubber corks. It is provided with inlet and outlet tubes for steam and with two other side tubes for inserting thermometers T,T. The material of which the coefficient of linear expansion is to be determined, is taken in the form of a rod R about a metre in length. It passes into the jacket through the top and stands vertically inside the jacket. The lower end of the rod rests firmly on a slab of marble or glass or on a vertical pivot fixed to the base board of the apparatus and is thus prevented from expanding in the downward direction. The upper end of the rod, which is free to expand, passes just outside the jacket and projects through a hole in a glass plate G placed horizontally at the top of the frame. A spherometer S is placed with its three outer legs on the plate G, while the central leg passes through the hole and can be screwed down to touch the upper end of R.

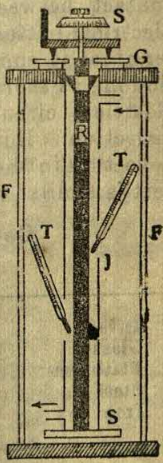

Fig. 12

The rod is taken out and its length is measured at the room temperature with a metre scale. It is then placed in position in the jacket tube and after some time its temperature is noted from the thermometers. If the two thermometers give different readings, the mean of the readings is to be taken which gives the initial temperature. The spherometer is now placed on the plate G and its central leg is screwed down just to touch the upper end of R. The reading of the spherometer is noted and the central leg is screwed up to make room for the rod to expand. Steam from a boiler is now passed through the jacket J. The temperature inside gradually rises and finally becomes constant, as would be shown by the thermometers. If, the steady temperatures as recorded by the thermometers, show slightly different readings, the mean of the two temperatures is to be taken as the final temperature. After waiting for some time in order to be sure that the rod has attained a constant temperature throughout, the central leg of the spherometer is again screwed down just to touch the upper end of R and the reading is noted. The difference between the two readings of the spherometer gives the increase in length of the rod for the rise of temperature during the experiment.

Let the initial length of the rod be $= l$
increase in its length, as given by the spherometer $= e$
and initial and final temperatures of the rod respectively $= t_1°$ and $t_2°$C.

Then from (14.2) it follows that $\alpha = \dfrac{e}{l(t_2 - t_1)}$.

Instead of a solid rod a hollow metal pipe with closed ends and having arrangements for circulating steam within it is sometimes used. On its outside the pipe is covered with non-conducting substance (such as felt or flannel) so as to prevent much heat from being lost. The expansion of a solid rod or a hollow pipe of the same length and of the same material is the same. To know the instant of contact between the rod and the spherometer, one terminal of an electrical battery is sometimes connected to the rod, while the other terminal is connected through a galvanometer to the spherometer. So long as there is an air gap between the central leg and the rod, the electrical circuit is not complete and the galvanometer shows no deflection. But the moment the two are just in contact, there is a deflection in the galvanometer. The sperometer is then read. The rest is similar to the experiment mentioned above.*

Co-efficients of Linear Expansions of Solids

(Mean Coefficients per °C between 0°C and *100°C)*

Invar	...0·0000009	Nickel	...0·0000130
Glass	...0·0000089	Copper	...0·0000167
Platinum	...0·0000089	Brass	...0·0000189
Steel	...0·0000110	Aluminium	...0·0000238
Iron	...0·0000116	Zinc	...0·0000293

16. Travelling Microscope Method—This is a very accurate method of measuring a small change in length or comparing two lengths which are nearly equal to each other. Hence this is sometimes called the *comparator method*. The specimen, whose linear expansion is to be measured, is taken in the form of a rod or a pipe. In Fig. 13. P represents a hollow pipe nearly one metre in length and closed at two ends by corks or rubber stoppers. Each stopper is provided with two holes, through which a thermometer and an inlet

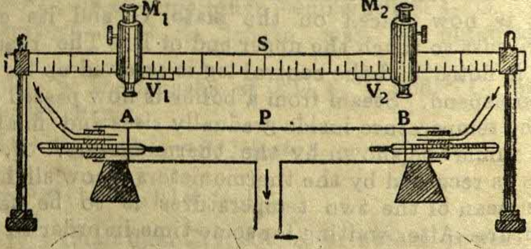

Fig. 13—Comparator Method

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*For further details of the experiment, vide J. Chatterjee's *Fundamentals of Practical Physics.*

glass pipe may be introduced. There is an exit pipe fitted at the central part of the pipe. The pipe with its equipments may be placed in a horizonatal position on two blocks of wood.

Two microscopes M_1 and M_2, which may be slided over a horizontal metal scale S, carry two verniers V_1 and V_2 or micrometer screws. Slight lateral movement of either microscope may be accurately measured with such attachments. Two marks A and B are made at the ends of the pipe. Thermometers and pipe fittings are made as shown and the inlet pipes are connected through rubber tubings to a boiler containing water at the room tempearture. To the nearest millimetre the length between the two marks A and B is measured with the scale S or any other metre scale. Let the length so read at the room temperature t_1°C be l cm. The microscopes are then focussed on the marks and their positions are read accurately with the vernier attachments. Suppose the respective initial readings of the microscopes are C_1 cm. and C_2 cm.

Then the boiler is heated. Steam on being formed circulates through the pipe and passes out through the exit. Thermometers record a rise in temperature. When they record a steady high temperature such as t_2°C, the microscopes are slided properly and the marks are again focussed. Let the new readings of the microscopes be C_3 cm. and C_4 cm. respectively. Then the elongation on one side is (C_3-C_1) cm. and on the other side (C_4-C_2) cm. Thus the total elongation is the sum of the two. Let it be x cm.

Then the Coefficient of Linear Expansion $\alpha=\dfrac{x}{l(t_2-t_1)}$. ...(16·1)

17. Coefficient of Superficial Expansion—It is defined to be *the increase in area per unit area of a surface for a rise of temperature of 1°C.*

Let the original area of the surface at 0°C be $=S_0$,

and the area of the surface at t°C $=S_t$

and coefficient of superficial expansion of the solid $=\beta$.

Then, by definition, $\beta=\dfrac{S_t-S_0}{S_0 t}$. ...(17.1)

$\therefore\ S_t-S_0=S_0\beta t$, or $S_t=S_0(1+\beta t)$. ...(17.2)

Relation between α and β—Consider a square lamina ABCD of a homogeneous solid of side l_0 cm. at 0°C (Fig. 14). Then its area at 0°C is given by $S_0=l_0{}^2$. At t°C let the square assume a larger shape as given by AEFG. Then the increase in area is given by the shaded part BCDGFE.

At t°C, the length of each side becomes $l_t=l_0(1+\alpha t)$.

$\therefore$ the area at t°C is given by $S_t=\{l_0(1+\alpha t)\}^2$.

Again, considering the area as a whole, $S_t=S_0(1+\beta t)$.

Now as α is very small, we can neglect powers of α higher than the first.

Hence, $S_0\,(1+\beta t)=\{l_0(1+\alpha t)\}^2=l_0{}^2(1+2\alpha t)$.

$\therefore$ since $S_0=l_0{}^2$, $1+\beta t=1+2\alpha t'$ or, $\beta=2\alpha$. ...(17.3)

Thus the coefficient of superficial *expansion*

$=2\times$ (*coefficient of* linear *expansion*).

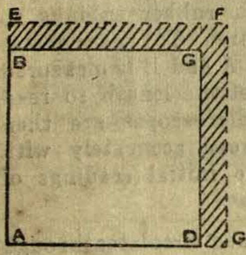


Fig. 14

It is evident from (17·3) that if the coefficient of linear expasion of a solid be known, its coefficient of superficial expansion may be readily found. For example, the coefficient of linear expansion α for brass is $1\cdot89\times10^{-5}$ and if a certain brass surface of unit area be raised to 100°C, its actual area $(1+\alpha t)^2$ becomes equal to 1·0037835 The quantity $(1+2\alpha t)$ is equal to 1·00378. These two values are equal up to the fifth place of decimals. Thus here also, as is to be expected, $\beta=2\alpha$.

18. Coefficient of Cubical Expansion of Solid—It is defined to be *the increase in volume per unit volume of a body for a rise of temperature of 1°C.* Let the original volume of the body at 0°C be V_o, the volume of the body at t°C be V_t and the coefficient of cubical expansion of the solid be γ.

Hence, by definition, $\gamma=\dfrac{V_t-V_o}{V_o t}$. ...(18.1)

$\therefore\ V_t=V_o+V_o\gamma t$, or $V_t=V_o\,(1+\gamma t)$. ...(18.2)

Relation between α and γ—Consider a cube of substance, the length of each side being l_0 cm. at 0°C. If V_0 be its volume at 0°C, $V_0=l_0{}^3$.

At t°C the length of each side becomes $l_t=l_0(1+\alpha t)$.

$\therefore$ the volume at t°C is given by $V_t=\{l_0(1+\alpha t)\}^3$.

Again, taking the volume as a whole. $V_t=V_0(1+\gamma t)$.

Neglect, as before, powers of α higher than the first, inasmuch as α is very small, Hence

$$V_0(1+\gamma t)=\{l_0(1+\alpha t)\}^3=l_0{}^3(1+3\alpha t).$$

$\therefore$ since $V_o=l_o{}^3$, $1+\gamma t=1+3\alpha t$, or, $\gamma=3\alpha$. ...(18.3)

Thus the *coefficient of* cubical *expansion*

$=3\times$ (*coeff. of* linear *expansion*).

It is evident from (18·3) that the coefficient of cubical expansion of a solid can be calculated, if the coefficient of linear expansion is known. It is to be remembered that for a particular rise of temperature a hollow body and a solid one of the same material and of *same shape* and volume increase by equal amount.

If the initial temperature of the solid be t_1°C instead of 0°C and if V_1 be the corresponding volume, it can be established exactly as in (14.1) that the volume V_2 at t_2 is given by

$$V_2 = V_1\{1+\gamma\, t_2 - t_1\}. \qquad \ldots(18.4)$$

$$\text{whence } \gamma = \frac{V_2 - V_2}{V_1(t_2 - t_1)}.$$

Examples :

1. The length of a copper rod at 50°C is 200·166 cm. and at 200°C it is 200·664 cm. Find its length at 0°C and the coefficient of linear expansion of copper.

Ans. Let the length of the rod at 0°C be l_0 cm. and the coefficient of linear expansion of copper be α.

Then $l_0\,(1+50\alpha) = 200{\cdot}166$ and $l_0\,(1+200\alpha) = 200{\cdot}664$.

On division, $\dfrac{1+200\alpha}{1+50\alpha} = \dfrac{200{\cdot}664}{200{\cdot}166}$, whence $\alpha = 0{\cdot}0000166$.

Substituting the value of α, $l_0(1+50\times 0{\cdot}0000166) = 200{\cdot}166$.

whence the length $l_0 = 200$ cm.

2. Railway lines are laid with gaps to allow for expansion. If the gap between steel lines, 66 ft. long, is 0·5 inch at 10°C, at what temperature will the lines just touch? (Coefficient of linear expansion of steel is 11×10^{-6}). [C. U.—1953 ; Gau. U.—1951]

Ans Each line can expand ·25 in, at each end making a total elongation of ·5 in, in order that consecutive lines may just touch other.

Now, elogation = ·5 in. = ·0416 ft.

If the required temperature be t°C, we find

$66\times11\times10^{-6}\times(t-10) = {\cdot}0416$, whence the temperature $t = 67{\cdot}3$°C.

3. An iron ring, which is 1 ft. in diameter, is to be shrunk on a pulley which is 1·005 ft. in diameter. If the temperature of the ring is 10°C, find the temperature to which it must be raised, so that it will slip on the circumference of the pulley (Coefficient of linear expansion of iron = 0·000012).

Ans. The iron ring of diameter 1 ft. has a circumferencial length of $\pi\times1$ or π ft. In order that it can just fit on a pulley whose circumferencial length is $\pi\times1{\cdot}005$ ft., the circumferencial elongation of the ring would be $\pi(1{\cdot}005-1) = \pi\times{\cdot}005$ ft. Now the elongation must be brought about by a change of temperature. Let the required temperature be t°C.

Thus $\pi\times1\times{\cdot}000012\times(t-10) = \pi\times{\cdot}005$,

whence the desired temperature $t = \left(\dfrac{5000}{12}+10\right)$°C $= 426{\cdot}6$°C

4. A zinc rod is measured by means of a brass scale, which is correct at 0°C, and is found to be 1·0001 metre long at 10°C. What is the real length of the rod at 0°C? (Mean coefficient of linear expansion of zinc is ·000029 and of brass is ·000019 per degree C.) [Nag. U.—1961 , Utkal U.—1964]

Ans. As the brass scale is correct at 0°C, 1 metre of the scale at 0°C would be $(1+{\cdot}000019\times10)$m, or 1·00019 m. at 10°C. But 1·00019 m. is apparently read by the scale at 10°C as 1 m. Hence the apparent reading of 1·000 m. at 10°C would correspond to (1·0001+1·00019) m. or 1·00029 m. in the correct scale at 0°C. Thus the length of the zinc rod at 10°C is 1·00029 metres.

If the length of the rod at 0°C be l_0 then by the stated condition,

$$l_0(1+\cdot000029\times10)=1\cdot00029, \text{ or, } l_0\times1\cdot00029=1\cdot00029.$$

whence the length $l_0=1$ m.

5. An iron scale is correct at 0°C and the length of a zinc rod as measured by the scale is 100 cm. when both the scale and the rod are at 0°C. What will be the length of the zinc rod as measured by the iron scale when both of them are at 100°C, the coefficient of linear expansion of iron and of zinc being ·000012 and ·000026 respectively per degree centigrade? [Pat. U.—1952]

Ans. At 0°C the length of the zinc rod = 100 cm.

At 100°C the true length of the zinc rod $=100(1+\cdot00002\times100)$ cm. When the zinc rod is measured with an iron scale at 100°C, each centimetre of the scale at 0°C becomes $(1+\cdot000012\times100)$ cm. at 100°C. Thus each true centimetre length at 100°C would apparently read $\frac{1}{1\cdot0012}$ cm. *i.e.*, ·9988 cm.

∴ the required length of the zinc rod at 100°C, which is 100·26 cm., would be apparently read by the iron scale at 100°C to be $=(100\cdot26\times\cdot9988)$ cm. = 100·14 cm.

6. A lump of iron has a volume of 10 cu. ft. at 100°C. Find its volume at 25°C. (α for iron $=0\cdot000012$).

Ans. We have γ for iron $=3\alpha=0\cdot000036$.

Let the required volume at 25°C be V_1 cu. ft.

Hence the relation $V_2=V_1\{1+\gamma(t_2-t_1)\}$ leads to

$$V_1\{1+0\cdot000036(100-25)\}=10$$

whence the required volume $V_1=9\cdot97$ cu. ft.

19. Practical Cases of Expansion of Solids—The expansion or contraction of solids due to temperature variation is practically utilised in a number of cases. In laying down rails on a railway track, small spaces are left between consecutive rails in order to allow room for expansion when they get heated. The slots in the fish-plates serve the same purpose. To allow for contraction in winter the telegraphic wires are given enough gap. The metallic water pipes and gas mains have telescopic joints to allow for change in length due to variations in temperature.

In the contruction of an iron bridge, each part is allowed freedom of expansion. The ends of the girders, for example, are not rigidly fixed on the supports but are placed upon rollers, so that they can expand or contract without causing any strain on the supports. The ends of the iron bars in a furnace are kept free; otherwise they would bend due to expansion on heating. When iron pipes are used for conveying hot water or steam, they are fitted with some device allowing for expansion and contraction.

On account of unequal contraction of different parts, metal castings may sometimes crack on cooling. Thus, in casting a wheel with thin rim and massive arms, the rim cools and solidifies much more rapidly than the arms and then opposes contraction of the latter. A great stress is thus set up which may break the rim or some of the arms. By making the arms bent this is got rid of.

If very hot water is poured into a thick-bottomed drinking vessel of glass, it would often crack. As glass is a bad conductor of heat, the inner and the outer layers of the thick portion do not rapidly pick up the same temperature and so expand unequally, causing the glass to crack. Similar effect may be observed when iced water is put into a glass tumbler, A vessel of fused quartz would stand quick variation in temperature, as its expansion is extremely small.

Sometimes metal wires are sealed into glass vessels for making air-tight joints, *e.g.*, in electric lamps. If a copper wire is fused into glass, the glass will crack during cooling, as copper contracts more than glass. But a platinum wire can be safely sealed into glass, as the coefficient of linear expansion of the latter is practically the same as that of the former. Nowadays for the sake of economy, a nickel-iron alloy, called platinite, is used for the construction of electric lamps.

When metal rods or chains are used as measuring instruments, temperature should always be noted and a correction applied for any variation. The *Standard Yard* is taken to be the distance, at 62°F., between two fine marks on gold or platinum plugs inserted into a bronze bar kept at the Standard Office. The standard metre is correct at 0°C.

The diameters of iron tyers of cart wheels are slightly smaller than those of the wheels. Such a tyre is very strongly heated so as to be easily put round the wheel. As the tyre cools, it contracts and fits tightly upon the wheel. Boiler plates are rivetted with red-hot rivets ; on cooling, the revets contract and grip the plates so tightly as to make the joints steam-proof. Big guns are often made by sliding several cylinders one above the other upon an inner tube. Each cylinder in a red-hot condition is put round the inner one and on cooling fits tightly upon the latter. By these means the enormous pressure developed due to the combustion of the powder is uniformly distributed throughout the body of the gun. Sometimes a stopper sticks tight into the mouth of a glass bottle ; if the bottle is slightly warmed at the neck, the neck gets heated earlier, expands and loosens the stopper.

Thermostat is an instrument in which the differential expansion of a compound bar of two dissimilar metals is utilised. It is used in fire alarms and in automatically controlling temperature in incubators. In constructing chronometers, watches and clocks, arrangements are to be made to avoid the effects of change of temperature on the time-keeping parts. For such purpose clocks are provided with compensated pendulums, and watches and chronometers with compensatad balance wheels.

20. Compensated Pendulum—The time T for one complete oscillation of a simple pendulum is given by the relation :

$$T=2\pi\sqrt{\frac{l}{g}}.$$

where l stands for the distance from the point of suspension to the centre of gravity of the bob, and g for the acceleration due to gravity.

The pendulum in a clock should be so designed that the time period may remain constant throughout the seasons. A change of temperature affects the length l and so also the time period T. With rise in temperature l increases : therefore T also increases and so the clock goes slow. When the temperature falls, l decreases ; consequently T also decreases, and so the clock goes fast. Hence with an ordinary pendulum a clock has a tendency to lose in hot seasons and gain in cold seasons In order that the clock may keep a correct time at all seasons the pendulum of a clock must be provided with special devices in order that the *effective* length of the pendulum remains unaltered in spite of the variations in temperature. Such a pendulum is termed a **Compensated Pendulum.**

In an ordinary clock pendulum there is a heavy metal disc, which can be slided up and down a rod. There is a small nut screwed at the end of the rod just under the disc. As temperature rises, the rod increases in length ; the disc or the bob moves down. so that the effective length increases. To effect compensation in this case, the nut is screwed up a bit so as to raise the centre of gravity of the pendulum as a whole to its former position and to make the effective length constant. When due to a fall in temperature the rod contracts, the nut is screwed down so as to effect the adjustment. Compensation by this method is not automatic but is to be done by trial. In one form of clocks, the *automatic* compensation is accomplished by the following arrangement :

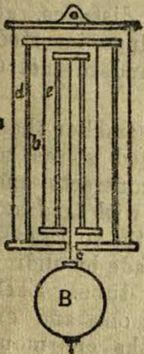


Fig. 15.

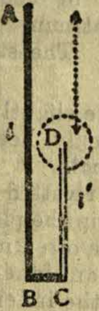


Fig. 16.

Harrison's Grid iron Pendulum—The painciple of this pendulum may be understood from the following explanation : Let A B and CD (Fig. 16) be two rods of different materials and of unequal lengths joined by cross-bar BC. The end A being fixed, the rod AB is allowed to expand downwards, while the rod CD being rivetted at C can expand upwards. If the lengths and the materials of the rods be so chosen that for an equal rise of temperature each increases by the same amount, the downward expansion of AB equals the upward expansion of CD. Hence the lowering of the end B is the same as the rise of the end D, so that the distance between A and D remains constant, whatever be the change of temperatures.

If the respective coefficients of linear expansion be α and α' and the lengths of AB and CD be l and l', then for a rise or fall of temperature through t°C we must have

$$l\alpha t = l'\alpha' t, \text{ or, } l\alpha = l'\alpha', \text{ or } \frac{l}{l'} = \frac{\alpha'}{\alpha}.$$

It is clear that the lengths of the rods must be inversely proportional to the respective coefficients of linear expansion. Evidently, the shorter rod CD should be of more expansible material than the longer one AB.

In an *actual grid-iron pendulum*, the bob is carried by a framework made of 9 parallel rods (Fig. 15), made alternately of brass and steel ; there are 4 brass rods (shown by double lines) and 5 steel rods (shown by single lines) including the central one which carries the bob B. The end of the rods are attached to cross-pieces in such a way that the steel rods are fixed at the upper ends and can expand downwards, thereby tending to lower the position of B, while the brass rods being fixed at the lower ends can expand upwards, thereby tending to raise the position of B. For the sake of compensation the total downward expansion of the steel rods must be equal to the total upward expansion of the brass rods. Again, save and except the central steel bar, all the bars occur in pairs. As each pair of similar rods made of the same metal and attached to the same cross-piece expands in the same direction by the same amount, determination of the displacements of the bob demands consideration of the expansion of one rod of each pair.

Let l be the total length of the three steel rods $(a,b,c,)$ as one-half of the frame, and l' the total length of the brass rods (de) in the same half If the coefficients of linear expasion of steel and brass be α and α', than $l\alpha = l'\alpha'$,

$$\text{or,}\quad \frac{l}{l'} = \frac{\alpha'}{\alpha} = \frac{19\times10^{-6}}{12\times10^{-6}} = \frac{19}{12},$$

so that for the sake of compensation, the effective lengths of the steel and brass rods are approximately in the ratio 3 : 2.

Nowadays the process of compensation is effected more easily by having the pendulum rods made of an alloy of steel and nickel prepared by M. Guillaum and named invar (invariable steel). As its coefficient of linear expansion is only 0·0000009, the change in length for ordinary variation of temperature is practically inappreciable. Standards of length also are made of invar. Due to temperature changes substances like fused quartz and silica also have very negligible expansion. Consequently due to sudden temperature variations vessels made of these substances have little chances of crack.

21. Compensated Balance Wheel of a Chronometer or a Watch—The motion of the hands of a watch or a chronometer is mainly controlled by the vibrating balance wheel, the period of oscillation of the wheel depending upon its diameter. An ordinary wheel would increase in diameter when temperature rises, and as a result its period of oscillation becomes larger. Hence the watch in hot seasons would go slower.

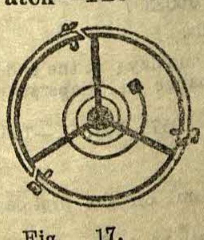


Fig. 17.

The *compensation* is secured in the following way. The rim of the wheel is divided into three sections, each of which is supported at one end by a spoke of wheel and carries a compensating mass screwed at the other (Fig. 17). The segments are made of strips of two dissimilar metals, the more expansible one remaining outside.

As temperature rises, the spokes increase in length and carry the ends of the respective segments of the rim outwards; but owing to the greater expansion of the outer metal of each strip, the other end of the segment moves inwards, thus bringing the compensating masses nearer the centre. If the wheel is so constructed that outward displacement of the masses due to expansion of the spokes is equal to the inward displacement due the curling of the segments, the average diameter of the wheel does not change; consequently the period of oscillation of the balance wheel is unaffected by changes of temperature and the watch keeps right time.

Examples:

1. A grid-iron pendulum is made of 4 brass rods, each of length 50 cm., and 5 iron rods. Find the length of each iron rod. (For iron $\alpha=0\cdot0000117$ and for brass $\alpha=0\cdot0000187$).

Ans. On each side of the central iron rod carrying the bob, there are one pair of iron and one pair of brass rods; but as the expansion of the rods of the same material on either side is the same, 3 iron rods and 2 brass rods are to be taken as doing the compensation.

Let the length of each iron rods at t, the rise of temperature, be l cm.

Here expansion of 3 iron rods = expansion of 2 brass rods.

$\therefore\ 3\times l\times0\cdot0000117\times t=2\times50\times0\cdot0000187\times t.$

Thus the l is 53·3 cm.

2. A clock, which keeps correct time at 25°C, has a pendulum made of brass. How many seconds will it gain per day, when the temperature falls to the freezing point? (Coefficient of linear expansion of brass is 0·0000187).

Ans. Let the length of pendulum at 0°C be l_0 and that at 25°C be l. Let the corresponding time periods be t_0 and t.

$$\therefore \frac{t}{t_0}=\sqrt{\frac{l}{l_0}}=\sqrt{\frac{l_0(1+25\times0\cdot0000187)}{l_0}}=(1+0\cdot0004675)^{\frac{1}{2}}$$

$$=1+\tfrac{1}{2}\times0\cdot0004675=1\cdot0002337.$$

But as at 25°C the pendulum keeps correct time, $t=1$ sec. and so at this temperature the pendulum makes $(60\times60\times24)$ or 86400 complete oscillations in a day of 86400 seconds.

At 0°C the time period is given by $t_0=\dfrac{1}{1\cdot0008337}$ sec.

$\therefore$ the number of oscillations made per day $=86400\div\dfrac{1}{1\cdot0003337}=86420\cdot2.$

$\therefore$ the clock gains by $(86420\cdot2-86400)$ secs. $=20\cdot2$ seconds.

3. An iron clock pendulum makes 86405 oscillations in one day; at the end of the next day the clock has lost 10 seconds. Find the change in temperature. (The coefficient of linear expansion of iron is 0·000117).

[A. B.—1952; U. P. B.—1957; Pat. U.—1964]

Ans. Let the change in temperature be 0°C.

With the usual symbols for a pendulum, $t=2\pi\sqrt{l/g}$.

Let l be the length and t the time period of a pendulum on the first day. Let l' be the length, and t' the time period on the next day.

A correct pendulum makes $24\times60\times60$, = 86400 oscillations a day. As the pendulum makes 86405 oscillations on the first day, $t=(86400\div86405)$ sec.

As the pendulum loses 10 seconds on the next day, it makes 86395 oscillations on the day, so that $t' = (86400 \div 86395)$ sec.

$$\therefore \frac{t'}{t} = \frac{86405}{86395}.$$

$$\text{Now } \frac{t'}{t} = \sqrt{\frac{l'}{l}} = \sqrt{\frac{l(1+\alpha\theta)}{l}} = \sqrt{1+\alpha\theta} = (1+\theta \times 0{\cdot}0000117)^{\frac{1}{2}}$$

Hence we find $(1+\theta \times 0{\cdot}0000117)^{\frac{1}{2}} = \frac{86405}{86395}$.

Solving this equation, the required temperature $\theta = 10^{\circ}{\cdot}6$ C.

EXERCISES ON CHAPTER II

Reference

Art. 12

1. Two long flat pieces of metals, one of copper and the other of iron, having the same dimension, are rivetted together at the two ends to form a single flat bar. When this bar is heated, it bends with its concavity towards the side of iron. Explain the phenomenon.

Arts. 13 & 18

2. Define coefficients of linear and cubical expansions. Do they differ when the lengths are measured in centimetres or feet? (Dac. U.—1963 ; C. U,—1954, '57 ; *Cf.* Nag. U—1956)

Art. 13

3. Define coefficient of linear expansion. Does it depend on (i) the unit of length, (ii) scale of temperature? (V. U.—1952)

18 & 13

4. Define the coefficient of linear and cubical expansions of a solid. Show that the latter is three times the former,

(E. P. U.—1951 ; *Cf.* C. U.—1955 ; P. U.—1962
Cf. Pat. U.—1962 ; Gau. U.—1961)

Art. 13

5. The distance between Allahabad and Delhi is 390 miles. Find the total space that must be left between the rails to allow for a change of temperature from 36°F in winter to 117°F in summer. Given expansibility of iron $= 12 \times 10^{-6}$ per degree centigrade.

Ans. 0·21 mile (nearly).

Art. 13

6. A glass rod when measured with a zinc scale, both being at 20°C, appears to be one metre long. If the scale is correct at 0°C. what is the true length of the glass rod at 0°C? The coefficient of linear expansion of glass is 8×10^{-6} and that of zinc is 26×10^{-6}.

Ans. 100·036 cm.

Arts 13 & 15

7. The height of a barometer appears to be 76·4 cm. according to a brass scale which is correct at 0°C. If the temperature at the time of reading is 30°C. What is the actual height of that mercury column? The coefficient of linear expansion of brass is 0·0000189. How is this quantity determined experimentally?

Ans. 76·44 cm.

Art. 13

8. A steel tyre, 4·0 feet in diameter, is to be shrunk on to a cart wheel, of which the average diameter is $\frac{1}{8}$ in greater than the inside diameter of the tyre. Assuming the coefficient of linear expansion of the material of the tyre to be 0·000012, calculate the necessary rise of temperature of the tyre in order that it may easily slip on the wheel.

Ans. 217°·02C.

Reference

Art. 13 9. A man wishes to fit an aluminium ring on a steel rod one inch diameter; but it is ·001 too small in diameter. How much should its temperature be raised before it will just slip on? Subsequently he wishes to remove it again; but now he has to heat both metals together. Through how many degrees must this be done? (Coeff. of expansion of aluminium = ·000010). (Gau. U.—1953)

Ans. 40°C; 66·7°C.

Art, 13 10. One end of a steel rod of length 61 cm. is fixed and the other presses against an end of a lever 10·5 cm. from the fulcrum. On being heated through 500°C; the rod turns the lever through 2°. Find the cofficient of linear expansion of rod. $(90^\circ = \pi/2$ radians).

Hints: $\dfrac{\text{Increase}}{10{\cdot}5} = \theta \text{ radians} = \dfrac{2 \times \pi}{180} \text{ radians.}$

$\therefore$ increase in length $-(10{\cdot}5 \times 2 \times \frac{22}{7} \times \frac{1}{180})$ cm. 0·366 cm.

Ans. 0·000012 per $^\circ$C.

Art. 15 11. Describe any method of finding the coefficient of linear expansion of a metal rod.

(Gau. U.—1963: C. U.—1952; Nag. U,—1951, '65; E. P. U. —1971)

Art. 18 12. Compare the density of lead at 100°C with that at -100°C. assuming that its coefficient of expansion remains constant within these limits of temperature. (Coefficient of linear expansions of lead = ·000028).

Ans. ·983 ; 1.

Arts. 13 & 18 13. Explain what is meant by the coefficient of linear expansion. State the relation it bears with the coefficient of cubical expansion. (C. U.—1943, '63

Art. 15 14. Describe any method of determining the coefficient of linear expansion of solid. (C. U.—1953; P. U.—1969; Dac. U.—1964; E. P, U.—1966; Pat. U.—1962; Gau. U.—1971)

Art. 17 15. A sheet of brass is 50 cm. long and 10 cm. broad at 0°C.: the area of the surface increases by 1·9 sq. cm. at 100°C. Find the coefficient of linear expansion of brass. (V. U.—1955)

Ans. 19×10^{-6} per °C.

Art. 18 16. The volume of a lead bullet at 0°C is 25 c.c. The volume increases at 98°C by 0·021 c.c. Find the coefficient of linear expansion of lead.

Ans. $28{\cdot}6 \times 10^{-6}$ per $^\circ$C.

Art. 13 17. A bar of steel is heated through 200°F and securely fixed at each end while at this increased temperature. What force per square inch of section is the bar capable of exerting, when it has cooled down to the original temperature, provided that fixings at both the ends remain rigid? Mean coefficient of expansion between 32°F and $212^\circ\text{F} = 6{\cdot}7 \times 10^{-6}$ and the modulus of elasticity = 14400 tons per sq. inch.

Ans. 18·76 tons-wt.

Reference

18. Two flat circular discs, one of platinum and the other of silver, of equal radius and each 2 mm. thick are securely fastened back to back when at 0°C. They are then suspended in water boiling at normal pressure. Show that they will be bent so that they form a section of a hollow sphere and determine their radius of curvature. Given the coefficient of linear expansion of silver is $18{\cdot}8 \times 10^{-6}$ per degree centigrade. Art. 17

Ans. Radius of curvature = 404 cm.

19. Describe the effects of varying temperature on the rate of a clock or a watch. Explain how chronometers are constructed so as to keep accurate time in spite of changes in temperature. (C. U.—1955 ; All. U.—1962) Art. 21

20. Why should the time of oscillation of a clock-pendulum change with the rise of temperature? What arrangement is made to make the clock keep correct time both in warm and cold weathers? Given that the coefficient of linear expansion of brass is 19×10^{-6}, and that of steel 11×10^{-9}, what must be the relative lengths of the bars of the metal used in grid-iron pendulum? Art. 20

Ans. 11 : 19.

21. What is compensated pendulum? Explain the principle of construction of Harrison's grid-iron pendulum. Art. 20

A gird-iron pendulum is made of 5 iron rods and 4 brass rods. Each of the brass rods is 50 cm. in length. Find the length of each iron rod. The linear coefficient of expansion of iron is 12×10^{-6} and that of brass is 18×10^{-6}. (C. U.—1968)

Ans. 50 cm.

22. The pendulum rod of a clock is made of wrought iron and the pendulum swings once per second. If the change of temperature is 25°C, find the alteration in the length of the pendulum. (Coefficient of linear expansion of the wrought iron is $11{\cdot}9 \times 10^{-6}$ and $g = 980$ cm./sec^2.) Art. 20

Ans. 0·029 cm.

23. Write explanatory notes on compensated clock pendulums and watch balance wheels. Give diagrams. Arts. 20 & 21

CHAPTER III

EXPANSION OF LIQUIDS

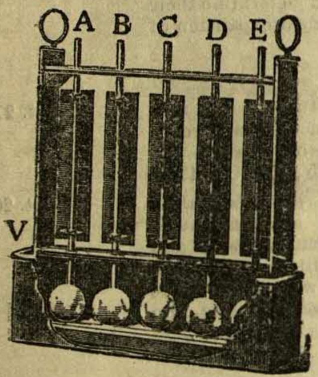


Fig. 18

22. Expansion of Liquids—Like solids liquids also expand when heated. But as liquids have no definite shape of their own but have fixed volumes, they possess volume or *cubical expansion* only. Expansion of a liquid, therefore, always means cubical expansion. For a given rise in temperature different liquids expand by different extents. Fig. 18 represents a number of glass bulbs A, B C, etc., of equal capacity each being provided with a thin stem and fixed vertically in a suitable frame V. All the bulbs are enclosed in a trough at the base of the apparatus. Different liquids are poured to the same height into the bulbs at the room temperature, so that they occupy equal volumes at this temperature. Into the trough hot water is now poured, so that each liquid is heated equally. But the rise of level in different vessels is different. This shows that expansibility differs with liquids. For an assigned rise of temperature, the volume expansion of a liquid is generally more than that of a solid.

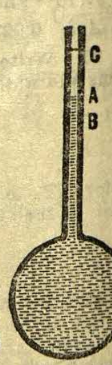


Fig. 19.

Real and Apparent Expansions—The expansion of the liquid due to a rise of temperature is always accompanied by an expansion of the container. Take a large glass bulb provided with a long stem of uniform narrow bore (Fig. 19). Fill the bulb and a part of the stem up to A with a liquid. If the bulb is now suddenly immersed into a bath of hot water, the level of the liquid momentarily comes down to a level B and then gradually goes up to G higher than A. This is so, because as the heat is supplied from outside, the vessel is warmed earlier than the liquid. The capacity of the vessel being increased, the level of the liquid falls. The heat then passes into the liquid which ultimately attains the temperature of the bath. As liquid is more expansible than glass, its level finally rises above A. In fact, we observe the expansion from A to C; the volume AC, therefore, measures the **apparent expansion** of the liquid. If we make a simple assumption that the vessel has first attained the temperature of the bath without importing any heat to the liquid contained and then the liquid has been warmed up to the temperature of the bath, then for the given rise of temperature the volume BC measures the real expansion and AB measures that of vessel. Figure 19 shows that AC = BC − AB.

∴ Apparent expansion = Real expansion − Expansion of the containing vessel,

i.e., *Real expansion = (apparent expansion) + (Expansion of the container)*

23. Coefficients of Real and Apparent Expansion.—The *coefficient of real (or absolute) expansion of a liquid* is defined to be the ratio of the increase in volume (of a liquid for a rise through 1°C) to the original volume. If γ be the coefficient of absolute expansion, then

$$\gamma = \frac{\text{real expansion}}{(\text{original volume}) \times (\text{rise of temperature})}.$$

The *coefficient of apparent expansion of a liquid* is given by

$$\gamma' = \frac{\text{observed expansion of the liquid}}{(\text{original volume}) \times (\text{rise of temperature})}.$$

Relation between γ and γ'—Suppose that the stem of the bulb in Figure 19 is graduated. Let the liquid stand at A when its temperature is 0°C, and let the level come to C when the temperature rises by $t°C$. Let V_0 be the volume of the liquid up to A at 0°C, and v, v_1, v_2 the volumes represented by BC. AC and AB respectively. Let γ, γ' be the respective coefficients of absolute and apparent expansions of the liquid. Denote the coefficient of cubical expansion of the material of the vessel by γ_0.

Then, for a rise of t^0C, $v = V_0\gamma t$, $v_1 = V_0\gamma' t$ and $v_2 = V_0\gamma_0 t$. Now since $v = v_1 + v_2$, it follows $V_0\gamma t = V_0\gamma' t + V_0\gamma_0 t$.

Hence $\gamma = \gamma' + \gamma_0$.

Thus the coefficient of real expansion = (coefficient of apparent expansion) + (coefficient of cubical expansion of the material of the vessel).

24. Determination of Coefficient of Apparent Expansion of a Liquid—

The Volume Dilatometer Method—Sppose the volume of the bulb in Figure 19 is known and the stem is graduated in equal divisions. The bulb and a part of the stem are filled with the given liquid. The bulb is then kept for some time in a bath at 0°C. From the position of the liquid surface the volume at 0°C is obtained. The bath is then raised to a fixed higher temperature, say, t^0C, and the position of the liquid level at this temperature is noted again.

If the area of cross-section of the stem be α sq. cm. and the distance, through which the liquid column moves, be l cm. the increase in volume is $l\alpha$ c.c.

Neglecting the expansion of the bulb, $\gamma' = \dfrac{l\alpha}{V_0 t}$.

The volume of the bulb can be measured at *any* temperature by weighing the vessel empty and then again weighing the bulb filled with mercury at the same temperature. The difference in weights gives the mass of mercury occupying the volume of the bulb. Dividing this by the density of mercury at the corresponding temperature, we obtain the volume of the bulb.

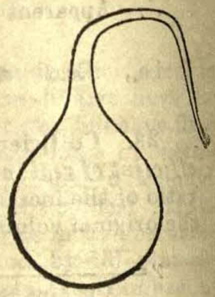


Fig.—20.

The Weight Thermometer Method—The weight thermometer consists of a glass bulb provided with a short bent capillary stem (Fig. 20).). The bulb is at first cleaned, dried and carefully weighed empty. The nozzle of the bent tube is kept under the liquid under experiment. By a process of alternate heating and cooling, it is completely filled with the given liquid. When quite full, the bulb is kept immersed for some time in a water bath (say, at the room temperature) with the open end still under the surface of the given liquid until the contents of the bulb acquire the temperature t_1°C of the water as noted by a thermometer. The weight thermometer is then removed, gently wiped dry and weighed again. It is now placed in the water bath which is kept well-stirred and gradually raised to a higher temperature. The temperature of the bath is kept constant for some time at some higher temperature t_2°C. The liquid in the bulb expands and a portion of it is forced out. The apparatus is then remeved from the bath, gently wiped dry, cooled to the initial temperature and weighed again As a matter of course, however, the liquid, that remains in the bulb, contracts to a smaller volume.

Let the mass of the weight thermometer, when empty, be $=$ W gm.,
its mass when filled with liquid at t_1°C $=\mathrm{W}_1$ gm.
and its mass with the liquid left at t_2°C $=\mathrm{W}_2$ gm.

The mass of the liquid filling the temperature at t_1°C

$$=(\mathrm{W}_1-\mathrm{W}_2)\ \text{gm.}=m_1\ \text{gm., say.}$$

Also the mass of the liquid left in the thermometer at t_2°C

$$=(\mathrm{W}_2-\mathrm{W})\ \text{gm.}=m_2\ \text{gm., say.}$$

Neglecting the expansion of the weight thermometer, we find that the volume occupied by a mass of m_1 gm. of the liquid at t_1°C is the same as that occupied by m gm. at t_2°C.

Let the density of the liquid at t_1°C. be $=\rho$.
Then the volume (of mass m_1 gm,) of the liquid at t_1C $=m_1/\rho$
and " (of mass m_2 gm.) " " at t_2°C $=m_2/\rho$.

Hence a volume m_2/ρ of the liquid at t_1°C occupies a volume m_1/ρ when raised to t_2°C,

$$\therefore \text{ the apparent expansion between } t_1{}^\circ\text{C and } t_2{}^\circ\text{C is}=\frac{m_1}{\rho}-\frac{m_3}{\rho}$$

$$=\frac{m_1-m_2}{\rho}$$

Hence the coefficient of apparent expansion of the liquid is given by

$$\gamma' = \frac{\dfrac{m_1 - m_2}{\rho}}{\dfrac{m_2}{\rho}(t_2 - t_1)} = \frac{m_1 - m_2}{m_2(t_2 - t_1)}. \qquad \ldots(24\cdot 1)$$

$$\text{Thus } \gamma' = \frac{\text{mass of liquid expelled}}{(\text{mass remaining}) \times (\text{rise of temperature})}.$$

As the coefficient of expansion is obtained directly from different weights, the method is known as that of *weight thermometer*. This method is *not* suitable for volatile liquids.

Examples :

1. A weight thermometer containing 100 gms. of mercury at 0°C is put in a liquid bath, when 4 gms. of mercury flow out. What is the temperature of the bath, if the apparent coefficient of expansion of mercury is ·00018 ?

Ans. We know $\gamma' = \dfrac{m_1 - m_2}{m_2(t_2 - t_1)} = \dfrac{4}{(100-4)(t_2 - 0)}$ and $\gamma' = \cdot 00018$,

$\therefore \dfrac{4}{96 \times t_2} = \cdot 00018$, or $t_2 = \dfrac{4}{96 \times \cdot 00018}$.

Using logarithms, log 4 = 0·6021 ;

$\log\ 96 = 1\cdot 9823$

$\log\ \cdot 00018 = \underline{\bar{4}\cdot 2553}$

$\bar{2}\cdot 2376$

$0\cdot 6021$

$\underline{\bar{2}\cdot 2376}$

$2\cdot 3645$

anti-log 2·3645 = 231·5

∴ the reqd. temp. t_2 of bath = 231·5°C

2. A glass weight thermometer has a mass of 6·34 gms. when empty and 153·81 gms. when filled with mercury at 0°C. If 2·08 gms. are expelled when it is heated to 100°C, find the coefficient of relative expansion of mercury in glass. [Raj. U.—1962]

Ans. The mass of mercury with the bulb at 0°C = (153·81 − 6·34) gms. = 147·47 gms.
and the mass expelled at 100°C = 2·08 gms.
and the mass remaining within the bulb = (147·47 − 2·08) gms = 145·39 gms.

Hence the apparent expansion γ' for mercury per degree centigrade

$$= \frac{2\cdot 08}{145\cdot 39 \times 100} = \cdot 000143.$$

3. What must be the radius of a cylindrical bulb of a thermometer of length 1 cm., in order that the distance between successive degree divisions may be equal to 1 mm. ? The internal diameter of the capillary is 0·1 mm. and the coefficient of expansion of mercury is 0·00018. [M. B. B.—1972]

Ans. This is a case of expansion of mercury in a vessel which is the thermometer itself. So it is a question of apparent expansion of mercury ; but as the coefficient of expansion of the containing vessel is not supplied, the value 0·00018 should be taken as the apparent expansion of mercury within the thermometer.

Let the radius of the cylindrical bulb be r cm. Then the volume of the bulb at 0°C is $(\pi r^2 \times 1) = \pi r^2$ c.c. If for 1°C rise of temperature, the mercury thread moves through 1 mm. in a bore whose radius is ·05 mm., the expansion in volume is $\{\pi \times (\cdot 005)^2 \times \cdot 1\}$ c.c.

On equating, we get $\pi r^2 \times \cdot 00018 = \pi \times (\cdot 005)^2 \times \cdot 1$,

or, $r^2 = \frac{(\cdot 005)^2 \times \cdot 1}{\cdot 00018} = 1\cdot 39 \times 10^{-2}$, whence the requisite radius $r = \cdot 118$ cm.

4. The coefficient of cubical expansion of glass and mercury being 25×10^{-6} and 18×10^{-5} per degree centigrade respectively, what fraction of the whole volume of a glass vessel should be filled with mercury in order that the volume of the empty part should remain constant when the glass and mercury are heated to the same temperature ? [Gau. U.—1962]

Ans. Let the total internal volume of the vessel at 0°C be V_0 and the volum filled with mercury be v.

Then the required fraction is $\frac{v}{V_0}$.

At t°C the volume of the vessel $= V_0(1+\gamma t) = V_0(1+25 \times 10^{-6} \times t)$

and volume of mercury $= v(1+18 \times 10^{-5} \times t)$.

$\therefore$ the volume of the empty part $= V_0(1+25 \times 10^{-6} \times t) - v(1+18 \times 10^{-5} \times t)$
$= V_0 - v + (V_0 \times 25 \times 10^{-6} - v \times 18 \times 10^{-5})t$.

As the volume of the empty part is to remain constant being equal to the volume $V_0 - v$, we get

$$V_0 - v + (V_0 \times 25 \times 10^{-6} - v \times 18 \times 10^{-5})t = V_0 - v,$$

or, $V_0 \times 25 \times 10^{-6} - v \times 18 \times 10^{-5} = 0$, whence $\frac{v}{V_0} = \frac{2\cdot 5}{18} = \frac{5}{36}$.

This gives the required fraction.

25. Effect of Heat on Density—The density of a body is its mass per unit volume. When a body is heated, it generally expands in volume. As the mass remains constant, the density must decrease with a rise of temperature.

Let the volume and density of a mass M of a liquid be V_0 and ρ_0 at 0°C, and V_t and ρ_t when raised to t°C.

Then $V_0\rho_0 = M = V_t\rho_t = V_0(1+\gamma t)\rho_t$,

or, $\rho_0 = \rho_t(1+\gamma t)$, .. (25.1)

whence $\rho_t = \rho_0(1+\gamma t)^{-1} = \rho_0(1-\gamma t)$, neglecting terms containing higher powers of γ.

Thus $\rho_t = \rho_0(1-\gamma t)$. ...(25.2)

The equation shows that *density decreases as temperature rises.*

From (25,2) it follows $\gamma = \frac{\rho_0 - \rho_t}{\rho_0 \times t}$.

If the temperature rises from t_1 to t_2 and if ρ_1 and ρ_2 be the corresponding densities, then

$$\rho_2 = \rho_1 \ \{1 - \gamma(t_2 - t_1)\}, \quad \text{whence } \gamma = \frac{\rho_1 - \rho_2}{\rho_1(t_2 - t_1)}. \qquad ...(25,3)$$

Hence by knowing the densities at two different temperatures t_1 and t_2, the value of the coefficient of cubical expansion γ can be *indirectly* determined.

Examples :

1. The density of mercury at 20°C is 13·546 and its coefficient of cubical expansion is 0·000182. Find the mass of 500 c.c. of mercury at 80°C. Also find the volume of 500 gms. of mercury at this temperature.

Ans. Let the density of mercury at 80°C be ρ' gms./c.c.

From (25.3) $\dfrac{13{\cdot}545\times\rho'}{13{\cdot}545\times(80-20)}=0{\cdot}000182$, whence $\rho'=13{\cdot}398$.

$\therefore$ at 80°C the mass of 500 c.c. of mercury $=(13{\cdot}398\times 500)$ gms. $=6699$ gms.

Also „ volume „ „ $=(500/13{\cdot}4)$ c.c. $=37{\cdot}3$ c.c.

2. The density of mercury is 13·6 gms./c.c. at 0°C and at 100°C it is 13·35 gms./c.c. Calculate the coefficient of absolute expansion of mercury.

[Utkal U.—1959]

Ans. In the standard formula (25, 2) put $\rho_0=13{\cdot}6$ and $\rho_1=13{\cdot}35$.

Then per °C, $\gamma=\dfrac{\rho_0-\rho_1}{13{\cdot}6\times t}=\dfrac{13{\cdot}6-13{\cdot}35}{13{\cdot}6\times 100}=\dfrac{\cdot 25}{1360}=1{\cdot}84\times 10^{-4}$.

3. A cylinder of iron, 30 cm. long floats vertically in mercury at 0°C. Calculate the increase in depth to which the cylinder sinks when the temperature is raised to 100°C. Density of mercury at 0°C $=13{\cdot}6$ gm./cm.^{3}; density of iron at 0°C $=7{\cdot}6$ gm./cm.^{3}; absolute coefficient of expansion of mercury $=1{\cdot}82\times 10^{-4}$ per deg. C, and coefficient of cubical expansion of iron $=3{\cdot}51\times 10^{-5}$ per deg. C.

Ans. Let the radius of the cylinder at 0°C be r cm. Then the volume of the cylinder $=(\pi r^2\times 30)$ c.c. and its mass is $=$ (volume) $\times$ (density) $=\pi r^2\times 30\times 7{\cdot}6$ gm. The cylinder displaces its own mass of mercury and if l cms. be the depth of the cylinder below mercury, the mass of mercury displaced is $(l\times\pi r^2\times 13{\cdot}6)$ gm.

Hence $l\times\pi r^2\times 13{\cdot}6=\pi r^2\times 30\times 7{\cdot}6$, whence $l=\dfrac{30\times 7{\cdot}6}{13{\cdot}6}=16{\cdot}77$

Now at 100°C the density of iron $=7{\cdot}6\times(1-3{\cdot}51\times 10^{-5}\times 100)$ gm./cm.^{3}

$=7{\cdot}6\times{\cdot}99649$ gm./cm.^{3}

and at 100°C density of mercury $=13{\cdot}6\times(1-1{\cdot}82\times 10^{-4}\times 100)$ gm./cm.^{3}

$=13{\cdot}6\times{\cdot}9818$ gm./cm^3.

Per deg. C, the linear expansion of iron $=3{\cdot}51\times 11^{-5}\div 3=1{\cdot}17\times 10^{-5}$.

A length of 30 cm. at 0°C becomes $30\times(1+1{\cdot}17\times 10^{-5}\times 100)$ cm.

$=(30\times 1{\cdot}00117)$ cm.

Now if the depth of the cylinder at 100°C be l_1 cm. and its radius be r_1 cm., then $l_1\times\pi r_1^2\times 13{\cdot}6\times{\cdot}9818=\pi r_1^2\times 30\times 1{\cdot}00117\times 7{\cdot}6\times{\cdot}99649$.

whence $l_1=\dfrac{30\times 1{\cdot}00117\times 7{\cdot}6\times{\cdot}99649}{13{\cdot}6\times{\cdot}9818}=17{\cdot}03$.

To calculate l_1 we consult the logarithmic table as below.

$$
\begin{array}{rl|rl|l}
\log 30 & =1{\cdot}4771 & \log & 13{\cdot}6=1{\cdot}1335 & \text{anti-log } 1{\cdot}2313 \\
\log 1{\cdot}00117 & =\ {\cdot}0005 & \log & {\cdot}9818=\bar{1}{\cdot}9921 & \quad =17{\cdot}03 \\
\log 7{\cdot}6 & =\ {\cdot}8808 & & \overline{\quad 1{\cdot}1256} & \\
\log {\cdot}99649 & =\bar{1}{\cdot}9985 & 2{\cdot}3569 & & \\
 & \overline{\quad 2{\cdot}3569} & 1{\cdot}1256 & & \\
 & & \overline{1{\cdot}2313} & &
\end{array}
$$

$\therefore$ the height submerged $=(l_1-l)$ cm. $=(17{\cdot}03-16{\cdot}77)$ cm. $={\cdot}26$ cm.

26. Determination of Coefficient of Real (or Absolute) Expansion of Liquid—(1) *Indirect Method*—If the coefficients of absolute and apparent expansions of a liquid by γ and γ' and if the coefficient of cubical expansion of the material of the containing vessel be γ_g, then from (20.9) we get

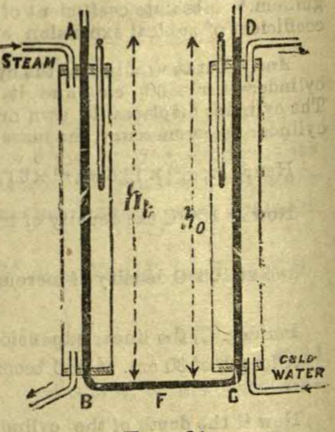
$$\gamma=\gamma'+\gamma_g.$$

Finding γ' by any of the methods given in the last article and taking γ_g as thrice the coefficient of linear expansion of the material of the vessel, we can calculate γ. It should, however, be noted that since different samples of glass have different expansibilities, the value of γ as calculated from γ_g is not very accurate.

(2) **Dulong and Petit's Method** – The coefficient of absolute expansion of mercury can be directly determined by this method.,

In its simplest form the apparatus consists of a glass tube open at both ends and bent twice at right angles in the form of a U-tube ABCD, (Fig. 21), of which the limbs AB and CD are vertical and the portion BC is horizontal. Each vertical limb is jacketted by a wider glass tube. The ends of each jacket are closed by cork, through which the vertical tube passes. The jacket tubes are provided with outlets at the top and at the bottom. Thermometers are inserted through holes in the upper cork.

Mercury is poured into the U-tube so that its level, equal in each limb, is just visible above the jacket tube. One of the jackets, say CD, is kept filled with a stream of ice cold water entering at the lower end and going out through the upper outlet. At the same time steam is passed into the other jacket, which enters at the upper end and passes out through the lower outlet. To prevent the passage of heat from the hot limb to the colder one, BC is kept covered with a piece of cloth or blotting paper moistened with cold water.



Fsg. 21.

A difference of temperatures is set up in the liquid in the two limbs. The level of mercury in the limb AB is found to rise above that in CD. When the liquid columns have attained steady positions, the heights of the mercury levels in AB and CD above the axis of the horizontal tube BC are measured and the temperatures in the two limbs are noted.

Let the heights of liquid column in AB and CD
$=h_t$ and h_0 respectively
temperatures of liquid in AB and CD $=t$ and $0°$C „ .
densities of mercury at $t°$C and $0°$C $=\rho_t$ and ρ_0 „ ,
and the coefficient of absolute expansion of mercury $=\gamma$.

Then at B the pressure $=P+g\rho_t h_t$ and at C the pressure $=P+g\rho_0 h_0$ where P is the atmospheric pressure.

As BC is horizontal; pressure at B = pressure at C.

$\therefore\ P+g\rho_t h_t = P+g\rho_0 h_0$, or $\rho_t h_t=\rho_0 h_0$.

Now by (25,1) $\rho_0=\rho_t(1+\gamma t)$ and $h_t\rho_t=h_0\rho_t(1+\gamma t)$,

whence $h_0\gamma t=h_t-h_0$, or $\gamma=\dfrac{h_t-h_0}{h_0 t}$. ...(26,1)

The increase in level is independent of the dimension or the expansion of the material of the vessel In the laboratory, instead of water at 0°C, water at room temperature is generally used. This demands modification of calculation. Let the heights of the cold and the hot columns be h_1 and h_2 respectively. Let the corresponding temperatures, at which the densities of mercury are ρ_1 and ρ_2 respectively, be t_1 and t_2,

Then $g\rho_1 h_1=g\rho_2 h_2$, or $\dfrac{h_1\rho_0}{1+\gamma t_1}=\dfrac{h_2\rho_0}{1+\gamma t_2}$.

$\therefore\ h_2(1+\gamma t_1)=h_1(1+\gamma t_2)$, or $\gamma(h_1 t_2-h_2 t_1)=h_2-h_1$,

whence $\gamma=\dfrac{h_2-h_1}{h_1 t_2-h_2 t_1}$. ...(26,6)

It is to be noted that Dulong and Petit's method gives the *real* expansion of the liquid, whereas a weight thermometer (or a dilatometer) gives *apparent* expansion,

Example :

In an experiment performed by Dulong and Petit's method the heights of the cold and hot columns of mercury are found to be 90·6 cm. and 92·2 cm. respectively. If the first column is at 0°C, find the temperature of the other (γ for mercury = $18{\cdot}2\times10^{-5}$).

Ans. Let the required temperature be t°C.

from (26, 1), $\dfrac{92{\cdot}2-90{\cdot}6}{90{\cdot}6\times t}=18{\cdot}2\times10^{-5}$, whence $t=97$°C (nearly).

Table of Coefficients of Real Expansion of Liquids

(mean value per °C between 10°C and 32°C)

Water (40° – 100°)	...0·00058	Aniline	...0·00085
Mercury	...0·00018	Terpentine	...0·00094
Glycerine	...0·00058	Ethyl Alcohol	...0·00120
Olive Oil	...0·00070	Methyl Alcohol	...0·00122
Benzene	...0·00124	Paraffin Oil	...0·00090

27. Anomalous Expansion of Water—We know that liquids expand on heating and contract on cooling. But in the case of water some anomaly is observed. If from the room temperature a quantity of water be gradully cooled, the volnme decreases until the temperature falls to 4°C. If the temperature is further lowered, he volume instead of diminishing increses. Water, therefore, occupies the smallest volume at 4°C and has consequently the *maximun density* at this temperature. This can be shown by either of the following methods.

Constant Volume Dilatometer—This consists of a glass vessel with a cylindrical bulb and a long graduated stem of uniform narrow bore (Fig. 22). In the bulb is put a quantity of mercury whose volume is one-seventh the capacity of the bulb. As the coefficient of cubical expansion of mercury is nearly seven times that of glass, it follows that for any change of temperature the expansion of the vessel is equal to the expansion of the mercury contained. Hence inside the vessel the volume, being indpendent of a change of temperature, remains constant.

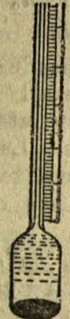


Fig. 22.

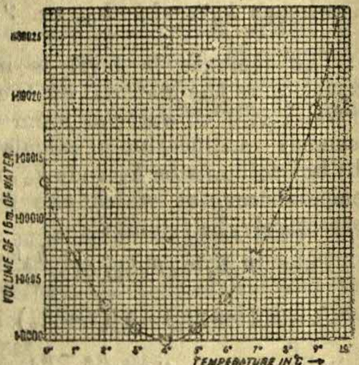


Fig. 23.

Into the dilatometer a quantity of water is put so as to occupy a certain level in the stem. The bulb is now placed in a water bath maintained at 0°C. The temperature is noted with a sensitive thermometer placed in the bath. When the position of the water column in the stem becomes steady, the volume of the water is noted. The temperature of the bath is then raised slowly, water being well-stirred. From the position of the water level in the stem the volume of the water at the corresponding temperature is noted.

If on a graph paper the volume be plotted against the csrresponding temperature, we get a curve, as in Figure 23. From the graph it is evident that as the temperature rises from 0°C, the volume of the water at first dereases, becomes *minimum* at 4°C and then goes on increasing for further rise of temperature. At 4°C, therefore, the volume of a given mass of water is minimum and so the density is maximum. It may be noted that exactly 1 c.c. of water at 4°C has a volume of 1·000127 c.c. at 0°C and 1·000265 c.c. at 10°C. Hence the variation in volume with a change of temperature is very small. It should be noted that due to the anomalous expansion of water between 0°C and about 8°C or 9°C, it cannot be used a thermometric liquid in this range, since in a thermometer we require a regular expansion of a liquid with temperature. It is possible to construct a water thermometer beginning from 8°C or 9°C, although such a thermometer has some other troubles such as capillary action, vapour pressure, etc.

Hope's Experiment—The apparatus consists of a tall glass cylinder G (Fig. 24), closed at the bottom and surrounded at the middle by a wide circular trough J. The cylinder has two lateral openings (one near the top and the other near the bottom) closed by corks, through which thermometers T; T are inserted horizontally. The cylinder is filled with water previously cooled to about 10°C, and the trough J is packet with a freezing mixture of ice and salt.

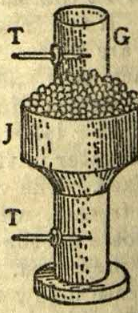


Fig. 24 – Hope's apparatus

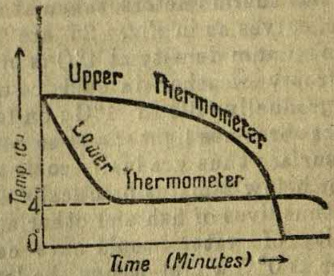


Fig. 25

From time to time the thermometers T,T are observed. Just at the beginning the two give practically the same reading. The lower thermometer then starts to fall and gradually comes down to 4°C at which it remains stationary. During all this time the reading of the upper thermometer remains practically unchanged. After some time it begins to fall gradually and comes to 0°C. If the cooling is continued for a long time small crystals of ice are found floating on the surface of the water, while the lower thermometer remains steady at 4°C. After a sufficiently long interval the lower thermometer records a fall of temperature from 4°C. This is an indication that water possesses the maximum density at 4°C.

We start with the assumption that water within the cylinder is initially at a uniform temperature of 10°C, when the freezing mixture is applied at its central part. Owing to the action of the freezing mixture the water in the central part of G cools, becomes denser and goes down. At the same time the warmer and lighter water from the bottom goes up, gets cooled and goes down again. Thus the water at the bottom gradually falls in temperature, as is shown by the lower thermometer. This continues till the whole mass of water in the lower part has come down to 4°C. On further cooling, the water near the middle part cools below 4°C but does not sink. This is evident from the constant reading of 4°C given by the lower thermometer. The water tends to ascend, but cannot do so, as it is heavier than the water in the upper portion. With continuation of cooling, the water near the middle gradually falls to 0°C and small crystals of ice are then formed. These tend to rise to the surface, melt and cool the water in the upper part, as is shown by a rapid fall in the readings of the upper thermometer. This continues till the whole mass of water in the upper portion comes to 0°C as indicated by the upper thermometer T, while all this time water in the lower part remains at 4°C. Finally, crystals of ice, which are now formed near the middle, float on its surface.

As the densest liquid occupies the lowest portion and as the lower thermometer indicates a temperature of 4°C, the conclusion is inevitable that *water at 4°C has the maximum density.* In other words, a *given mass of water occupies the smallest volume at 4°C.* The density of water at 4°C is taken to be unity. If the readings of the two thermometers, taken at regular intervals, are plotted against time, curves as in Fig. 25 are obtained. The fact that water has its maximum density at 4°C is of great practical importance in nature, In frosty weather in cold countries the water in lakes, rivers. etc. gradually falls to 4°C ; on further cooling, the water in the upper layers becomes lighter and so remains on the surface. The water on the surface thus gradually cools to 0°C and finally freezes, while the water below remains practically at 4°C. The ice floats on the surface and thus lives of fish and other aquatic animals are preserved. If the density of water would have continued to decrease below 4°C, the water at 0°C would occupy the lowest layer and ice would be formed from below upwards, so that the whole mass of water would have changed into a solid block of ice.

***28. Thermostat**—One of the most useful practical applications of the expansion of a solid or liquid is utilised in the construction of an instrument which maintains a substance at a known constant temperature. Such an apparatus is called a *thermostat* or a thermo-regulator.

Fig, 26 represents a liquid thermostat, in which the glass bulb B contains some such liquid as toluene, benzene or alcohol possessing large coefficient of expansion. Sealed into the bulb, a glass tube P nearly reaches its bottom and is provided with a side tube N having a stop-cock. The tube P is bent twice at right angles and is terminated with a bulb C having an opening. At the top of C is fused a short glass pipe reaching nearly to its bottom. As shown in the diagram, an inter-connecting adjustable tap S is very often fitted as shown. The end F is connected to a gas burner.

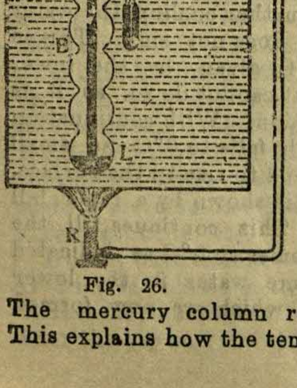


Fig. 26.

When the temperature of the bath exceeds a certain critical limit, the liquid inside forces the mercury column contained in P through the pipe so as to close the end of the pipe E and there the supply of gas to the burner through the pipe is stopped temporarily. The gas supplied through the by-pass tube is just sufficient to maintain a small flame at the burner. The heat supplied by the burner to the bath becomes insufficient and the temperature of the bath tends to fall and produce a small decrease in volume of toluene. The mercury column recedes, thereby opening the gas supply. This explains how the temperature of the bath is not allowed to cross assigned limit. Within the funnel N the critical temperature may be adjusted by the mercury column.

***29. Correction for Barometric reading for Temperature—** In a mercury barometer the pressure of the atmosphere is expressed in terms of the height of a column of mercury, which is measured usually by a brass scale. The scale is correct at 0°C and the liquid column is initially adjusted at 0°C. Hence the observed height of a barometer at t°C is to be reduced to the *equivalent* height at 0°C. In order to fulfil this requirement it is necessary—

(1) to make correction for expansion of the scale between 0°C and t°C

and (2) to make correction for the change in density of mercury.

With these set purposes let

H = the observed height of the barometer at t°C.
H_0 = the equivalent *corrected* height at 0°C.
ρ_0 and ρ = densities of mercury at 0°C and t°C respectively
and α = coefficient of linear expansion of brass scale.

(i) *Correction for the expansion of the scale*—At t°C the scale increases in length, so that each centimetre of the correct scale becomes $(1+\alpha t)$ cm. Recorded by this increased scale, the observed reading H is, therefore, less than the reading given by the correct scale at 0°C.

$$\text{Thus} \qquad h = H(1+\alpha t). \qquad \ldots(29, 1)$$

(ii) *Correction for the change in density of mercury*—The reading h requires correction for the change in density of mercury. As the atmospheric pressure remains the same,

$$g\rho h = g\rho_0 H_0 \quad \text{or} \quad \rho h = H_0 \rho(1+\gamma t),$$

$$\text{whence } H_0 = \frac{h}{1+\gamma t} = h(1+\gamma t)^{-1} = h(1-\gamma t), \qquad \cdots(29, 2)$$

higher powers of γ being neglected.

Relations (29,1) and (29,2) combine to form

$$H_0 = H(1+\alpha t)(1-\gamma t) = H[1-(\gamma-\alpha)t], \qquad \ldots(29,3)$$

neglecting $\gamma\alpha t^2$ which is small in comparison with αt or γt.

The following Table supplies the variation of density of water with temperature ;

Density of Water with Temperature

Temp. (in °C)	Density (gm./c.c.	Temp. (in °C)	Density (gm./c.c.)	Temp. (in °C.	Density (gm./c.c.)
0	·99987	10	·99973	24	·99732
1	·99993	12	·99953	26	·99681
2	·99997	14	·99927	28	·99626
3	·99999	16	·99897	30	·99567
4	1·00000	18	·99862	32	·99505
5	·99997	20	·99823	34	·99440
6	·99988	22	·99780	36	·99371

Example.

1. A barometer is provided with a silver scale which reads correctly at 0°C. What is the true height of the mercury corresponding to an apparent height of 761·05 mm. at 15°C? What height would the barometer register at 0°C? (Given coeff. of cubical expansion of mercury = ·000182 ; coeff. of linear expansion of silver = ·000019) [Utkal U.—1954]

Ans. As the silver scale increases in length at 15°C, the apparent height of mercury would be less than the true height h cm., for by reason of the standard formula we have

$$h = 761·05 \times (1 + ·000012 \times 15),\ \text{whence}\ h = 761·27\ \text{mm.}$$

But the real height of the mercury column would be obtained after correcting for change of density of mercury at 0°C. If the true height be H_0 cm., then

$$H_0 = h(1 - \gamma t) = 761·37 \times (1 - ·000182 \times 15),\ \text{whence}\ H_0 = 759·19\ \text{mm.}$$

Thus the desired height of the barometer is 759·19 mm.

***30. Correction for exposed stem in the Thermometer—.** When a thermometer is used for measuring the temperature of a body the upper part of the stem is at a temperature considerably lower than that of the bulb. The divisions of the scale in the lower part do not, therefore, expand equally and so the observed temperature requires correction.

Let t_1 be the *observed* temperature of a body whose *real* temperature is t. Suppose there are n divisions of the thermometer exposed to air at an average temperature t_2 If the thermometer were throughout at t, these n divisions would cover a space equivalent to $n\{1 + \gamma(t - t_2)\}$ divisions, being the coefficient of apparent expansion of mercury in glass.

Hence the correct number of divisions in the exposed column $= n\{1 + \gamma(t - t_2)\} - n = n(t - t_2)\gamma$.

∴ the true temperature of the body is given by

$$t = t_1 + n(t - t_2)\gamma. \qquad \ldots(30,2)$$

EXERCISES ON CHAPTER III

Reference

Arts. 22 & 25 — 1. Define the coefficients of apparent and real expansions of a liquid. State the relation between the two. (P. U.—1970 ; C. U.—1963 ; R. P. B.—1955 ; Anna. U.—1961 ; Gau. U.—1965 ; *Cf.* Del. H. S.—1954)

Arts. 22 & 23 — 2. Show that the absolute coefficient of cubical expansion of a liquid is the sum of its apparent cofficient of expansion and the coefficient of cubical expansion of the material of the container.
(Utk. U.—1963 ; P. U.—1962 ; Pat. U.—1972 ; Gau. U.—1965, A B.—1953 ; Raj. U,—1955)

Reference

3. A weight thermometer weighs 1·8760 gms when empty. It is filled with mercury at 0°C and then heated to 100°C, when 0·2351 gm. of mercury is expelled. The thermometer with the remaining mercury weighs 16·8625 gms. Calculate the coefficient of apparent expansion of mercury. (Utk. U.—1953) Art. 24

Ans. 16×10^{-5} per °C.

4. A specific gravity bottle holds 50 gms. glycerine at 30°C. How much will it hold at 100°C, assuming the coefficient of cubical expansion of glycerine to be ·00051 and that of glass to be ·00008 ? (R. P. B.—1960) Art. 24

Ans. 48·37 gm.

5. Describe the weight thermometer method of determining the apparent expansion of a liquid. (E. P. U.—1970 ; Dac. U.—1963 ; R. P. B.—1967 ; U. P. B.—1962) Art. 24

6. A glass vessel, which holds exactly 1000 gms. of mercury at 15°C. is put into boiling water at 100°C. What weight of mercury is expelled ? [Coeff. of absolute expansion of mercury is 0.00018 and coeff. of linear expansion of glass is 0·00001.]. (Gau. U.—1965) Art. 24

Ans. 12·6 gms. (nearly).

7. A weight thermometer contains 510 gms. of mercury at 15°C. On placing it in a hot oil bath only 500 gms. are left in it. Find the temperature of the oil bath. [Coeff. of absolute expansion of mercury = ·00018 and coeff. of linear expansion of glass = ·00001.] Art. 24

Ans. 148·3°C.

8. The bulb of an alcohol thermometer upto 0°C mark has a volume of 2 c.c. and the area of cross-section of the stem is ·008 sq. cm. When the thermometer is raised in temperature from 0°C to 50°C, the liquid rises 13·1 cm. in the stem. Calculate the coefficient of expansion of alcohol relative to glass. (V. U.—1953] Art. 24

Ans. ·001048 per °C.

9. A glass weight thermometer has a mass of 6·34 gms. when empty and 151·73 gms. when filled with mercury at 99°C. If 2·08 gms. have been expelled in changing the temperature from 0°C to 99°C, determine the coefficient of relative expansion of mercury in glass. Art. 24

Ans. 14.5×10^{-5} per °C.

10. A weight thermometer weight 25 gms. when empty and 475 gms. when filled with mercury at 0°C. On heating it to 100°C, 655 gm. of mercury is expelled. Calculate the coefficient of linear expansion of glass, the coefficient of real expansion of mercury being ·00182 per °C. (Pat. U.—1962) Art. 24

Ans. ·000011 per °C.

11. A weight thermometer contains 82 gms. of a liquid at 10°C and on heating it to 85°C 2 gms of liquid are expelled. Find the coefficient of absolute expansion of the liquid. If the coefficient of linear expansion of glass is 00009. (E. P. U.—1970) Art. 24

Ans. ·00036 per °C.

12. Explain how to find the coefficient of real expansion of a liquid by weight thermometer. Why is it called a thermometer ? (Raj. U.—1964 ; E. P. U.—1972) Art. 24

Reference

Art. 24 13. A thermometer is made by forming a bulb at the end of a glass capillary tube of which the area of cross-section of the bore is 0·0004 sq. cm. The apparent coefficient of expansion of mercury in glass being $15·5 \times 10^{-5}$ per degree centigrade, what must be the volume of the bulb, if the degree marks are to be 0·2 cm. apart ?

Ans. 39·2 c.c.

Art. 24 14. A weight thermometer weighs 40 gms. when empty and 490 gms. when filled with mercury at 0 C. On heating to 100°C, 6·85 gms. of mercury escape. Calculate the cofficient of linear expansion of glass, the coefficient of real expansion of mercury being 0·000182. (R. P. B.—1942, '53 : U. P. B.—1948)

Ans. 9×10^{-6} per degree C.

Art. 24 15. Find the mass of mercury which must be introduced into a glass bulb of 20 c.c. capacity in order to meke the volums of the unoccupied portion in the bulb constant between 20°C and 100°C. (α for glass $= 8 \times 10^{-6}$; ρ of mercury at 20°C and 100°C are 13·546 and 13·352 respectively.)

Ans. 35·5 gms. (nearly).

Art. 24 16. A graduated glass tube of uniform bore contains mercury upto 100 divisious at 0°C. Find the temperature a t which the mercury occupies 101·5 division (Coeff. of absolute expansion of mercury is 0 00018 ; α for glass is 8×10^{-6})

Ans. 96°·15C.

Art. 24 17. The coefficient of linear expansion of glass is 8×10^{-6} and that of cubical expansion of mercury is $1·8 \times 10^{-4}$ per deg. C. What volume of mercury must be placed in a specific gravity bottle in order that the volume of the bottle not occupied by the mercury shall be the same at all temperature ?

Ans. $\frac{4}{15}$ of the volume of the bottle.

Art. 24 18. The bulb of a dilatometer has a volume of 19·8 c.c. What volume of mercury must be introduced into it, so that the volume of the free portion becomes independent of charges in temperature ? (α for glass $= 8 \times 10^{-6}$ and γ for mercury $= 18 \times 10^{-5}$).

Ans. 2·64 c.c.

Art. 25 19. How does the density of a substance change with temperature ? Find the relation between the two.

(E. P. U.—1963)

Art. 26 20. Explain how to determine the coefficient of absolute expansion of a liquid by Dulong and Petit's method.

(E. P. U.—1962)

Art. 26 21. In an experiment with a liquid the column stood at 60 cm. at 32°F and stood 5 cm. higher at 212°F. Find in degree centigrade the coefficient of real expansion of the liquid.

(E. P. U.—1969)

Ans. ·00083 per °C.

Art. 26 22. 1 c.c. of water weighs 0·99874 gm. at 0°C and 1 gm. at 4°C. Find the mean coefficient of absolute expansion of water between 0°C and 4°C.

Ans. 32×10^{-6} per degree C.

Reference

23. In an experiment to find the real expansion of mercury with Dulong and Petit's apparatus, the temperature of hot column was 100°C and that of the cold column was 0°C. The heights of the mercury columns were found to be 51 cm. and 50·1 cm. respectively. Calculate the coefficient of real expansion of mercury. Art. 26

Ans. $17 \cdot 9 \times 10^{-5}$ per degree C.

24. In an experiment two columns of mercury, one at a temperature of 14°C and the other at a temperature of 100°C, balance each other. If the height of the shorter column is 86·4 cm, find that of the other, (Coefficient of cubical expansion of mercury $= 0 \cdot 00018$). Art. 26

Ans. 87·7 cm.

25. Water is said to have its maximum density at 4°C. Explain what this means. In what respects is the behaviour of mercury different from that of water, when both are gradually warmed from 0°C ? (C. U.—1957) Art. 27

26. "Water is not all suitable as a thermometer liquid between 0°C and 8°C, but the above this. the peculiar property of water saves the lives of animals in very cold countries." Explain this statement as clearly as possible. (Pat. U.—1955) Art. 27

27. Describe Hope's Apparatus for determination of maximum density of water. Explain step by step how and why the reading of its thermometers changes as the temperature and density of water change. What importance does the fact that the maximum density of water is at 4°C bear in the economy of nature ? (Utkal U.—1964) Art. 27

28. Show how you would correct the reading of a barometer for the expansion of the mercury and the scale. Art. 29

(Utkal. U.—1954 : R. P. B.—1953, '62 ; Pat. U.—1966)

29. A barometer with a brass scale correct at 62°F reads 30 inches at 50°F. Find the reading of the barometer when corrected to 32°F. (Given that α for brass $= 18 \times 10^{-6}$ per °C and γ for mercury $= 18 \times 10^{-6}$ per °C.) Art. 29

Ans. 20·91 inches.

80 A mercury barometer with brass scale and correct at 0°C reads 76 69 cm. at 17°·8C. Reduce the reading to 0°C. (α for brass $= 19 \times 10^{-6}$: γ for mercury $= 18 \times 10^{-5}$). Art. 29

Ans. 76·47 cms.

31. A brass scale of barometer was correctly graduated at 15°C. At what temperature would the observed reading require no temperature correction ? (Coefficient of linear expansion of brass = 000019 ; coefficient of absolute expansion of mercury = ·00018). (R. P. B.—1966) Art. 29

Ans. 1°·8C (nearly).

CHAPTER IV

EXPANSION OF GASES

31. Expansion of Gases—Due to changes in temperature gases like solids and liquids change in volume but at a much greater rate. In considering the changes in volume of solids or liquids the effect of pressure is not taken into account, for even with a great change of pressure the change in their volumes is negligibly small. But in the case of gases a slight change in pressure produces an appreciable change in volume. To state the condition of a gas, it is necessary, therefore, to mention its *volume* (V), *pressure* (P) and *temperature* (T), A change in any one of these factors affects any other to a marked degree. Such changes are governed by the *Gas Laws* given below ;

(1) change of volume by change of pressure, the temperature remaining constant (*Boyle's Law*).

(2) change of volume by change of temperature, the pressure remaining constant (*Charles' Law*),

and (3) change of pressure by change of temperature, the volume remaining constant (*Law of Pressure*),

The first of these showing the relation between the volume (V) and the pressure (P) under constant temperature (T) is given by **Boyle's Law** which states that *at constant temperature, the volume of a given mass of* any *gas varies* inversely *as the pressure to which it is subjected*

Thus $V \propto \frac{1}{P}$, when t remains constant.

or, $PV = k$, where k is a constant, depending on the mass and temperature of the gas. This equation is called the *isothermal equation* of state. This has been dealt with in General Physics (Chap. XI).

If no heat is allowed to enter into, or leave, the mass of the gas during a change of volume, the temperature does not remain constant and the changes are then known as **adiabatic changes** of volume. Under an adiabatic condition the relation between pressure and volume is given by the equation :

$$PV^{\gamma} = \text{constant}, \qquad \ldots(31,1)$$

when γ is the ratio of the specific heat of the gas at constant pressure to that at constant volume.

32. Expansion of Gases at Constant Pressure—John Dalton first established in 1801 that all gases when heated expand equally, It was found by Charles that *at constant pressure the volume of a given mass of* any *gas increases for each degree centigrade rise of temperature by a constant fraction* ($\frac{1}{273}$) *of its volume at* $0°C$. This is known as **Charles' Law**. This relation was afterwards verified by Gay-Lussac and more accurately by Regnault and is sometimes called Gay-Lussac's Law.

If the volumes of a given mass of gas at 0°C be V_0 c.c. and if γ_p be the fractional increase of volume per °C, then for a rise of temperrture of t°C the increase in volume is $V_0\gamma_p t$. Hence the volume (V_t) of the gas at t°C is given by

$$V_t = V_0 + V_0\gamma_{pt} = V_0(1+\gamma_p t), \qquad \ldots(32.1)$$

$$\text{whence } \gamma_p = \frac{V_t - V_0}{V_0 t}. \qquad \ldots(32.2)$$

This constant fraction γ_p may, therefore, be taken as the *coefficient of cubical expansion of* any *gas at constant pressure* or simply as the *volume coefficient of the gas*. For real gases the volume of coefficient has been experimentally found to be very nearly $\frac{1}{273}$ or 0·00366.

$$\text{Hence from (32.1),} \qquad V_t = V_0\left(1+\frac{t}{273}\right). \qquad \ldots(32.3)$$

The above relation is similar to that in the case of solids and liquids. But the following important points of difference are to be noted—

(1) As a change of pressure considerable affects the volume of a gas, the pressure must be kept constant.

(2) The coefficient of expansion for a gas is much greater than that for a solid or a liquid.

(3) The value of this coefficient is approximately the *same* for *all gases* and *not* appreciably different from one another as in the case of solids and liquids.

(4) As the expansion in the case of a gas for a slight change of temperature is much bigger than that in the case of a solid or a liquid, the volume at 0°C must always be taken, in applying the formula for expansion,

It is interesting to study the equation (32.2), for given a fixed mass of gas, its volume V_0 at 0°C is constant. If the temperature t of the gas is continuously increased, its volume V_t also continuously increases. The relation between V_t and t is that of a *straight line*, if their values are plotted on a graph paper.

33. Determination of the Coefficient of Expansion of a Gas at Constant Pressure The coefficient of expansion of a gas at constant pressure can be determined and hence Charles' Law can verified by Constant Pressure Thermometers of different forms.

(1) Constant Pressure Thermometer of a simple from—This consists of a glass tube about 50 cm. long and with a uniform bore of about 1 mm. in diameter. Dry air is passed through the tube for some time, after which one of the ends is sealed with a blow-pipe flame. The tube is then heated gently with the open end dipping in pure dry mercury. As the tube cools, the air inside contracts and small pellet H of mercuury (Fig. 27) is sucked in ; this serves as an index. The amount of air expelled from inside should be such the mercury pellet stands sbout midway along the tube at the room temperature.

A temperature T is tied along the body of the tube. The tube A is now held inside a wide jacket tube G of glass, with its open end projecting out of the jacket. The two ends of the jacket are also closed by stoppers and through each one there passes a short glass tube in order to serve as an inlet or outlet tube.

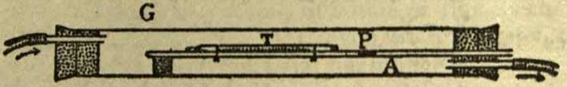


Fig. 27.

Ice-cold water is at first circulated through the jacket till the thermometer T records a constant temperature of 0°C. The gas within the glass tube also attains the temperature of 0°C and its volume becomes constant : so the pellet P comes to a steady position. After waiting for some time the distance of the inner end of the pellet from the closed end is measured. Let the length of the air column at this stage be l' cm. The water is let out and steam is now passed through the jacket : the temperature rises and ultimately becomes constant. After some time the position of the inner end of the pellet is noted again ; let the length of the air column be now l' cm., and the temperature t°C.

If the internal cross-section of the tube be a, the volume of the enclosed air at 0°C and t°C are respectively la and $l'a$ the small expansion of the glass tube being neglected. If in course of the experiment the inclination of the tube be kept fixed, its mouth being always open to the atmosphere, the pressure, to which that air inside is subjected, remains the same during the period of experiment.

Hence the coefficient of cubical expansion (γ_p) of the air at constant pressure is given by

$$\gamma_p = \frac{l'a - la}{la(t-0)}, \text{ i.e., } \gamma_p = \frac{l'-l}{lt},$$

At all constant pressures the mean value of the coefficient γ_p is found to be the same By any other gas the air in the tube may be replaced and the coefficient of expansion for the gas may be determiend as above. The coefficient of expansion as found above is, in reality, the coefficient of apparent expansion of the gas with respect to glass. But as the expansion of the glass is very small in comparision with that of the gas, it is not ordinarily necessary to apply correction for that.

(2) **Constant Pressure Air Thermometer** (*Regnaults apparatus*)—This consists of a U-tube AB (Fig. 28) with one limb graduated and ending in the bulb A, while the limb B is open to atmosphere. At the bottom of the U-tube is attached a short tube having a stop-cock G at its end. The U-tube is placed with in a wide glass jacket J, the bottom of which is closed by a rubber cork ; through this the short tube just mentioned passes. There is a copper tube S bent in the form of an inverted U passing through the cork with both ends outside.

The jacket is filled with water so as to keep the bulb immersed.

Close to the bulb, a thermometer T is suspended in the water. There is also a stirrer placed inside the jacket. Sulphuric acid is housed in the lower part of AB, while the bulb contains some air, the volume of which can be directly read from the graduations on the stem attached to the bulb. By pouring in more of the liquid through the open limb or by letting out some through the opening C the liquid in both the limbs can always be brought to the same level. The pressure of the enclosed air in A is then equal to the atmospheric pressure at the time.

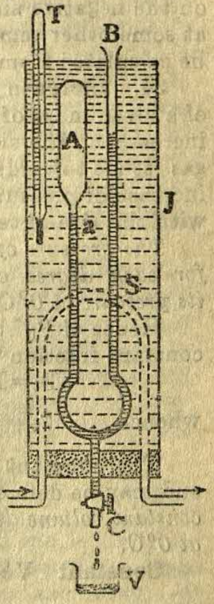


Fig. 28.

At the room temperature t_1 as recorded by the thermometer T sulphuric acid in the U-tube is brought to the same level and the volume of air in A is noted. Let the volume at the room temperature be V_1. Steam is now passed through the copper tube S which heats water in the jacket. To maintain uniformity of temperature water is kept constantly stirred. With the temperature rising, the air expands and forces down the liquid which, therefore, rises in the other limb. By regulating the steam, temperature of water at some higher value is maintained constant for some time, during which the air within the bulb acquires the temperature of hot water. The stop-cock is opened and some liquid is let out, till the level again becomes the same in both the limbs, The volume of the air is noted. Let the volume be V_2, and its temperature t_2.

Then $V_1 = V_0(1+\gamma_p t_1)$ and $V_2 = V_0(1+\gamma_p t_2)$

$$\therefore \frac{V_2}{V_1} = \frac{1+\gamma_p t_2}{1+\gamma_p t_1} \text{ or } \gamma_p = \frac{V_2 - V_1}{V_1 t_2 - V_2 t_1}. \qquad \ldots(33.2)$$

Thus by knowing V_1, V_2, t_1 and t_2 the value of γ_p can be calculated, For greater accuracy in result. the temperature of the bath is increased at successive steps and the corresponding volumes are noted. By taking temperature as the abscissa and the corresponding volume as the ordinate, we plot (fig. 29) a graph, which is readily found to ba a *straight line* That the graph should be a right line is evident from a mathematical standpoint as well. In Fig. 29 the straight line (such as AB) represents the

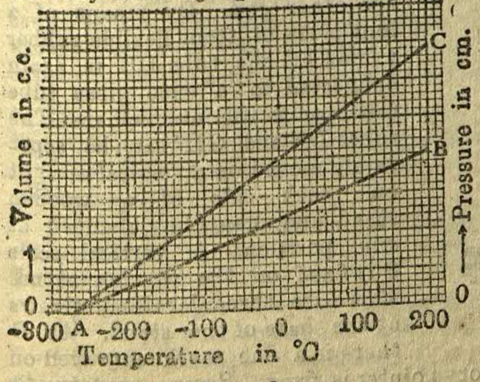


Fig. 29.

relation between the volumes of a given mass of a gas and the corresponding temperatures. The stright line is, of course, produced much on the negative side of the temperature. The volume at 0°C and that at some other temperature t^0C are found from the graph and so γ_p can be calculated from equation (32.2).

34. Increase of Pressure at Constant Volume—If the volume of a given mass of a gas is kept constant while it is heated, its pressure increases. The simple reason is that due to a rise of temperature the gas tends towards expansion which, on being prevented, results in an increase of pressure. It is found that pressure increases uniformly with the rise of temperature and obeys the following law :

The pressure of given mass of any gas at constant volume increases for each degree centigrade rise of temperature by a constant fraction of its pressure at 0° C.

Hence if P_0 be the pressure at 0^0C; P_t that at t^0C, and γ_v the constant fraction, the increase in pressure is $P_0\gamma_v t$ and so

$$P_t = P_0 + P_0\gamma_v t = P_0(1+\gamma_v t),$$

whence
$$\gamma_v = \frac{P_t - P_0}{P_0 t}.$$

The constant fraction is called the **pressure coefficient of a gas** and may be defined as *the ratio of the increase of pressure of the gas at constant volume due to a rise of temperature through 1°C to its pressure at 0°C.*

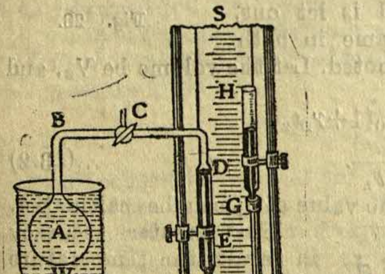


Fig. 80

Constant Volume Air Thermometer—The relation between the temperature and the pressure of a gas at conssant volume can be studied by an apparatus known as the *constant volume air thermometer.* It consists of a large glass bulb A (Fig. 30) at the end of a thick-walled narrow tube BCD, bent twice at right angles and enlarged into a wider tube DE at the other end. By means of a long piece of stout flexible rubber tubing FG the tube DE is again connected to a wide glass tube GH open to the atmosphere. The tubes DE and GH can be separately moved on a vertical upright on each side of a wooden stand and can be clamped at *any* position. A vertical scale S is fixed on the wooden stand. There are three levelling screws at the base of the stand. At D there is a fine mark engraved on the tube : or sometimes an ivory pointer is fixed. Some apparatus is provided with a three-way tap at C, through which the bulb A can be exhausted of air and filled with any other gas. A part of each of the tubes DE and GH and the whole of the rubber tube are filled with pure dry mercury, while the bulb A and the narrow tube up to D contain dry air or any other gas.

The bulb A is now kept immersed into a bath of ice-cold water and the tube DE is clamped at any position. By raising or lowering the tube GH the level of the mercury in DE is adjusted exactly to the mark D. The difference of mercury levels in the two tubes is noted. By adding to this the difference in the barometric pressure at the time of experiment the pressure of the enclosed air is obtained. Denote the pressure by p_0.

The water-bath is now heated and is kept stirred well. The temperature is raised to some higher value which is ready by a thermometer held in the bath. By adjusting the flame the temperature of the bath is kept constant for some time. During this period the gas attains the temperature of the bath, The pressure of the enclosed air is then read. Let the pressure at some higher temperature $t^\circ C$ be p. Then the value of the pressure coefficient is calculated from the equation :

$$\gamma_v = \frac{p - p_0}{p_0 t} \qquad \text{...(34.2)}$$

For air the value of γ_v is found to be 0·00366. In the laboratory, temperature of the enclosed gas is raised usually from the room temperature in steps and the corresponding pressures are noted. A *graph* is plotted by taking the temperature as the abscissa and the corresponding pressure as the ordinate. The graph is found to be a *straight line* (Fig. 29, AC), This shows that *the pressure increases uniformly with temperature* From the graph the pressure p_0 at 0°C and that at some convenient higher temperature may be obtained. From these the value of the pressure coefficient is more accurately calculated from the relation (34.2).

Advantages of a Gas Thermometer :—

(*i*) As gases are more expansible, a gas thermometer is much *more sensitive* to temperature variation than a liquid thermometer. Further, except when very great accuracy is demanded, the effect due to the expansion of the containing vessel need not be taken into consideration.

(*ii*) Under the same condition and over a large range of temperature the rate of expansion of all gases is practically the *same.*

(*iii*) The *expansion* of all gases is very approximately *uniform* and *regular*.

(*iv*) A gas thermometer can be used for a *much wider* range of temperatures so as to record very low as well as very high temperatures.

Disadvantages of a Gas Thermometer :—

(i) A gas thermometer is bulky and so is not easily portable.

(ii) As a change of atmospheric pressure affects the readings of the instrument, a barometer also is to be carried along with the gas thermometer.

(iii) It is too large to be used in calorimetric work.

(iv) The instrument cannot be furnished with fixed scales, inasmuch as the atmospheric pressure varies from time to time.

(v) For quick changes of temperature a gas thermometer is insensitive.

Examples :

1. A uniform vertical glass tube, open at the top and closed at the bottom, contains air and a pellet of mercury 3·0 cm. long. The lower and of the pellet is 30·5 cm. above the bottom of the tube when the tube is at temperature of 5°C. How far will the pellet rise, if the tube is heated to 100°C ? [Utkal U.—1952, '54]

Ans. Since the volume coefficient of air is considerably larger than the cubical expansion of glass, the latter may be neglected in comparison with the former. Hence the bore of the glass tube may be assumed not to change with temperature insofar as the problem is concerned and so the length of the air column within the glass tube at any temperature is proportional to the volume of air at that temperature. Further the pressure on the column is due to the atmospheric pressure and the pellet of mercury. Consequently, if the inclination of the tube is not changed during the experiment, the *pressure* on the air column remains *constant*, whatever might be the inclination of the tube and the length of the mercury pellet.

Let the length of the air column at 0°C be $l_\circ$ cm. Then the volume of air column at 0°C is $kl_\circ$ c.c. where k is a constant. At 5°C the volume is $(k\times 30{\cdot}5)$ c.c. Then we have

$$k\times 30{\cdot}5=kl_\circ\times(1+{\cdot}00366\times 5)\qquad\text{whence } l_\circ=29{\cdot}36.$$

If at 100°C the length of the column be l cm., we must have

$$kl=k\times 29{\cdot}96\times(1+{\cdot}00366\times 100),\qquad\text{whence } l=40{\cdot}39\text{ cm.}$$

2. What is the temperrture of a gas whose pressure is 136·63 cm. of Hg. if its pressure at 0°C is 100 cm. of Hg. the volume remaining constant? Given the pressure coefficient of the gas = 0·003663. [V. U.—1954]

Ans. Here P_1 = pressure at t°C = 136·63 cm., $P_\circ$ = pressure at 0°C = 100 cm. and γ = coefficient of pressure increase of the gas = ·003663.

The usual formula $P_1=P_\circ(1+\gamma t)$ leads to $100(1+{\cdot}003663\times t)=136{\cdot}63$,
or, $1+{\cdot}003663\times t=1{\cdot}3663$, or ${\cdot}003663\times t=3663$, whence $t=100$

Hence the temperature is 100°C.

3. To a town ten million cubic feet of a gas are supplied per week under a pressure of 3 inches of water over the atmospheric pressure at 5 shillings per 1000 cubic feet. When the barometer reads 31 inches, the gas company can make no profit at all. Find the profit per week when the average height of the barometer is 29 inches. (Density of mercury = 13·6)

Ans. For the decrease of pressure let the increase in volume of 10^7 cu. ft. of the gas be v. cu. ft. Then the company can save this v cu. ft. of gas per week.

The pressure due to 3 inches of water is equal to that if $\frac{3}{13{\cdot}6}$ inch of mercury.

Utilising the relation PV = P′V′, we find

$$\left(31+\frac{3}{1{\cdot}36}\right)\times 10^7=\left(29+\frac{3}{13{\cdot}6}\right)(10^7+v),\text{ whence } v=685\times 10^3,$$

Hence the profit made by the company $=\left(\frac{6{\cdot}185\times 10^6}{10_3}\times 5\right)$ s. = £171.5s.

35. Absolute Scale of Temperature—The volume of a given mass of a gas at any temperature can be obtained from Charles' Law which gives the equation :

$$V_t = V_0\left(1+\frac{t}{273}\right).$$

For instance. the volume at -10°C is given by

$$V_{-10} = V_0\left(1-\frac{10}{273}\right).$$

According to this equation it is interesting to note is that the apparent volume at -273°C is given by $V_{-273} = V_0\left(1-\frac{273}{273}\right)=0.$

Evidently -273°C is the lowest temperature that can be theoretically indicated by the gas thermometer. For, at this temperature the volume of any gas obeying Charles' Law appears to occupy no space. Indeed all known gases liquefy, and even solidify, before this temperature is reached. Further. the pressure coefficient of a gas having also a value $\frac{1}{273}$, we can show from (34.1) that at -273°C a gas would have *no pressure*. The same conclusion may be derived from the volume temperature (AB) and the pressure-temperature (AC) graphs (Fig. 29). Both the straight lines when produced to meet the abscissa representing zero volume and zero pressure lines meet at the point A which is -273°C. We are thus led to consider -273°C as the lowest temperature conceivable and so it is called the **absolute zero** of the gas thermometer.

If we now have a scale of temperature starting from the absolute zero, *i.e.*, from -273°C, as its zero and having each degree equal to that of the Centigrade scale, that scale is called the **absolute scale of temperature**. Temperature when reckoned on the absolute scale are termed *absolute temperatures* (in degree absolute), According to the scale the melting point of ice (0°C) is 273° A and the boiling point of water (100°C) under normal pressure is 373°A. In general, if t° of the Centigrade scale corresponds to T° of the absolute scale, then

$$T^\circ \mathrm{A} = t^\circ \mathrm{C} + 273,$$

i.e., *Absolute Scale reading = Centigrade reading* + 273.

As Kelvin defined the absolute scale, the temperatures on the absolute scale are sometimes termed Degrees Kelvin in place of degrees absolute ; thus $100^\circ\mathrm{C} = 373^\circ\mathrm{K}$. To be more accurate, absolute zero is 273·18 degrees below the Centigade zero.

36. Charles' Law and the Law of Pressure expressed in different forms—From Charles' Law, if V and V′ are the volumes of a gas at *Centigrade temperatures t* and *t′*, then

$$V = V_0\left(1+\frac{t}{273}\right) \text{ and } V' = V_0\left(1+\frac{t'}{273}\right)$$

$$\therefore \quad \frac{V}{V'} = \frac{t+273}{t'+273} = \frac{T}{T'}, \text{ or } \frac{V}{T} = \frac{V'}{T'} \qquad \ldots(36.1)$$

where T and T' are absolute temperatures respectively corresponding to $t°$C and t'C.

Hence $\frac{V}{T}$ *is constant, when* P *is constant.*

From this relation Charles's Law may be stated as follows ; *The volume of a given mass of a gas at constant pressure is* directly *proportional to the* **absolute** *temperature of the gas.*

Again, the relation $P=P_0\left(1+\frac{t}{273}\right)$ readily gives

$$\frac{P}{P'}=\frac{t+273}{t+273}=\frac{T}{T'} \qquad \ldots(36.2)$$

Hence $\frac{P}{T}$ *is constant, when V is constant.*

In other words, *the pressure of a given mass of a gas at constant volume is* directly *proportional to the* **absolute** *temperature of the gas.*

Gas Laws combined—A simple general relation can be established between the pressure, volume and temperature of a given mass of a gas. Let V be the volume of a given mass of a gas at pressure P and absolute temperature T. We are to find the volume V' of the same mass at pressure P' and absolute temperature T'. First, let the pressure change from P to P', the temperature remaining constant. It the volume thereby changes to V_1, then by Boyle's Law

$$PV=P'V_1, \quad \text{or } V_1=\frac{PV}{P'}.$$

Pressure remaining constant at P', let the temperature now change from T to T' If in consequence the volume changes from V_1 to V', Charles' Law gives

$$\frac{V_1}{T}=\frac{V'}{T'}, \text{ or } \frac{PV}{P'T}=\frac{V'}{T'}$$

Therefore, $$\frac{PV}{T}=\frac{P'V'}{T'} \qquad \ldots(36.3)$$

In other words, $$\frac{PV}{T}=\text{a constant. say } k, \text{ i.e., } PV=kT. \qquad \ldots(36.4)$$

Thus *the product of the pressure and the volume of a given mass of a gas varies* directly *as its* absolute *temperature.*

Equation (36.4) is referred to as the *gas equation* or the **equation of state**, for by knowing any two of the three quantities P, V and T, the third can be found and the state of the gas can be completely determined. The value of the constant k depends on the mass of the gas taken. If a gramme molecule of a gas be taken, the value of k is then the same for all gases. It is then called the *universal gas constant* and is denoted by R. Hence for a gramme-molecule (usually called the *mole*) of a gas,

$$PV=RT, \qquad \ldots(36.5)$$

where R is a constant.

Let M be the mass of the gas occupying a volume V at pressure P and at absolute temperature T. Under the same conditions of temperature and pressure the volume of a gramme-molecule of the gas having a molecular/weight m is $\frac{mV}{M}$.

Consequently, $$P\frac{mV}{M}=RT, \text{ whence } PV=\frac{M}{m}RT=nRT, \qquad \ldots(36.6)$$

when n is the ratio of the mass of the gas under examination to its gramme-molecular weight.

As the volume of a gas is affected by the pressure and temperature, a gas is always to be referred to at a definite temperature and pressure. A volume of a gas at 0°C and at a pressure of 760 mm. of mercury is said to be at **normal temperature and pressure (N.T.P.).**

To find the value of the gas constant (R).—One litre of hydrogen at N.T.P. weighs 0 0896 gm. Find the value of R. considering one gramme-molecule o the gas. [Mys, U.—1952 ; R P. B.—1954]

Solution. The volume of 0 0896 gm. of hydrogen at N.T.P. is 1000 c.c,

$\therefore$ „ „ „ 2 gm. „ „ is $\frac{2000}{0\ 0896}$ c.c. $=22321$ c.c.

The normal atmospheric pressure is $76\times13{\cdot}6\times981$.

By reason of the formula $PV=RT$, we get

$$(75\times13{\cdot}6\times981)\times22321=R\times273.$$

whence $$R=8{\cdot}3\times10^7 \text{ ergs per degree Centigrade.}$$

37. Variation of the density of a gas—Led ρ be the density, and V the volume, of a mass of a gas at a pressure P and absolute temperature T, Let ρ' be the density, and V′ the volume, of the same mass of the gas at a pressure P′ and absoulte temperature T′.

As the density of a gas at a given pressure and temperature varies inversely as its volume, we have from equation (36,4)

$$\frac{P}{\rho T}=\frac{P'}{\rho' T'} \qquad \ldots(37.1)$$

If the temperature suffers no change. $T=T'$, so that (37.1) reduces to

$$\frac{P}{\rho}=\frac{P'}{\rho'}. \qquad \ldots(37.2)$$

Hence *the density of a gas at a constant temperature varies* directly *as the pressure.*

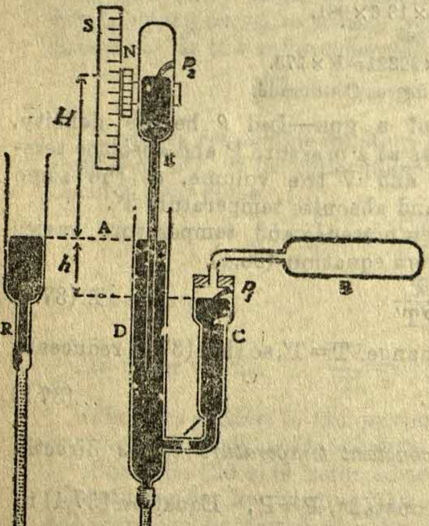
Again, if the pressure remains constant, $P=P'$, Equation (37,1) in consequence boils down to

$$\frac{1}{\rho T}=\frac{1}{\rho' T'}, \text{ or } \rho T=\rho' T'. \qquad \ldots(37.3)$$

Hence *the density of a gas at constant pressure varies* inversely *as the absolute temperature.*

***38. Standard Gas Thermometer**—An ideal thermometer is one in which equal rise of temperature would be indicated by graduations of its scale by equal increment. This can only be theoretically attained by the absolute Thermo-dynamic Scale as proposed by Lord Kelvin (Art. 35). No property of a substance varies *exactly informly* with temperature ; as a result. if any meterial property be utilised for thermometric measurement, equal changes of magnitude of that property will *not* indicate *exactly equal* changes of temperature. The nearest approach to the Absolute Thermo-dynamic scale is obtained by employing the volume coefficient or the pressure coefficient of a real gas as a thermometric substance. Hence so as to indicate the temperature with the least possible correction either a constant pressure or else a constant volume gas thermometer is to be used. The constant volume thermometer is, however, preferable, as it is susceptible to easier adjustment and as the volume of excess of a gas within the connecting tube, which always remains outside the temperature bath, is very small and constant. That was why in 1887 the International Committee of Weights and Measures decided to adopt as a practical standard the centigrade scale of a constant volume hydrogen thermometer described hereunder.

Constant Volume Hydrogen Thermometer—The constant volume hydrogen thermometer was originally designed by Chappuis in 1887. Figure 31 represents a hydrogen thermometer of a simpler form which consists of a bulb B about 100 cm. long and of capacity 1 litre. The bulb is held horizontally and. can be placed in any temperature bath To withstand a high temperature and pressure this bulb in the original form was made of platinum-iridium. By a narrow bore the bulb is connected to a wider tube C to which it is sealed air-tight. A little ivory pointer p_1 projects vertically at the top of C. There is another vertical side tube D havfng communication with C at the lower part. A barometer tube E is kept above the open top of D and is provided with an identical pointer p_2 near its top. The combination of the tubes C and D together with the barometer forms a manometer with which the gas pressure of the bulb can be measured. By an air-tight pressure tubing a mercury reservoir R is connected to the lowest point of the manometer. The barometer and the reservoir can be slided up and down a vertical metre scale and fixed at any



Fig, 31
Constant Volume Thermometer

position. The heights of the mercury columns in C, E and R can be accurately read with verniers attached to the main scale or with a cathetometer.

So as to attain a temperature of 0°C the bulb B is kept within a temperature bath of melting ice for a considerable time. Then the reservoir R is raised or lowered until the mercury surface in C touches the pointer p_1. The barometer tube is then adjusted so that mercury surface in it touches p_2. More than one adjustment may be necessary to make the mercury surface in the two tubes exactly touch the pointers. This being done, the volume of the enclosed gas becomes always constant and the pressure exerted by the gas is the sum of the mercury level difference in R and C plus the barometric pressure. Consequently the pressure P_0 is the difference of vertical heights of p_2 and p_1, which may be determined with reference to the vertical scale attached.

Next the bulb is kept in the steam of boiling water. When the temperature becomes steady, the difference of heights of p_2 and p_1 is found. Thus with proper correction of the boiling point the pressure of the gas at 100°C is determined. Call this prssure P_{100}. If now the bulb be kept in an unknown temperature bath at t°C and if when the temperature becomes steady, the pressure recorded be P_t, it is clear that

$$\frac{t}{100}=\frac{P_t-P_0}{P_{100}-P_0}, \text{ whence } t=100\times\frac{P_t-P_0}{P_{100}-P_0} \qquad \ldots(38.1$$

Within the bulb hydrogen is kept such that at 0°C the pressure exerted by it is one metre of mercury. A hydrogen thermometer may be used for temperatures ranging between −200°C and 500°C. Above 500°C hydrogen diffuses through platinum and attacks glass or porcelein. For temperatures lying between 500°C and 1500°C nitrogen is used in place of hydrogen. Hydrogen is replaced by helium for temperatures below −200°C

Examples.

1. Collected over mercury in a graduated tube, a quantity of gas is found to occupy 25 c.c. at 27°C. The level of the mercury inside stands 15 cm. higher than the level outside, while the barometer stands at 75 cm. Find the volume that the mass of the gas would occupy at a pressure of 74·5 cm. of mercury and at a temperature of 32°C.

Ans. The initial pressure P of the gas is $=(75-15)$ cm., $=60$ cm. of mercury.

Let the required volume be V′ c.c. From the relation $\frac{PV}{T}=\frac{P'V'}{T'}$ we get

$$\frac{74{\cdot}5\times V}{32+273}=\frac{60\times 24}{27+273}, \text{ whence the required volume } V'=20{\cdot}5 \text{ c.c.}$$

2. At 22°C and at a pressure of 74 cm. the volume of a given mass of gas was found to be 54·02 c.c. On cooling to 0°C the volume became 49·3 c.c., the pressure having risen to 75 cm. Find the coefficient of expansion of the gas.

Ans. Denote the coefficient of expansion of the gas by γ.

The volume of the gas at 0°C and 75 cm. pressure $=49{\cdot}3$ c.c.

From Boyle's Law the volume at 0°C and 74 cm. pressure $=\frac{49{\cdot}3\times 75}{74}$ c.c.

$=49{\cdot}97$ c.c.

So by Charles' Law the volume at 22°C and under 74 cm. $=49{\cdot}97\times(1+22\gamma)$. Hence by the problem $49{\cdot}97\times(1+22\gamma)=54{\cdot}02$, whence $\gamma=0{\cdot}00368$.

3. The internal volume of a glass vessel is 1 litre at 0°C. It is filled with hydrogen at N. T. P. and is then heated to 27°C. Find the mass of hydrogen that would escape, if the flask is now opened under a pressure of 74 cm. (1 litre of hydrogen at N.T.P. weighs 0·09.gm.)

Ans. Suppose the volume, that 1 litre of hydrogen at N. T. P. would occupy at 27°C and 74 cm. pressure, is V litres.

Employing Charles' and Boyle's Laws, we get

$$\frac{74\times \mathrm{V}}{27+273}=\frac{76\times 1}{273}\text{, whence } \mathrm{V}=\frac{300\times 76}{273\times 74}.$$

So the volume of hydrogen that escapes $=\left(\dfrac{300\times 76}{273\times 74}-1\right)$ litres.,

At N. T. P. the above volume becomes $=\dfrac{2598}{22800}$ litres.

$\therefore$ the required mass of hydrogen $=\left(\dfrac{2598}{22800}\times 0{\cdot}09\right)$gm. $=0{\cdot}0102$ gm.

4. What is the height of a barometer when a milligramme of air at 27°C occupies a volume of 25 c.c. in a tube over mercury, the mercury standing 70 cm. higher inside the tube than outside? (1 litre of dry air at N. T. P. weighs 1·293 gr.)

Ans. Let the required height of the barometer $=h$ cm.

$\therefore$ the pressure of air in the tube over mercury $=(h-70)$ cm.

Also the vol. of 1 mg. of air at this pressure and 27°C is 25 c.c.

Again, at N. T. P. the volume of 1·293 gm. of air is 1000 c.c.,

so that at N. T. P. the vol. of 1 gm. of air $=\dfrac{1000}{1293}$ c.c.

Hence from the relation $\dfrac{\mathrm{PV}}{\mathrm{T}}=\dfrac{\mathrm{P'V'}}{\mathrm{T'}}$ we get $\dfrac{(h-70)\times 35}{27+273}=\dfrac{76\times 1000}{273\times 1293}$,

whence $h-70=2{\cdot}58$, so that the desired height h is $72{\cdot}58$ cm.

5. The density of dry air at N. T. P. is 0·001293 gm. per c.c. At what temperature will a litre of dry air weigh 1 gm. at a pressure of 75 cm.?

Ans. Denote the required temperature by t°C and the density of dry air at t°C at a pressure of 75 cm. by ρ.

Then by virtue of the relation $\dfrac{\mathrm{P}}{\rho\mathrm{T}}=\dfrac{\mathrm{P'}}{\rho'\mathrm{T'}}$, we have

$$\frac{76}{0{\cdot}001293\times 273}=\frac{75}{\rho(t+273)}\text{, whence } \rho=\frac{75\times 0{\cdot}001293\times 273}{76(t+273)}$$

Now since by definition the product of density and volume of a body is its mass, which is 1 litre, *i.e.*, 1000 c.c., we find:

$$\frac{75\times 0{\cdot}001293\times 273\times 1000}{76(t+273)}=1\text{, whence } t=75{\cdot}35.$$

Hence the required temperature is 75°·35C.

***39. Kinetic Theory of Gases**—A mass of a gas consists of a vast assemblage of molecules, which are extremely small in dimension. An idea of smallness of a molecule may be made by imagining the molecules to be arranged side by side just as a row of balls of equal size. Were it so, a hundred million molecules could be packed in a length of one centimetre. In fact, in proportion to the volume occupied by a quantity of a gas, the volume actually occupied by the molecules is so small that for a free and random movement of any molecule there is enough of space surrounding it. The space between any to molecules is called *intermolecular space* and it is in the these spaces that the molecules keep on moving continuously in different directions. Figure 32 represents a box having walls A, B and C and containing a number of molecules as shown by black dots, They are moving in various directions shown by arrowheads. It is apparent that due to such motfons some might collide with each other, while some others might strike against the walls.

However tiny they might be, molecules possess some mass and hence a certain amount of inertia. Hence a molecule continues to move in a straight line until it knocks against the walls of the containing vessel or with another molecule. Is is only when a collision occurs that there is a deflection of the path of a molecule. Against the walls of the containing vessel a large number of knocks thus occur every second. Such continual hailstorm of particles tends to push the wall out. This results in a *steady pressure*, which is the pressure exerted by the gas. By virtue of being in continuous motion the kinetic energy possessed by the molecules manifests itself as the temperature of the gas. Such an idea regarding the mechanism of gas pressure was advanced by Joule in 1853. which was later on modified by Maxwell'

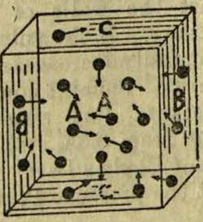


Fig. 32

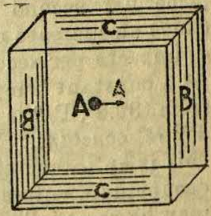


Fig. 33

Suppose there are N molecules enclosed in a cubical box, the length of each edge being a. For the sake of simplicity assume that all the molecules have an equal speed c and that each of them behaves as a perfectly elastic sphere of mass m. Since the number of molecules is extremely large, we can imagine that at any instant one-third of the total number of molecules moves perpendicular to the pair of faces (A,A), one-third to that of the faces (B,B) and one-third to that of the faces (C,C). Take the case of a molecule which is moving at right angles to the pair of faces (B.B) (Fig. 33). Starting from any wall of the pair, it must travel a distance $2a$ before impinging on the same wall, hence the time taken between any two consecutive hits on the same wall is $2a/c$ and so the number of hits per second on any wall is $c/2a$.

Again, since after a normal impact on any wall a molecule retraces its path, its velocity changes from $+c$ to $-c$ Thus, considering the impact to be perfectly elastic, the change in velocity is $2c$ and the change of momentum of any molecule for one impact is $2mc$, In unit time the change of momentum for a molecule is, therefore.

$$=\frac{c}{2a}\times 2mc=\frac{mc^2}{a}.$$

Now by assumption $\frac{1}{3}$N molecules move prependicular to any pair of faces. Therefore such molecules the rate of change of momentum is $\frac{1}{3}\mathrm{N}.\frac{mc^2}{a}$, which represents the force on an area a^2,

Thus the pressure $\mathrm{P}=\frac{\text{force}}{\text{area}}=\frac{1}{3}.\ \frac{\mathrm{N}mc^2}{a^3}=\frac{1}{3}.\ \frac{\mathrm{N}mc^2}{\mathrm{V}}$,
where V represents the volume of the gas.

$$\therefore \qquad \mathrm{PV}=\frac{1}{3}.\ \mathrm{N}mc^2. \qquad \text{...(39.1)}$$

But V c.c. of the gas contains N molecules, each of mass m. Thus we have $\frac{\mathrm{N}m}{\mathrm{V}}=\rho$, where ρ is the density of the gas.

we have $\frac{\mathrm{N}m}{\mathrm{V}}=\rho$, where ρ is the density of the gas.

$$\text{In other words.} \qquad \mathrm{P}=\frac{1}{3}\rho c^2. \qquad \text{...(32.2)}$$

Boyle's Law and Kinetic Theory of Gases—If the mass of a gas be compressed into a smaller volume, while the average kinetic energy of the molecules remains unchanged, a large number of impacts take place per second against the walls of the contaning vessel and cause an increase of pressure. If the volume is increased, the number of impacts per second naturally decreasesand sothe pressure decreases. At a constant temperature c^2 remains constant and so, as is evident from (39.2), P/ρ is a constant. But since ρ varies as 1/V, the product PV is *constant*. Viewed from the standpoint of Kinetic Theory, this is an interpretation of Boyle's Law If M be the mass of a gas occupying a volume V at the pressure P, then since $\rho=\mathrm{M/V}$, (39.2) may be remodelled as

$$\mathrm{PV}=\frac{1}{3}\mathrm{M}c^2. \qquad \text{..(39.3)}$$

Physical Interpretation of the Absolute Zero—If there are N molecules, each of mass m contained in a volume V of a gas and if c^2 be their mean square velocity, mean *kinetic energy* of this volume of the gas is $=\frac{1}{2}.\ \mathrm{N}mc^2=\frac{1}{2}\mathrm{M}c^2$. But from (39.2) and (39,3) it follows that:

$$\text{Kinetic Eenery}=\frac{1}{2}\,\mathrm{M}c^2=\frac{3}{2}\mathrm{PV}=\frac{3}{2}n\mathrm{RT}. \qquad \text{...(39.4)}$$

Hence the K, E, of a given mass of a gas is proportional to the absolute temperature. Thus at the absolute zero degree the K. E. possessed by the molecules tends to vanish. This may be taken as the *physical interpretation* of the Absolnte Zero. (For further evidences in favour of Kinetic Theory vide, HEAT, Appendix F, at the end of Chapter IV.)

EXERCISES ON CHAPTER IV

Reference

Art. 32 1. A flask with a capacity of 500 c.c. is heated to 87°C at atmospheric pressure and then corked up. It is then inverted and immersed in water at 24°C. The cork is removed. What mass of water will enter the flask. if the pressure remains the same as above ? (Density of water at 24°C is 0·9973).

Ans. 87·26 gm.

Art. 33 2. Describe an experiment to find the coefficient of expansion of a gas at constant pressure.
(C. U—1940, '49 ; Pat. U—1932 ; Nag. U.—1951)

Art. 36 3. Enunciate Charles' Law. Describe a simple experiment for the verification of the law. (Cal., Pre. U.—1961)

Arts 33 & 34 4. Explain how thermal expansion of air can be utilised as a convenient means of measuring temperature. What are the advantages and disadvantages of a gas thermometer ? (Gau. U.—1951)

Art. 34 5. Describe the constant volume air thermometer and explain how you will use it to find the temperature.
(P. U.—1951 ; U. P. B.—1954 ; R. P. B.—1944, '46)

Arts. 34 & 35 6. State and explain the law of the variation of pressure of a gas with its temperature at constant volume. Hence define the absolute zero of temperature. Describe an experiment to verify the law. (V. U—1954)

Art. 34. 7. Describe a form of an air thermometer and explain how this could be used to determine the melting point of napthalene. Why has gas thermometer been chosen as standard ? (R. B.—1941)

Art. 34 8. The pressure in the bulb of a constant volume air thermometer is 74 cm. at 0°C, 101·2 cm. at 100°C and 52·8 cm. when the bulb is surrounded by solid carbon dioxide. Calculate the temperature of solid carbon dioxide. (Del, H. S.—1949)

Ans. —78°4C.

Art. 35 9. What is meant by absolute temperature ? Find the value of absolute zero on the Fahrenheit scale.
(Gau. U.—1951 ; C. U.—1938, '49)

Ans. —449°·4F.

Art. 36 10. Establish PV=RT for a perfect gas, (E. P, U.—1953) ;
C. U.—1950 ; U. P. B,—1951, '53, Mys. U.—1952 ;
Raj. U.—1949, '55 ; M. B B.—1952 ; Nag. U.—1952, '53)

Art. 36 11. Calculate the value of the gas constant for carbon dioxide, given that 22·4 litres of the gas at N. T. P. weighs 44 gms. (U. P. B.—1947 ; R. P. B.—1945)

Ans. $8{\cdot}32 \times 10^7$ ergs/°C.

Art. 36 12. A gas at 18°C has its temperature raised, so that its volume is doublod, the pressure remaining constant. What is its final temperature ?

Ans. 299°C.

Art. 34 13. The pressure of air in a vessel is 10^6 dynes per sq. cm. at 30°C. Find in centigrade degrees the value of the temperature at which the pressure will be doubled, given that the pressure of air increases by ·0036 of its value at 0°C per degree rise of temperature, (Ignore the increase in volume of the vessel). (V. U.—1952)

Ans. 333°C.

Reference

Art. 36

14. A bicycle is standing in shade where the temperature is 80°F. It is then taken out into the sun where the temperature is 140°F. Find the percentage increase of pressure within the tyre.

Ans. 9·9%.

Arts. 36

15. A litre of hydrogen at N. T. P. weighs 0·9 gm. What is the weight of a litre of this gas at 27°C and 76 cm. pressure? (E. P. U.—1952)

Ans. 0·8 gm.

Art. 36

16. One litre of air at N. T. P. weighs 1·293 gm. Find the value of R for air ($g=980$ cm./sec.^{2} ; density of mercury $=13{\cdot}6$). (U. P. B.—1951)

Ans. $8{\cdot}03\times10^7$ ergs/°C.

Art. 36

17. A vessel of capacity 210 litres contains gas at N. T. P. The vessel is heated to 26°C ; but there being a slight leak, the pressure remains constant. If the original mass of the gas were 185 gms., find the mass of gas remaining in the vessel. (The volume of the vessel may be taken to remain unchanged).

Ans. 168 gms.

Art. 36

18. The measurement of a room is 10 ft. × 30 ft. × 25 ft. If the temperature of the room be increased from 20°C to 25°C, calculate what percentage of the original volume of air will be expelled from the room, the pressure remaining constant. (U. P. B.—1949)

Ans. 1·7.

Art. 36

19. Containing a little air at 27°C. a barometer tube is fixed inverted over a deep cistern of mercury. From the mercury surface in the cistern the height of the mercury column in the tube is 61 cm., while the barometer reads 76 cm. Find by how much the level of mercury in the cistern must be raised so that the position of the mercury meniscus in the tube may remain the same when the temperature of the closed air is raised to 87°C.

Ans. 8 cm.

Art. 36

20. A flask containing dry air is corked up at 20°C, the pressure being 76 cm. of mercury. Calculate the temperature at which the cork would be blown out, if this occurs when the pressure inside the flask is 1·7 atmospheres. (Raj. U.—1955)

Ans. 22°·1C.

Art. 36

21. Under a pressure of 770 mm. mercury and at a temperature of 20°C a corked bottle contains some air. It stands near a fire and the temperature rises to 200°C. If the surface of the cork exposed to the air in the bottle be 9 sq. cm., in area what will be the force on the cork urging it out? ($g=980$ cm. per sec.^{2} ; density of mercury $=13{\cdot}6$).

Ans. Total force = (pressure) × (area) = $5{\cdot}67\times10^6$ dynes.

Art. 36

22. Given that the universal gas constant R is $8{\cdot}315\times10^7$ ergs per gramme-molecule per degree Centigrade, calculate the volume occupied by 6 gms. of oxygen at—30°C and 90 cm. Find also the molecular weight of a gas, 60 gms. of which occupy a volume of 10 litres at 200°C and 35 cm.

Ans. 3155 c.c. ; 491.

Reference

Art. 36 23. Calculate the change in volume during expansion of 1 gm. of hydrogen when heated from 25°C to 26°C against the atmospheric pressure of one million dynes per cm^2. Given gas constant $R = 8{\cdot}3 \times 10^7$ ergs/gm. molecule/degree centigrade.
(R. P. B.—1951)

nns. 41·5 c.c.

Art. 36 24. At a pressure of 15 lbs. wt. per sq. in. and at a temperature of 20°C a cylinder contains 6 cu. ft. of gas. The volume of the gas in the cylinder is kept constant and the pressure is changed to 150 lbs. wt. per sq. in. What should be the temperature of the gas, if the volume is found to be 2·5 cu. ft.

Ans. 947°·8C.

Art. 36 25. A vessel contains 1000 litres of a gas at 0°C and at 740 mm. pressure. Find the pressure of the gas when the temperature rises to 27°C. (U. P. B.—1953)

Ans. 813·2 mm.

Art. 36 26. Twenty c.c. of hydrogen are at a pressure equal to the pressure at 50 cm. below the surface of water exposed to the atmosphere when the barometer reads 75 cm. The temperature of the gas is 27°C. What would be its volume under N. T. P.? (Density of mercury = 13·6)

Ans. 18·84 c.c.

Art, 36 27. A litre of gas at 10°C is heated until both volume and pressure are doubled. What is the temperature then?
(C. U.—1947)

Ans. 859°C.

Art. 36 28. A litre of oxygen at N. P. weighs 1·429 gm. Find the mass of 10 litres of oxygen at—35°C and 77 cm. pressure.

Ans. 16·59 gm.

Arts. 33 & 34 29. Prove that for a perfect gas the volume and the pressure coefficients are equal.
(Nag. U.—1953; R. P. B.—1949; U. P. B.—1947; Del. U.—1952)

Art. 37 30. Compare the density of air at 10°C and 750 mm. pressure with that at 15°C and 760 mm. pressure.

Ans. 5400; 5377.

Art. 37 31. On a certain day the barometer reads 75 cm. and the temperature is 10°C. On being taken to the bottom of a mine where the temperature is 27°C, the barometer reading increases by 4 cm. Find the ratio of the density of the air at the bottom of the mind to that of air on the ground level.

Ans. ·9993; 1

Art. 38 32. Describe the standard constant volume hydrogen thermometer. Explain how it may be used to measure temperature. Mention the advantages of a permanent gas over mercury as a hermometric substance. (Raj. U.—1952, '54)

Art. 39 33. How do you account for the pressure of a gas in a closed space and on what factors does it depend?
(Pat. U.—1932)

Art. 39. 34. Write short notes on the molecular motion in gases,
(C. U.—1949)

CHAPTER V

CALORIMETRY

40. Quantity of Heat : Unit of Heat—It has already been stated that heat is a form of energy and as such, it is a *meaeurable* quanitity. Thus when a body is heated, it gains heat from the heater, and when cooled, it gives away heat to the neighbouring bodies. Again, a given mass of a substance requires a quantity of heat for its rise of temperature. If the mass is doubled, the quantity of heat required for the same rise of temperture also is doubled ; the two quantities of heat are here added. Quantities of heat, therefore, obey arithmetical laws of addition and subtraction.

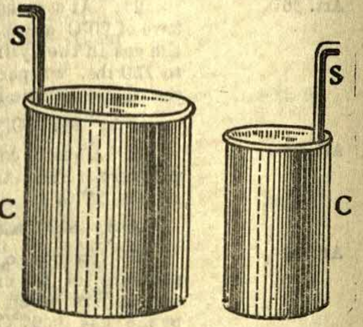


Fig. 34—Calorimeters

Calorimetry deals with the measurment of quantities of heat. The vessel, which is ordinarily used in such measurement, is known as a *calarimeter*. It is a cylindrical vessel C usually made of copper and provided with a stirrer S of the same material. Figure 34 represents two calrimeters of different sizes. The **unit of heat** is defined to be *the quantity of heat required tc raise one gramme of pure water throuph* 1°C, This unit is called **calorie** of sometimes gramme-degree calorie′,

It is found experimentally that the quantity of heat required to raise one gramme of water through 1°C is *not* exactly the same all over the scale of temperature. It decreases from 0°C to minimum value at about 40°C and then increases. As the variation of heat equivalent over the range from 0°C to 100°C is extremely small, the difference need not be taken into consideration unless a great accuracy is required.

Although a calorie is that unit most commonly used, other units also are prevalent. Termed the *kilogramme calorie*, that continental Engineer's Unit is the amount of heat required to raise 1 kilogramme of water through I°C. The British Thermal Unit (B Th. U.) is the quantity of heat required to ralse 1 lb. of water throngh 1°F. The *therm* is a special unit used by gas companies and is the quantity of heat required to raise 100 lb. of water through 100°F and so equals 100,000 B Th, Untis, The heat required to raise 1 lb. of water through 1°C is also used as a unit in Great Britain and is known aa the *Centigrade unit* (lb, deg, C.) The quanitity of heat required to raise *m* gms. of water throught *t*°C is, therefore, *mt* calories. Conversely, *mt* calories of heat are given out when *m* gms. of water cool through *t*°C.

Since 1 lb. = 453·6 gm. and $1^\circ\text{F} = \frac{5}{9}$ of 1°C, it follows that :

$$1 \text{ B. Th. U.} = \frac{5}{9} \times 453{\cdot}6 \text{ calories} = 252 \text{ calories}$$

and 1 Pound Degree Centigrade(C.H.U.) = 1·8B.Th. U = 453·6 calories.

Principle of Measurement of Heat—If two bodies at different temperatures be mixed, there will be sharing of the heat possessed by the bodies until they attain a common temperature. The heat given out by one body during the period of its cooling is taken up by the other body during the same period in reaching the common temperature. Assuming that no heat is received from or imparted to any body outside the system and that no chemical action occurs in the mixture, we obtain :

heat lost = heat gained,

41, **Specific Heat**—It is found that equal masses of the *same* substance require the same quantity of heat so as to be heated through the same range of temperature. Conversely, it is also true that equal masses of the same substance give out the same quantily of heat when cooled through the same range of temperature. But it is found that equal masses of *different* substances take in (or give out) *different* quantities of heat when heated (or cooled) through the same range of temperature. This can be demonstrated by a simple experiment described below.

Take a number of small balls of different materials (say, lead, tin copper and iron) but of the *same* mass. Either in steam or in any suitable bath heat them to the same temperature. Remove the balls quickly by means of strings attached to them and place them simultaneously on a thick cake of paraffin wax (Fig. 35).Observe that each ball melts different amounts of wax and consequently sinks to different extents into the cake. It is clear that different balls, although of the same mass, give out different quantities of heat in cooling through the same range of temperature, *i.e.*. from the temperature of the bath to that at which wax melts. The above fact is due to a difference in some specific property of the materials of the balls and is expressed by saying that different materials possess different *specific heats*.

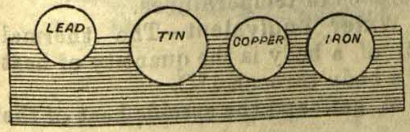


Fig. 35

The **specific heat** of a substance is the *ratio of the quantity of heat required to raise a given mass of the substance through a range of temperature to the quantity of heat required to raise an equal mass of water through the same range of temperature.*

Let the specific heat of a substance of mass m gm. be s.

$$\text{Then } s = \frac{\text{heat required to raise } m \text{ gms. of the substance through } t^\circ\text{C}}{\text{heat required to raise } m \text{ gms. of water through } t^\circ\text{C}}$$

$$= \frac{\text{heat required to raise 1 gm. of the substance through } 1^\circ C}{\text{heat required to raise 1 gm. of water through } 1^\circ C}$$

$$= \frac{\text{heat required to raise 1 gm. of the substance through } 1^\circ C}{\text{1 calorie}}$$

Hence we can also state that *the specific heat of a substance is equal to the number of units of heat in calorie required to raise the temperature of 1 gm. of the substance through 1° C*. The very statement, therefore, that the specific heat of copper is 0·1 means that the heat required to raise 1 gm. of copper through any range of temperature is $\frac{1}{10}$th of the quantity of heat required to raise 1 gm. of water through the same range of temperature ; in other words, the heat required to raise 1 gm. of copper through 1°C is one-tenth of a calorie. As the specific heat is taken as a ratio. it is a mere **number** and is independent of the unit of heat chosen. Nevertheless, the specific heat is sometimes expressed in calories per gm. per degree centigrade.

From the definition of specific heat it is easy to see that the :

heat required to raise m gm. of the substance through $t^\circ C$

$= s \times$ heat required to raise m gm. of water through $t^\circ C$

$= s \times mt$ calories $= mst$ calories.

Conversely, heat given out by m gm. of the substance in cooling through $t^\circ C$ is mst calories. Similarly, the heat required to raise m pounds of the substance through $t^\circ F$ is $= mst$ B. Th. U. Also the heat given out by m lbs. of the substance in cooling through $t^\circ F$ is $= mst$ B. Th. U.

∴ heat taken in or given out

= mass × specific heat × difference of temperatures.

42. Thermal Capacity and Water Equivalent—The thermal capacity (or the *capacity of heat*) of a body is the quantity of heat required to raise the temperture of a body through 1°C.

Let the mass of a body be m gm. and the specific heat of the material of the body be s.

Then the thermal capacity of the body per 1°C $= ms$ calories,

Hence the Specific Heat of a substance is sometimes defined as its *thermal capacity per unit mass* and is referred to as the *specific thermal capacity* of the substance. Further, the *thermal capacity per unit volume* is the amount of heat required to raise unit volume of the substance through 1°C and is, therefore, equal to the product of the density and the specific heat.

The **water equivalent** of a body is the quantity of water (in grammes) which would be raised through 1°C by the amount of heat required to raise the temperature of the body through 1°C.

Let the mass of the body be m gm. and the specific heat of the material of the body be s.

Then heat required to raise temp. of the body by 1°C $= ms$ calories.

Now this is the quantity of heat that raises the temperature of ms gm. of water through 1°C.

∴ the water equivalent of the body $= ms$ gm.

Hence the *numerical portions* of the expressions for the thermal capacity and the water equivalent of a body are the same.

Thus for a body of mass 100 gms. and made of copper $(s=0{\cdot}1)$

$$\text{the water equivalent}=(100\times0{\cdot}1)\text{ gm.}=10\text{ gm.}$$

$$\text{and the thermal capacity}=(100\times0{\cdot}1)\text{ cal.}=10\text{ cal. per }^\circ\text{C}$$

43. Regnault's Calorimeter—In experiments on measurement of heat two bodies, one being hot and the other cold, at *known* temperatures are very often mixed together in a calorimeter. When the exchange of heat is complete the resulting temperature is measured with a thermometer. For this purpose an apparatus, originally designed by Regnault, is very useful. It consists of a calorimeter C placed on a non-conducting support inside a larger wooden vessel (Fig. 36). The calorimeter is provided with a stirrer R whereby any liquid within it may be well-stirred. Fixed to a stand, a thermometer T can be lowered into the calorimeter so as to read the temperature of its contents. For insertion of the thermometer and stirrer the calorimeter can be covered with a suitable lid D with a slit. The purpose of placing the calorimeter in a closed chamber is to reduce the loss of heat from it to a minimum. A sliding gate P stands as a partition wall between the calorimeter and the steam-heater S. Whenever necessary, the gate may be slided up and the chamber containing the calorimeter may be brought below the stand just under the steam-heater.

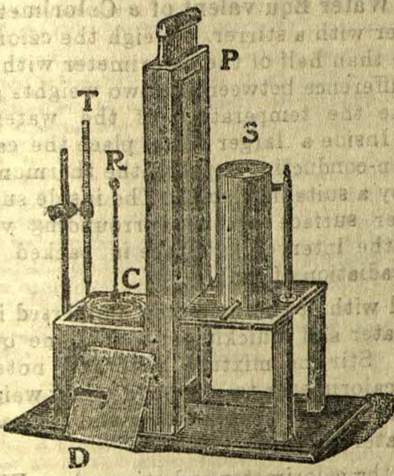


Fig. 36

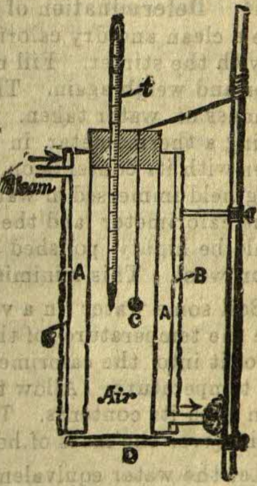


Fig. 37

Steam Heater—The steam-heater S is a mechanism in which a small body may be heated by steam *not* by direct contact. In Figure 37 it is represented in section. It consists of two coaxial hollow metal cylinders (A, B), usually of copper, the air space between them being closed at both the ends. Serving as the inlet and the outlet, the pipes are connected to the annular space. A boiler containing water, is connected with a rubber tubing to this inlet pipe at the top. On heating the boiler steam passes through the space between the two cylinders and escapes through the outlet pipe at the bottom. By aid of a cork at the top and a movable shutter O at the bottom the inner cylinder is closed. Inside the heater the body C is suspended by a thread passing through a hole in the cork. With its bulb close to the solid the thermometer t is inserted through another hole. The solid is thus indirectly heated by steam.

44. Determination of the Water Equivalent of a Calorimeter—Take a clean and dry calorimeter with a stirrer. Weigh the calorimeter with the stirrer. Fill more than half of the calorimeter with cold water and weigh again. The difference between the two weights gives the mass of water taken. Note the temperature of the water by putting a thermometer in it. Inside a larger vessel place the calorimeter with its contents on a non-conducting support, the thermometer being held immersed in water by a suitable clamp. The inside surface of the calorimeter and the inner surface of the surrounding vessel should be highly polished and the intervening space is packed with cotton wool. This minimises radiation of heat.

Boil some water in a vessel with a thermometer immersed in it. Note the temperature of the water and quickly pour off some quantity of it into the calorimeter. Stir the mixture well and note the final temperature. Allow the calorimeter to cool and then weigh it again with its contents. The difference between this and the second weight gives the mass of hot water added.

Let the water equivalent of the calorimeter and stirrer be $= W$ gm,
,, mass of cold water taken $= m$ gm.,
,, initial temperature of water and calorimete $= t_1$°C,
,, temperature of the hot water added $= t_2$°C,
,, final temperature of the mixture $= t$°C,
,, mass of hot water added $= m'$ gm.

Then in cooling from t_2°C to t°C heat lost by m' gm. of water $= m'(t_2 - t)$ cal.,

and in rising from t_1°C to t°C heat gained by cold water $= m(t - t_1)$ cal,
and heat gained by calorimeter and stirrer $= W(t - t_1)$ cal.
Assuming that no heat is exchanged with any external body,

heat lost = heat gained.

$$\therefore \quad m'(t_2 - t) = m(t - t_1) + W(t - t_1).$$

$$\text{or} \quad W(t - t_1) = m'(t_2 - t) - m(t - t_1),$$

$$\therefore \quad W = m'\frac{t_2 - t}{t - t_1} - m. \qquad \ldots(44{\cdot}1)$$

All the quantities on the right side of (44·1) being known, W can be calculated. The result can be verified from the relations $W = Ms$, when M is the mass of the calorimeter with stirer and s is the specific heat of the material of the calorimeter. (For a method of radiation correction in calorimetric experiments vide Practical Physics, All India Edition by J. Chatterjee.

Examples :

1. A calorimeter contains 30 gms. of water at 28°C. A quantity of 44 gms. of water at 57°C is poured into the calorimeter and the contents are well stirred, after which the resulting temperature is found to be 43°C. Find the water equivalent of the calorimeter.

Ans. Let the water equivalent of the calorimeter be W gm.

Now heat lost by hot water = 44 × (57 − 43) calories
and heat taken by calorimeter and water in it = {(W + 30) × (43 − 28)} calories.
Hence (W + 30)(43 − 28) = 44 × (57 − 43), whence W = 11·07 gm.

2. A Dewar flask is filled about three-quarters by water at a high temperature. The water is rolled about the flask until a uniform temperature of 95°C is attained. The hot water is emptied away and 50 gms. of cold water at 15°C are poured in. After shaking, the resulting temperature is observed to be 31°C. Calculate the water equivalent of the flask. [Gau. U.—1952]

Ans. A Dewar flask is a contrivance wherein either a hot substance or a cold one can be preserved without any appreciable loss or gain of heat from the surrounding atmosphere (Chap. VIII). The object of rolling hot water inside the flask is to raise its temperature. When hot water is thrown away, the temperature of the flask is found to be 95°C, which is thus its *initial high temperature* when empty. If the water equivalent of the flask be W heat lost by the flask = (95 − 31) W cals = 64 W cals.

Again, in rising from 15°C to 31°C heat gained by 50 gms. of cold water = 50 × (31 − 15) cals = (16 × 50) cals.

On equating, the required water equivalent $W = \frac{16 \times 50}{64} = 12{\cdot}5$ gms.

3. A piece of lead at 99°C is placed in a calorimeter containing 200 gms. of water at 15°C. After stirring, temperature stands at 21°C. The calorimeter weighs 40 gms. and is made of a material of specific heat 0·1. Calculate the thermal capacity of the piece of lead.

Ans. Let the required thermal capacity be W calories.
Heat lost by lead in cooling from 99°C to 21°C is = (99 − 21) W cal = 78 W cal.
Heat gained by the calorimeter = {40 × 0·1 × (21—15)} cal = (4 × 6) cal.
Heat gained by water inside = {200 × (21 − 15)} cal, = (200 × 6) cal.,
so that the total heat gained = (4 × 6 + 200 × 6) cal. = (204 × 6) cal.
Hence we derive
78 W = 204 × 6. whence the required capacity W = 15·7 cal.

4. The densities of two substances are as 2 : 3 and their specific heats are 0·12 and 0·09 respectively. Compare the thermal capacities per unit volume.

Ans. Let the densities of the two substances be 2ρ and 3ρ respectively.
Then the mass of unit volume of the first substance = 2ρ gm.
and mass of unit volume of second substance = 3ρ gm.
Hence the thermal capacity per unit vol. of the first substance = (2ρ × 0·12) cal.
and that of the second substance = (3ρ × 0·09) cal.

$$\therefore \frac{\text{thermal capacity per unit vol. of the first substance}}{\text{thermal capacity per unit vol. of the second substance}} = \frac{2\rho \times 0{\cdot}12}{3\rho \times 0{\cdot}09} = \frac{8}{9}$$

45. Determination of the Specific Heat of Solid—(by the *Method of Mixture*)—Take a clean dry calorimeter and weigh it with a stirrer of the same material. Pour a suitable quantity of water into the calorimeter and weigh it again. Difference between these two weights gives the mass of water taken. Note the temperature of the water. Take a suitable piece of the solid whose specific heat is to be determined and weigh it. Heat the solid in a **steam-heater**. Note the temperature of the inside space of the steam-heater when it has become steady. Then bring the calorimeter under the heater, move away the shutter D quickly drop the solid into the calorimeter and immediately remove the calorimeter. Stir the water well and note the final temperature of the contents of the calorimeter.

Let the combined mass of the calorimeter and strirrer be = W gm.,
,, specific heat of the material of the calorimeter $=s'$.
,, mass of water taken $=m$ gm.,
,, intial temperature of the water and calorimeter $=t_1$°C,
,, mass of solid = M gm.
,, steady temperature of the solid in steam-heater $=t$°C.
,, final temperature of the mixture $=t_2$°C,
and ,, the required specific heat of the solid $=s$.

Now in cooling from t°C to t_2°C heat lost by the solid.
$=Ms(t-t_2)$ cal.

Also, in rising from t_1°C to t_2°C heat gained by calorimeter, stirrer and water $=\{Ws'(t_2-t_1)+m(t_2-t_1)\}$ cal. $=(Ws'+m)\,t_2-t_1)$ cal.

Assuming that no heat was exchanged with any outside body.

heat lost = heat gained

so that $$Ms(t-t_2)=(Ws'+m)(t_2-t_1) \qquad \ldots(45.1)$$

$$\text{whence } s=\frac{Ws'+m}{M}\cdot\frac{t_2-t_1}{t-t_2}.$$

Sources of Errors and Precautions.

(1) In being transferred from the heater to the calorimeter some heat is lost by the solid. This makes the observed specific heat *too low*. This is minimised by dropping the hot solid directly into the calorimeter after bringing the latter under the heater.

(2) Some heat is lost by *conduction* and *radiation* from the calorimeter to the surrounding media when the former is at a higher temperature than the latter. This will make the observed specific heat *too low* and can be minimised in the following way :—

(*i*) The calorimeter is placed on *non-conducting* supports inside a deeper vessel, the intervening space being sometimes packed with cotton-wool or some other *bad conducting* material, so that the loss of heat by *conduction* is made very little.

(*ii*) The outer surface of the calorimeter and the inner surface of the outer vessel are *polished*. This minimises loss of heat by *radiation*.

The water in the calorimeter is previously cooled as much below the room temperature as the final temperature is above it. By this means the average temperature of the calorimeter and its contents is nearly the same as the room temperature, so that the gain of heat (by the calorimeter and its contents from outside due to *radiation* during the first half of the experiment is practically compensated for by the loss during the second half. This is known as Rumford's *Compensation Method* (A detailed process of correction for radiation is given in *Intermediate Practical Physics*, All-India Edition, by J. Chatterjee).

(3) The changes of temperature should be very accurately and carefully determined by a *sensitive thermometer*.

(4) In the case of a bad conducting solid it should be broken into small fragments, so that heat can evenly diffuse through the mass and the different pieces may be raised to the same temperature.

Examples :

1. A body of mass 100 gm. at 120°C is plunged into 300 gm. of water at 20°C contained in a copper calorimeter of mass 50 gm. The final temperature attained is 30°C. Find the specific heat of the material of the body. (Sp. ht. of copper = 0 09).

Ans. Denote the required specific heat by s.

Now in cooling from 120°C to 30°C heat lost by the body $=\{100\times(120-30)\}s$ cal.

Also in warming from 20°C to 30°C heat gained by the calorimeter and water $=\{(50\times0{\cdot}09+300)\times(30-20)\}$ cal.

Since heat lost = heat gained, we get $9000\,s=3045$, or $s={\cdot}34$,

Thus the required specific heat is ·34.

2. Equal volumes of mercury and glass have the same capacity for heat. Calculate the specific heat of piece of a glass of specific gravity 2·5, if the specific heat of mercury is ·033 and its specific gravity 13·6.

Ans. Let the sp. ht. of glass be s and the vol. of the glass piece be V c.c.

∴ mass of the glass piece $=(2{\cdot}5\times V)$ gms.

Also mass of mercury taken $=(13{\cdot}6\times V)$ gms.

Hence of V c.c. of glass the capacity for heat $=2{\cdot}5\times V\times s$.

Also „ „ mercury „ „ $=13{\cdot}6\times V\times{\cdot}033$.

On equalisation, $2{\cdot}5\times V\times s=13{\cdot}6\times V\times{\cdot}033$, whence $s=0{\cdot}18$.

This gives the specific heat sought for.

45. Determination of the Specific Heat of a Liquid—(by the *Method of Mixture*)—Take a clean dry calorimeter and weigh it together with a stirrer of the same material. Take some amount of the given liquid in the calorimeter and weigh again. The difference between these two weights gives the mass of the liquid taken. Note the temperature of the liquid and the calorimeter.

Take a piece of a solid of *known* specific heat and weigh it. It must be ensured beforehand that the solid is insoluble in and chemically unaffected by the given liquid.

In a steam heater heat the solid to a steady high temperature and note a temperature. Bring the calorimeter with the liquid under the heater, drop the solid into the liquid and quickly remove the calorimeter. Stir the liquid well and note the final temperature of the contents of the calorimeter.

Let the mass of the calorimeter and stirrer be $=W$ gm.,
„ specific heat of the material of the calorimeter $=s_1$.,
„ mass of the liquid taken $=m$ gm.,
„ initial temperature of the liquid and calorimeter $=t_1$°C.
„ mass of the solid $=M$ gm.,

Let the specific heat of the material of the solid $= s_2$
„ steady temperature of solid in the steam-heater $= t°\text{C}$.
„ final temperature of the contents of the calorimeter $= t_2°\text{C}$
and „ the required specific heat of the liquid $= s$.

Now in the cooling from $t°\text{C}$ to $t_2°\text{C}$ the heat lost by the solid

$$= \text{M}s_2(t - t_2) \text{ cal.}$$

Also in warming from $t_1°\text{C}$ to $t_2°C$ the heat gained by calorimeter, stirrer and the liquid $= \{\text{W}s_1(t_2 - t_1) + ms(t_2 - t_1)\}$ cal. $= (\text{W}s_1 + ms) \times (t_2 - t_1)$ cal.

Assuming that no heat is exchanged with any outside body.

heat lost = heat gained,

whence $\text{M}s_2(t - t_2) = (\text{W}s_1 + ms)(t_2 - t_1)$,

or, $ms(t_2 - t_1) = \text{M}s_2(t - t_2) - \text{W}s_1(t_2 - t_1)$, ...(46.1)

$$\text{whence } s = \frac{\text{M}s_2(t - t_2) - \text{W}s_1(t_2 - t_1)}{m(t_2 - t_1)}$$

In the case of a liquid, which has no *chemical action with water* the specific heat may be determined by heating a quantity of it and then pouring it into a known mass of water in a calorimeter. If *there be a chemical action between the liquid and water*, the hot liquid may be poured directly into a thin-walled metallic vessel (preferably of a good conductor) of known water-equivalent, placed in a known quantity of water contained in a calorimeter.

In the case of a *volatile* liquid (such as, alcohol) a quantity of the liquid may be enclosed in a thin-walled vessel of a good conductor; the vessel is almost filled with liquid and a thermometer is introduced through the cork. The vessel with its contents is heated to a known temperature and then transferred into water contained in a calorimeter. The vessel itself may be used as a stirrer and the final temperature is given by the thermometer. Then proceeding as usual the specific heat of the liquid is obtained. The specific heat may also be conveniently determined by previously cooling the liquid to known low temperature and then dropping directly into it a piece of heated solid of given mass and specific heat.

Examples:

1. Milk of volume 200 c.c. and of density 1·03 at 30°C contained in a brass vessel of thermal capacity equal to that of 8 gms. of water is mixed with 200 gms. of water at 98°C. The temperature of the mixture is 64°C assuming that there is no loss of heat by radiation, find the specific heat of milk.

Ans. Let the required specific heat of milk $= s$.

Now the mass of the milk taken $= (200 \times 1{\cdot}03)$ gms. $= 206$ gms.,
heat lost by water $= \{200 \times (98 - 64)\}$ cal.
$= (200 \times 34)$ cal.,

heat gained by the milk $= \{200 \times 1{\cdot}03 \times (64 - 30)s\}$ cal.
and heat gained by the brass vessel $= \{8 \times (64 - 30)\}$ cal $= (8 \times 34)$ cal.

There being no loss of heat, we obtain
$(206s + 8) \times 34 = 200 \times 34$,
or, $206s + 8 = 200$, whence the required sp. ht. $s = 0{\cdot}98$.

2. The specific gravity of certain liquid is 0·8, and that of another liquid is 0·5. It is found that the heat capacity of 3 litres of the first is the same as that of 2 litres of the second. Compare their specific heats.

Ans. Let the specific heats of the two liquids be s_1 and s_2 respectively.

Now the mass of 3 litres of the first liquid $=(3000\times0{\cdot}8)$ gms. $=2400$ gms.
and „ 2 litres „ second „ $=(2000\times0{\cdot}5)$ gms. $=1000$ gms.

Also the heat capacity of the first liquid is given by $H_1=2400\times s_1$
and „ „ second „ „ $H_2=1000\times s_2$.

As $H_1=H_2$, we have $2400s_1=1000s_2$, whence $s_1 : s_2=5 : 12$.

47. Determination of the Specific Heat of a Liquid—(by the *Method of Cooling*)—If a given mass of liquid is kept in a certain vessel and is heated to a known temperature, then on allowing it to cool in an enclosure at a temperature lower than that of the liquid, the *rate* of *loss of heat* is found to depend on (*i*) the temperature of the liquid, (*ii*) the temperature of the enclosure and (*iii*) the nature and extent of the surface of the containing vessel. The *rate of loss of heat* does not, therefore, depend on the nature of the liquids. Hence, if allowed to cool in such a way that the above condition does exactly hold good in each case, different liquids will *lose* heat at the *same* rate. This principle is applied in comparing specific heats of liquids by the *method* of *cooling*.

A copper calorimeter blackened on the outside, is taken with a stirrer of the same material. Weigh the combination. With hot water fills about two-thirds of the calorimeter at a temperature of 25° to 30° higher than the room temperature. Place the calorimeter on non-conducting supports inside a large copper vessel blackened on the inside. To introduce thermometer and the stirrer the outer vessel may be covered with a lid with small holes. Start a stop-watch and stir the water gently. For about 20 minutes note the temperature at intervals of one minute. Then weigh the calorimeter with its contents, when cool ; the difference between this weight and that of the empty calorimeter with a stirrer gives the mass of water taken.

Now throw away the water from the calorimeter. Heat a suitable quantity of the given liquid in another vessel to about the same temperature as that of the water previously taken and fill the calorimeter with the hot liquid up to the *same* level as before, the calorimeter being placed as before. Stir gently. For almost an equal range of time note the temperatures at intervals of one minute. Then weigh the calorimeter with its contents. The difference between this weight and that of the empty calorimeter with stirrer gives the mass of the liquid taken. From these data draw on a graph paper the time-temperature curves for the two liquids and thence find the times taken by the water and the liquid to cool through an equal range of temperature.

Let the mass of water taken be $=m$ gm.,
„ water equivalent of the calorimeter $=M_1$ gm.
„ time taken by water to cool from θ_1°C to θ_2°C $=t_1$ sec.,

Let the mass of liquid taken $= \mathrm{M}$ gm.

,, time taken by the liquid to cool from $\theta_1{}^\circ$ to $\theta_2{}^\circ = t_2$ sec,

,, specific heat of the liquid $= s$.

$\therefore$ in cooling during t_1 sec. heat lost by water $= m(\theta_1 - \theta_2)$

$\therefore$ rate of loss of heat from water and calorimeter $= \dfrac{(m+\mathrm{M}_1)(\theta_1 - \theta_2)}{t_1}$

Again in t_2 sec. heat lost by the liquid $= \mathrm{M}s(\theta_1 - \theta_2)$.

$\therefore$ rate of loss of heat from liquid and calorimeter $= \dfrac{(\mathrm{M}s+\mathrm{M}_1)(\theta_1 - \theta_2)}{t_2}$

Hence from the principle stated above we can readily obtain

$$\frac{(m+\mathrm{M}_1)(\theta_1 - \theta_2)}{t_1} = \frac{(\mathrm{M}s+\mathrm{M}_1)(\theta_1 - \theta_2)}{t_2}.$$

whence $s = \dfrac{(m+\mathrm{M}_1)t_2}{\mathrm{M}t_1} - \dfrac{\mathrm{M}_1}{\mathrm{M}}$.

Examples :

1. 50 gms. of water and an equal volume of alcohol (relative density $= 0{\cdot}8$) are placed one after the other in the same calorimeter. They are found to cool from 60°C to 55°C in two minutes and one minute respectively. Find the specific heat of alcohol. The water equivalent of the calorimeter is 2 gm.

[U. P. B.—1951]

Ans. Heat lost by 50 gm. of water inside a calorimeter of water equivalent 2 in cooling from 60°C to 55°C in two minutes $= \{(50+2)(60-55)\}$ cal $=$ 260 cal.

$\therefore$ rate of loss of heat from water and calorimeter $= \dfrac{160}{120} = \dfrac{13}{6}$ cal./sec.

Again 50 gms. of water occupy a volume of 50 c.c. Therefore the volume of alcohol taken is 50 c.c. and its mass is $(50 \times 0{\cdot}8)$ gms. $= 40$ gms Let the sp. ht. of alcohol be s.

Heat lost by 40 gms. of alcohol and the calorimeter in cooling through the same range in one minute is

$= \{(40s+2)(60-55)\}$ cal. $= 10(20s+1)$ cal.

$\therefore$ rate of loss of heat from alcohol and calorimeter $= \dfrac{10(20s+1)}{60}$ cal./sec.

$$= \frac{20s+1}{6} \text{ cal./sec.}$$

The law of cooling then gives $= \dfrac{20s+4}{6} = \dfrac{13}{6}$,

whence the required specific heat $s = \dfrac{12}{20} = 0{\cdot}6$.

The following Table supplies the specific heats of some common substances :

Specific Heats of Common Substances

Substance	*Sp. Ht.*	*Substance*	*Sp. Ht.*	*Substance*	*Sp. Ht.*
Brass	·088	Gold	·03	Mercury	·04
Copper	·094	Glass	·16	Ice	·50
Iron	·119	Marble	·22	Mustard oil	·50
Lead	·031	Turpentine	·42	Castor oil	·44
Silver	·055	Olive Oil	·47	Bromine	·51

48. Specific Heat of Gases—The specific heat of a gas depends on whether the quantity of the gas is heated at constant volume or at constant pressure. Gases, therefore, have two specific heats :

The *specific heat of a gas at constant volume* (C_v) is the number of units of heat required to raise the temperature of 1 gm. of the gas through 1°C, the volume being kept constant. The *specific heat of a gas at constant pressure* (C_p) is the number of units heat required to raise 1 gm. of the gas through 1°C, the pressure being kept constant.

The specific heat of a gas at constant pressure (C_p) *is always greater than* the specific heat of the gas at constant volume (C_v). For, if a unit mass, say 1 gm., of the gas is heated through 1°C at a constant volume a quantity of heat is required for the purpose. But if the mass is heated through 1°C at constant pressure, then in addition to the heat required to raise the temperature through 1°C some amount of heat must be necessary to supply the energy for the work done during expansion against the external pressure. In the case of diatomic gases (*e g.*, nitrogen, hydrogen, etc,) the ratio of C_v to C_p is 1·41 and for monatomic gases it is about 1·6.

Determination of the Specific Heat of a Gas at Constant Pressure—(*Regnault's Method*)—The specific heats of various gases at *constant pressure* were determined by Regnault. The gas is compressed in a reservoir R (Fig. 38) kept in a constant temperature bath, from which a steady stream of it is made to flow out through the pipe P, A valve V which regulates the amount of flow, keeps the pressure of the gas constant. The pressure is read by a manometer M. The gas then passes through a long spiral tube immersed in an oil bath H and is thus heated to a known high temperature as recorded by the thermometer. It then passes through a copper coil which is immersed in water contained in a calorimeter C. In the process of its circulation through the copper spiral within the calorimeter the gas gives up its heat to the water and finally escapes into the air. The rise of temperature of the water in the calorimeter is noted. From the change of pressure in the reservoir the mass of the gas. which has come out, is calculated. On equating the heat lost by the heated gas to the heat gained by the calorimeter and its contents, the specific heat of the gas at constant pressure is calculated.

If the temperature of the oil bath be T_2, and if the initial and final temperatures of the calorimeter and its contents be t^0C and $t_2{}^0C$, we

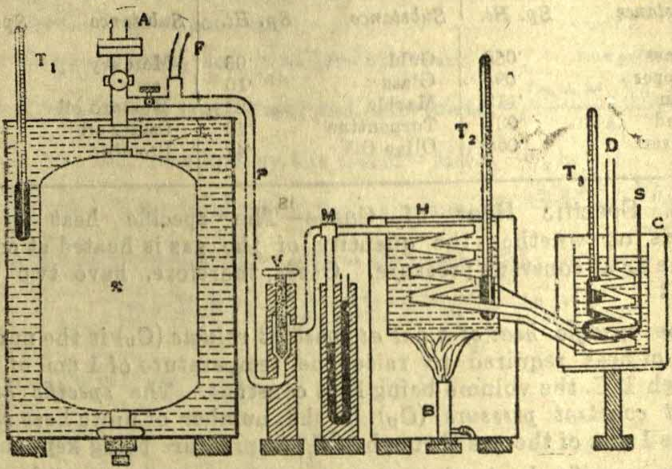


Fig. 38—Regnault's Apparatus

may suppose that the fall of temperature of the gas is $\{T_2-(t_1+t_2)/2\}$. If m be the mass of the gas which has left the reservoir and if C_p be its specific heat at constant pressure, the heat lost by the gas is $mC_p \times \left(T_2-\frac{t_1+t_2}{2}\right)$. The heat gained by the calorimeter and its contents is $(M+m)(t_1-t_2)$ where M and m are the water equivalents of the calorimeter and the mass of water contained. Equating the two terms C_p may be found.

Determination of the Specific Heat of a Gas at Constant Volume—The specific heat of a gas at *constant volume* has been determined by Joly with *steam calorimeter*. (Vide Art. 56.) Ths gas is enclosed in a copper sphere which is suspended from the pan of a balance within an enclosure. By weighing the sphere again when it is empty and once again when it is filled with the gas, the mass of the enclosed gas is known. When steam is passed into ths enclosure, it condenses on the sphere until the temperature of the latter with its content comes to that of the steam. From the mass of steam condensed the amount of heat taken by the sphere and the gas to rise through the temperature is known. From this the heat required by the mass of the enclosed gas for the rise of temperature is obtained and hence the specific heat of the gas at constant volume is found.

49. Observation on Specific Heat of Water—It is found that of solids and liquids, *water has the highest specific heat.* This means that a given mass of water absorbs more heat in warming and gives out more heat in cooling through a given range of temperature than an equal mass of any solid or any other liquid. It is for this reason that water is preferably used in hot water bottles, in foot warmers and in hot water pipes for heating rooms in cold countries. Again, when kept in the sun for some time, a quantity of water becomes less hot than an equal mass of any other material.

The use of *water as a calorimetric substance*, although almost universal, is found inconvenient for the following reasons :

(1) Owing to the high specific heat of water. variation of its temperature due to small quantities of heat cannot be appreciably measured.

(2) Owing to the variation of the specific heat with temperature water is not suitable for very accurate work.

Again, land is more easily warmed or cooled than water. The temperature of an island is, therefore, much influeneed by that of the surrounding sea. In fact, the tempetature of an island is an average between those of the land and the sea. Thus, during summer when the sun is above the horizon for a long period, the temperature of island is lower than that of the continent ; again, during winter when the sun is above the horizon for shorter period, the temperature of the island is higher than that of the continent. Further, during day the sea is less heated than the continent : but during night water cools more slowly than the land and so maintains the temperature of the island which does not, therefore, cool so much as the continent. It follows, therefore, that extremes of heat and cold are controlled by the sea, so that an island or a sea-side place has a more equitable climate than an inland place of the same latitude ; the sea may thus be called a *moderator of climate.*

50. Measurement of High Temperature by the Calorimeter Method—A *high* temperature (say, that of a flame or a furance) may be found by (1) Platinum Resistance thermometer, (2) Thermoelectric couple and (3) Calorimetric method. The first two are dealt with in the Part on Voltaic Electricity.

Calorimetric Method—A suitable piece of a solid of known mass and specific heat is placed in contact with the source of the high temperature till the mass acquires this temperature. A calorimeter with sufficient water is taken, the heated solid is thrown into the water and the final temperature is carefully noted.

Let the water equivalent of calorimeter and stirrer be $=W$ gm.,
,, mass of water taken $=m$ gm.,
,, mass of the solid taken $=M$ gm.,
,, specific heat of the solid $=s$.
,, initial and final temperature of the water $=t_1$ and t_2
,, temperature of the source $=t$.

Then as before $Ms(t-t_2)=(W+m)(t_2-t_1)$,

whence
$$t=\frac{(W+m)(t_2-t_1)}{Ms}+t_2.$$

Calorific value of a Fuel—The *calorific value* of a fuel is the amount of heat given out, when a definite mass (say, 1 lb) of the fuel is completely burnt. Thus when we say that *the heating value of a sample of coal is 10,000 B.Th. U. per pound*, we mean that the heat produced by the complete combustion of 1 lb. of coal of the particular sample is 10,000 B. Th. U. The calorific value of a fuel is usually determined by the *Bomb Calorimeter*. A small quantity of this fuel powdered and dried, is placed inside a strong steel bomb. The bomb is provided with gas-tight cover and is filled with oxygen at a high pressure. The bomb is kept immersed in water contained in a calorimeter : the fuel is fired by heating a wire in contact with it by means of electric current. The heat produced by combustion passes into the water, the initial and final temperatures of which are noted. Water equivalents of the calorimeter and the bomb as also the mass of water taken being known, the amount of heat produced by the combustion can be calculated and the calorific value is found therefrom.

Example :

A ball of platinum, whose mass is 80 gm., is kept for some time in a furnace and when it has acquired the temperature of the furance, it is immediately transferred to a vassel of water at 15°C. The temperature rises to 20°C. If the weight of water together with the water equivalent of the calorimeter be 400 gms., what is the temperature of the furnace ? (Sp. ht. of platinum = ·0365).

[Gau. U.—1953]

Ans. Let the temperature of the furnace be T°C which is the initial temperature of the platinum ball. The final temperature of the ball being 20°C., the heat lost by it is $\{80\times \cdot 0365\times (T-20)\}$ calories.

Now heat gained by calorimeter and water $=\{400\times (20-15)\}$ cal.

$=(400\times 5)$ cal.

On equating and re-arranging, we easily find

$$T=\frac{400\times 5}{80\times 0365}+20=705\ ^\circ C$$

The required temperature is, therefore, 705°C.

51. Latent Heat—When heat is continuously supplied to a mass of a solid, its temperature gradually rises until at a certain temperature it begins to melt. More and more of the solid melts on further heating ; but the temperature remains *constant* till the whole mass of the solid has completely changed into the liquid state. Conversely, if a mass of a liquid is continuously cooled, its temperature gradually falls, until, at a certain temperature the liquid begins to solidify. On continuing the process, more and more of the liquid solidifies ; but the temperature remains *the same* till the whole mass of the liquid gets solidified.

Similarly, if a mass of liquid be continuously heated, its temperature at first gradually rises till at a certian temperature depending on the super-incumbent pressure the liquid begins to boil and change into the vaporous state. More and more of the liquid is vaporised on further heating ; but the temperature remains *constant* until the whole mass of the liquid has evaporated. Conversely, if a mass of vapour is gradually cooled, its temperature falls until at a certain temperature the vapour begins to condense, which remains *constant* until the whole mass of the vapour has passed into the liquid state.

Hence when a piece of solid melts (or when a mass of liquid is being vaporised), the heat supplied does not manifest itself in producing a rise in temperature but is solely utilised in bringing about the change of state. The heat absorbed during a change of state remains as if *latent* or hidden within the mass of the liquid (or the vapour) formed and reappears only during the reverse process. The heat thus utilised in bringing about a change of state is called **latent heat,** as distinguished from *sensible heat* which produces a rise in temperature.

52. Latent Heat of Fusion—*The latent heat of fusion of a solid is the quantity of heat required to change a unit mass of the substance at its melting point from the solid to the liquid state without any change of temperature.* It is also equal to the quantity of heat given out by a unit mass of the substance at its freezing point in changing from the liquid to the solid state without any change of temperature.

"The latent heat of fusion of ice (or the latent heat of ice) is 80 calories per gm." This statement means that 80 calories of heat are required to change 1 gm. of ice at 0°C into water at 0°C. Conversely 80 calories of heat are given out when 1 gm. of water at 0°C in being changed into ice at 0°C. Similarly, by the statement that the latent heat of fusion of lead is 5·7 calories per gm. we mean that the amount of heat required to convert 1 gm. of lead at its melting point into the liquid state is 5·7 calories.

In the pound-degree centigrade unit the latent heat of fusion of ice is also 80 heat units (pound degree). In the British Thermal Unit (pound degree Fahrenheit) the value becomes larger in the proportion 9 : 5 and hence in B. Th. U. per pound the latent heat of fusion of ice is $=80 \times 9 \div 5$, *i.e.*, 144. If the latent heat of fusion of the solid be L, the amount of heat absorbed by m gm. of the solid in passing from the solid to the liquid state is equal to mL heat units.

53. Latent Heat of Vaporisation—*The latent heat of vaporisation of a liquid is the quantity of heat required to change a unit mass of the liquid at its boiling point from the liquid to the vaporous state without any change of temperature.* The same quantity of heat is also given out by a unit mass of the vapour in condensing to the liquid state at the boiling point *without any change of temperature.*

It is experimentally found that the quantity of heat required to convert 1 gm. of water at 100°C into steam at the *same temperature* is 537 calories. The same quantity of heat is also given out by 1 gm. of steam at 100°C in condensing into water at 100°C in other words, the latent heat of vaporisation of water (or the latent heat of steam) at 100°C is 537 calories per gm. The value of the latent heat of steam in the pound degree Centigrade (C.H.U) unit is also 537 **heat units.** Its value in the B. Th.U. (pound-degree Fahrenheit) per lb. $= \left(\frac{9}{5}\times 537\right)$, *i. e.*, 966·6 B. Th. U.

Thus when ice melts, it first absorbs the latent heat, and when steam condenses, it first gives out the latent heat. Hence ice-cream appears so much colder than iced water and more severe burns are caused by steam than by boiling water at the same temperature.

54. Determination of Latent Heat of Fusion of ice—Take a clean, dry calorimeter and weigh it together with a stirrer of the same material. A stirrer with a piece of wire gauge fitted to the loop should preferably be taken. Fill about two-thirds of the calorimeter with water and weigh again. The difference between the two weights gives the mass of water taken. With a sensitive thermometer note the steady temperature of the water and the calorimeter.

Inside a large vessel on a non-conducting support put the calorimeter with the stirrer and the thermometer. Take some pieces of ice of suitable size. Dry them carefully with blotting paper and drop them one by one into the water in the calorimeter, care being taken to hold the pieces with blotting paper while dropping them into water. Keeping the pieces of ice under water with the wire-gauge stirrer, stir the water well all the time. Note down the lowest temperature reached. Then removing the thermometer, weigh the calorimeter again with its contents. The difference between the last two weights gives the mass of ice added.

Let the mass of the calorimeter and the stirrer be $= W$ gm.,
,, specific heat of the material of the calorimeter $= s$.
,, mass of water taken $= m$ gm.
,, initial temperature of water and calorimeter $= t^\circ$C.
,, final temperature of the mixture $= t_1{}^\circ$C.
,, mass of ice added $= M$ gm.
,, required latent heat of fusion of ice $= L$.

Now in cooling from t to t_1 heat lost by the calorimeter (with stirrer and the water) $= Ws(t-t_1)+m(t-t_1)$ cal $=(Ws+m)(t-t_1)$ cal. Next in meling to water at 0°C heat gained by M gm. of ice at 0°C is ML cal. Also in rising to t_1C heat attained by the M gm. of water formed $= Mt_1$ cal. Thus the totality of heat gained $= M(L+t_1)$ calories.

On the assumption that no heat is exchanged with any external body, so that

heat lost = *heat gained*,

we get $(Ws+m)(t-t_1)=M(L+t_1)$,

or, $ML=(Ws+m)(t-t_1)-Mt_1$.

This determines L. The value of L is found to be nearly 80 calories per gramme.

Sources of Errors and Precautions :

(1) Should some water happen to stick to a piece of ice, the mass of ice would really be less than the mass of this water, so that the quantity of heat required for fusion will be less. This makes the observed value of L *too low*. Hence just before being dropped into water, the pieces of ice should be carefully dried with blotting paper.

(2) A water-gauge stirrer should be used, so that the pieces of ice added are always kept under water and *not* allowed to float ; otherwise the portion of ice above the surface of water will, while melting, consume heat not only from the water but also from the surrounding air.

(3) By radiation some heat is gained by the calorimeter and its contents from outside. This will make the observed value of L *too low*. This is remedied by taking the initial mass of water a few degrees (say, 5°C) above the room temperature, and ice is added till to the same extent the final temperature of the mixture goes below the room temperature. This goes by the name of the *method of compensation*.

(4) The quantity of ice added must be small, as otherwise the lowering of temperature of the calorimeter and its contents may be so much that there may be actual condensation of moisture on the outer walls of the calorimeter. This *considerably* lowers the observed velue of the latent heat of ice.

Examples

1. Find what happens when 5 gm. of ice at 0°C is placed into 20 gm. of water at 45°C.

Ans. In cooling to 0°C heat lost by 20 gm. of water at 45°C $=(20\times45)$ cal $=900$ cal.

To melt 5 gm. of ice at 0°C to water at 0°C heat required $=(5\times80)$ cal. $=400$ cal.

This shows that all the ice melts.–

Let the resulting temperature be t°C.

Heat lost by 20 gm. of water at 45°C to come t°C is $=20\times(45-t)$ calories.

Heat gained by 5 gm. of ice in melting to water at 0°C is $=400$ calories.

Heat gained by 5 gm. of water (melted ice) to rise to t°C is $=5t$ calories.

Hence we have $400+5t=20\times(45-t)$, whence $t=20$.

∴ there will remain 25 gm. of water at 20°C.

2. In 50 gm. of water at 40°C are dissolved 15 gm. of ice at -20°C and the resultant temperature is found to stand at 10°C. If the latent heat of fusion of ice is 80, find its specific heat.

Ans. Let the required specific heat of ice be $=s$.

So as to cool from 40°C to 10°C heat lost by 50 gm. of water $=(50\times10)$ cal.

Heat gained by 15 gm. of the ice at -20°C to reach 0°C is $=(15\times20)\times s$ cal.

Also „ „ „ to melt to water at 0°C is $=(15\times80)$ cal.

Further, to come to 10°C heat gained by 15 gm. of water $=(15\times10)$ cal.

Hence we have $15\times20\times s+15\times80+15\times10=50\times30$,

or $300s=150$, whence $s=0{\cdot}5$. which is the required specific heat.

3. Calculate the amount of heat necessary to convert 1 gm. of ice at -10°C into steam at 100°C. (Sp. ht. of ice $=0{\cdot}5$; latent heat of water $=80$ cal./gm. and latent heat of steam $=536$ cal./gm.) [Anna. U.—1950]

Ans. Heat necessary for raising 1 gm. of ice from $-10°C$ to $0°C$ is $= (1 \times \cdot 5 \times 10)$ cal.
" " " water from $0°C$ to $100°C$ is $= (1 \times 100)$ cal.
" " " converting 1 gm. of ice into water at $0°C$ is $= (1 \times 80)$ cal.
and " " " water at $100°C$ to steam $= (1 \times 536)$ cal.
∴ total amount of heat $= (5 + 80 + 100 + 536)$ cal. $= 721$ cal.

4. In an experiment to determine the latent heat of fusion of ice a carefully insulated calorimeter of water equivalent 20·1 gm. containing 300 gm. of water at 19°·5C is taken. Into this is dropped a lump of pure ice *not previously dried.* The mass of the calorimeter with its contents is found to have increased by 17·9 gm. and the resultant temperature comes to 14°·7C. The correct value for latent heat of fusion of ice is 80 calories per gramme. What mass of water was sticking to the ice?

Ans. Let the mass of the water adhering be $= m$ gm.
∴ mass of ice added $= (17 \cdot 9 - m)$ gm.
Heat lost by the calorimeter and water
$= \{20 \cdot 1 \times (19 \cdot 5 - 14 \cdot 7) + 300 \times (19 \cdot 5 - 14 \cdot 7)\}$ cal. $= (320 \cdot 1 \times 4 \cdot 8)$ cal.
Heat taken up by the ice to melt $= \{(17 \cdot 9 - m) \times 80\}$ cal.
Also " " " water to rise to $14° \cdot 7C$ is $= (17 \cdot 9 \times 14 \cdot 7)$ cal.
Hence we derive $(17 \cdot 9 - m) \times 80 + 17 \cdot 9 \times 14 \cdot 7 = 320 \cdot 1 \times 4 \cdot 8$, whence $m = 1 \cdot 98$.
Thus the mass required is 1·98 gms.

55. Ice Calorimeters.—For determining the specific heat of a substance the fact that a certain quantity of ice in melting always absorbs a definite quantity of heat has been utilised in the application of ice calorimeters.

Black's Ice Calorimeter—It consists merely of a large block of pure ice which with a hole, about an inch in diameter and an inch deep, is scooped out into it (Fig. 39). The mouth of the hole is covered by a slab of ice, which serves as a lid over the cavity. A portion of the solid, of which the specific heat is to be determined. is taken and weighed. In a steam heater it is then heated to a definite temperature. With blotting paper or sponge the inside of the cavity is thoroughly dried. The heated solid is then dropped into the cavity and the lid is replaced quickly. By conducting heat, which melts a certain quantity of ice, the body within the cavity reaches the temperature of 0°C. After a few minutes the water in the cavity as well as on the surface of the body is collected by means of a small pipette and a piece of sponge or blotting paper previously weighed (and cooled to 0°C). The mass of water formed is determined by direct weighing.

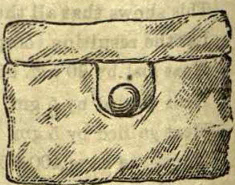


Fig. 39

Let the mass of the solid be $= M$ gm.
" initial temperature of the solid $= t°C$,
" mass of the melted ice $= m$ gm.
and " specific heat of the material of the solid $= s$.
Now heat lost by M gm of the solid in cooling to 0°C is $= Mst$ cal.
and heat gained by m gm. of ice in melting $= mL$ cal.

As no heat from outside can reach the solid, we must have

$$Mst = mL. \text{ whence } s = \frac{mL}{Mt}.$$

The method is *not accurate*, since (*a*) the water formed by the melting of ice cannot be completely removed and (*b*) some ice may melt by absorbing heat from the atmosphere during the period intervening the drop of the solid inside and the closure of the lid.

The *advantages* of the method are (1) that it is a ready method, that (2) there is no loss of heat by radiation and that (3) no delicate temperature is necessary for noting the final temperature.

Bunsen's Ice Calorimeter—The action of the instrument depends on the fact that on melting, a given mass of ice diminishes in volume to a definite extent, so that by accurately measuring the change in volume the weight of ice melted (and hence the quantity of heat required to melt) can be found.

Bunsen's ice calorimeter consists of a large thin-walled test tube A (Fig.40) fused into a large glass vessel B, which at the bottom extends into a narrow tube CD, bent twice at right angles, as shown in the figure. At the other end of CD there is a collar D closed by a cork, through which passes a bent piece of capillary tubing T of uniform bore. To the horizontal part of the tube T a finely graduated scale S is attached, so that the *volume* between any two divisions is accurately known. The lower part of B, the tube CD and a part of the capillary tube T are filled with pure dry mercury, while the upper part of B is filled with pure distilled water, which has been boiled so as to rid it of dissolved air.

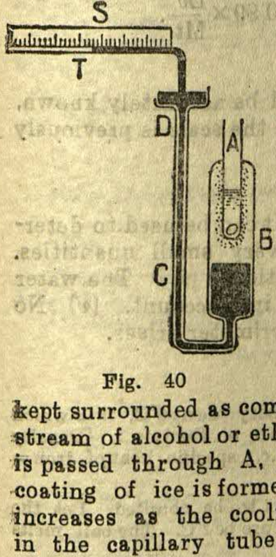


Fig. 40

Before being used, the instrument is kept surrounded as completely as possible with pure melting ice and a stream of alcohol or ether, previously cooled by a freezing mixture, is passed through A, till owing to the intense local cooling, a thin coating of ice is formed round the lower part of A. This gradually increases as the cooling is continued, while the mercury meniscus in the capillary tube is found to move forward. The freezing mixture is then removed from A and a quantity of water already cooled at 0°C is placed in it. After a sufficient interval the whole of the instrument with the contents comes to 0°C and the mercury meniscus assumes a steady position as read off from the scale.

To determine the specific heat of a metal a small lump of it is taken and weighed. The lump is then heated to a constant high temperature and quickly dropped into the water in A which is at once corked. The heat given out by the solid is communicated by the water to the ice surrounding A and melts some of the ice. This continues until everything again is at 0°C. If the ice melts, a contraction in the volume takes place, which causes the mercury meniscus to move inwards. When it has been steady, its position is read again. The cross-section of the capillary tube having been previously known, the contraction in volume is easily found, whence the quantity of ice melted (and so the quantity of heat supplied) can be calculated.

Let the mass of the solid be $= M$ gm.,
„ specific heat of the material of the solid $= s$,
„ initial high temperature of the solid $= t^\circ C$
and „ contraction in volume of the ice $= v$ c.c.
Now contraction of 1 gm. of ice at 0°C on melting $= 0\cdot091$ c.c.
∴ mass of ice melted in the present case $= v/0\cdot091$ gm.
Heat absorbed in the melting of ice $= vL/0\cdot091$ cal.
where L is the latent heat of fusion of ice.

Also heat lost by the solid in cooling from t°C to 0°C is $= Mst$ cal.
As this heat is wholly utilised in melting the ice, we get

$$Mst = \frac{vL}{0\cdot091}, \quad \text{or} \quad s = \frac{vL}{0\cdot091 \times Mt} = 10\cdot99 \times \frac{Lv}{Mt}.$$

This gives the required specific heat s.

As v is very small, the cross-section should be accurately known, and the contraction carefully noted. Sometimes the scale is previously calibrated directly in calories.

Advantages of Bunsen's Ice calorimeter :

(*i*) It is very sensitive and accurate. (*ii*) It can be used to determine the specific heat of solids available in very small quantities. (*iii*) There is no loss of heat due to radiation. (*iv*) The water equivalent of the calorimeter is not to be taken into account. (*v*) No necessity for a thermometer along with the calorimeter arises.

Examples :

1. A spherical iron ball is placed on a large block of dry ice at 0°C, into which it sinks until half submerged. What was the temperature of the iron ? (Density of iron = 7·7 gm. per c.c. and that of ice = 0·92 gm. per c.c., specific heat of iron = 0·12 and latent heat of fusion of ice = 80 cal./gm.)

Ans. Evidently the loss of heat by radiation is not to be counted. Let the volume of the iron ball be V c.c. and its initial temperature be t°C. The ball melts a volume of ice equal to half its volume and falls to 0°C.

Now the mass o the ball $= (7\cdot7 \times V)$ gm. and the mass of ice melted

$$= 0\cdot92 \times \frac{V}{2} \text{gm.}$$

∴ heat lost by the ball $= (7\cdot7 \times V \times 0\cdot12 \times t)$ cal.

And heat gained by the ice $= \left(0\cdot92 \times \frac{V}{2} \times 80\right)$ cal.

Hence we have $7\cdot7 \times V \times 0\cdot12 \times t = 0\cdot92 \times \frac{V}{2} \times 80$, whence the desired temperature $(t) = 39\cdot8$C.

2. If a gram of ice at 0°C contracts by ·091 c.c. in melting, calculate the

specific heat of the substance when 40 gms. at 60°C dropped into an ice calorimeter cause a change in volume of ·273 c.c. (Given latent heat of fusion of ice = 80 cals./gm).

Ans. The mass of melted ice $= \frac{\cdot 273}{\cdot 091}$ gm. = 3 gm.

Heat gained by 3 gm. of ice in melting $= (3 \times 80)$ cal.

Again, heat lost by 40 gms. of the substance at 60°C, when dropped into the ice calorimeter $= (40 \times 69 \times s)$, where s stands for the specific heat.

No heat being lost or gained, we get

$$40 \times 60 \times s = 3 \times 80, \text{ where } s = 0{\cdot}1,$$

Thus the specific heat is 0·1.

3. A mass of 0·96 gm. of substance heated to 100°C is dropped into a Bunsen's calorimeter. The thread of mercury recedes through a distance of 8·3 mm. in the capillary tube of 1 sq. mm. section. Calculate the specific heat of the substance, given that 1 gm. of water on freezing expands by 0·0905 c.c. and evolves 80 calories. [Nag. U.—1952]

Ans. Effected by the movement of mercury thread by 8·3 mm. through a capillary tube of cross-section 1 sq. mm. the volume contraction $= (8{\cdot}3 \times 1)$ cu. mm. = 8·3 cu. mm. = ·0083 c.c.

This volume contraction is due to some ice melting into water at 0°C.

Now contraction is ·0905 c.c., when the mass of ice melting into water at 0°C is 1 gm.

∴ The contraction is 1 c.c., when the mass of ice melting is $\frac{1}{\cdot 0905}$ gm.

∴ " " ·0083 c.c., " " " " " is $\frac{\cdot 0083}{\cdot 0905}$ gm.

Again, in melting to water, 1 gm. of ice absorbs 80 calories.

∴ " $\frac{\cdot 0083}{\cdot 0905}$ gm. " $\frac{80 \times 83}{905}$ "

This amount of heat is given out by 0·96 gm. of the substance in cooling through 100°C.

If s be its specific heat, then

$$0{\cdot}96 \times 100 \times s = \frac{80 \times 83}{905}, \text{ whence } s = \frac{80 \times 83}{905 \times 96} = {\cdot}075.$$

56. Determination of Latent heat of Vaporisation of Water

(*Latent heat of Steam*)—Take a clean, dry calorimeter and weigh it together with a stirrer made of the same meterial. Fill the calorimeter about two-thirds with water and weigh again. The difference between the two weights gives the mass of water taken. With a sensitive thermometer note the steady temperature of the water and the calorimeter.

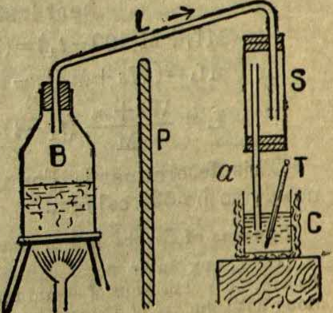


Fig. 41

Boil some water in a vessel B (Fig. 41) with its mouth closed by a cork, through which a bent glass tube passes. The open end of the bent tube is connected to a *water-trap* or the so-called **steam-trap** S. This is a contrivance whereby the steam is freeed form any water accompanying it. It consists of a wide glass tube fitted with steam-tight corks, through each of which there passes a glass tube reaching nearly up to the opposite cork. The upper tube is connected to the delivery tube from the flask. Steam enters into the trap by the upper tube and the water formed by the condensation of any steam is deposited at the bottom of the wide tube, while the steam goes up and comes out by the exit tube *a* which is practically free from water.

With its contents and a thermometer T, put the calorimeter C, on non-conducting supports inside a large vessel. Bring it under the steam-trap, quickly put the nozzle of the exit tube well within the water and pass steam for some time, the water being kept stirred all the while. Place a screen P between the calorimeter and the boiler. Take away the nozzle quickly and note the highest temperature attained by water. Allow the calorimeter to cool to the room temperature and remove the thermometer, it being ensured that no water clings to it. Weigh the calorimeter with contents. The difference between the last two weights gives the weight of steam condensed.

Let the mass of the calorimeter and stirrer be $= W$ gm.,
,, specific heat of material of the calorimeter $= s$,
,, mass of water taken $= m$ gm.,
,, initial temperature of water and calorimeter $= t^\circ$C,
,, final temperature of the mixture $= t_1{}^\circ$C
,, temperature of the steam $= 100^\circ$C,
,, mass of steam condensed $= M$ gm,
and ,, required latent heat of steam $= L$.

Now in condensing to water at 100°C heat lost by steam $= ML$ cal. and in cooling from 100°C to $t_1{}^\circ$C heat lost by water $= M(100 - t_1)$, cal.

Also in rising from t°C to $t_1{}^\circ$C heat gained by calorimeter with stirrer and water $= \{Ws(t_1 - t) + m(t_1 - t)\}$ cal,

No heat being assumed to be exchanged with any outside body.

heat lost = heat gained

$$\therefore \quad ML + M(100 - t_1) = (Ws + m)(t_1 - t).$$

$$\text{or} \quad ML = (Ws + m)(t_1 - t) - M(100 - t_1),$$

$$\therefore \quad L = \frac{Ws + m}{M}(t_1 - t) - (100 - t_1),$$

This determines L. For steam at 100°C the value of L is found nearly to be 537 cal./gm.

Sources of Errors and precautions :

(1) If any water accompanies steam, the observed value of L would be *too low.* To avoid condensation of steam the delivery tube should be covered with cotton wool along with the use of a steam-trap.

(2) Any loss of heat by radiation from the calorimeter and its contents will make the observed value of L *too low.* To reduce the effect of radiation the water in the calorimeter should be initially cooled a few degrees below the

room temperature and steam should be passed till the temperature of the water rises as much above the room temperature as it was below.

(3) The temperature of the steam may not be 100°C and so it should be determined in each case.

(4) To prevent the calorimeter from gaining any heat directly from the burner or the boiler by radiation, a screen is to be placed in between.

(5) The issue of steam should not be rapid; otherwise some water may be lost by splashing, which will make the observed value *too low.*

(6) The temperature of the calorimeter and its contents should not be allowed to rise more than 30°C above the room temperature, for in that case much vapour would be given off, causing considerable loss of heat.

57. Joly's Steam Calorimeter—Joly in 1886 devised a very accurate method of determining the specific heat of a substance by the use of condensation of steam on it. The appartus consists of a metal enclosure S called the *steam chamber*, in which steam is supplied by a boiler B (Fig. 42) The steam chamber is enclosed within a box D, on the top of which is placed a sensitive balance. From one arm of the balance is suspended a fine wire B, which passes down through a hole into the steam chamber. At its other end the wire carries a scale pan A.

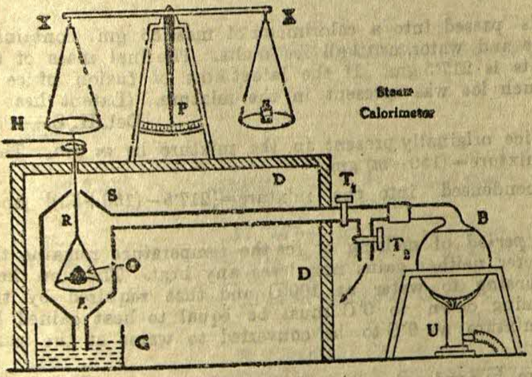


Fig. 42—Steam Calorimeter

The body, O whose specific heat is to be found, is placed on the pan within the steam chamber *when steam is not passed through it.* By placing standard weights on the other pan, the body is counterpoised. This gives the mass of the body in air at the room temperature t°C. Let the mass be m, gm. Then steam is passed from the boiler into the chamber and when a *steady high temperature* of the chamber is recorded by the thermometer, 'weights' are placed on the other pan so as again to counterpoise the body and the condensed steam. If this be M_1 gm., this gives the total mass of the body, the steam condensed on the body and the pan and the suspension wire. Then the body is taken out and again steam is passed over the empty pan. The mass of steam condensed on the pan and its accessories is found by the method of weighing. Call it M_2 gm. Then the mass of steam condensed on the body is (M_1-M_2) gm. Let the temperature of steam be T°C.

If the specific heat of the body be s, the heat absorbed by the body, when its temperature is raised from t°C, to T°C, is $ms\,(T-t)$.

The heat given out by steam in condensation over the body is $(M_1 - M_2)\,L$, where L represents the latent heat of steam at T°C.

No heat being lost or gained.

$$ms(T-t)=(M_1-M_2)\ L,\ \text{whence}\ s=\frac{(M_1-M_2)L}{m(T-t)} \qquad \cdots \quad (57.1)$$

In order that the steam might not condense on the upper left pan of the balance, an electric heater coil H is placed in front of the slit of the chamber D.

Examples :

1. Steam at 100°C is passed into a calorimeter of mass 55 gm. containing 150 gm. of mixture of ice and water, until all ice melts. The final mass of the calorimeter and its contents is 217·5 gm. If the latent heat of fusion of ice is 80 cal./gm., find how much ice was present in the mixture. (Latent heat of steam = 540 cal./gm.)

[Del. H. S.—1953]

Ans. Let the mass of ice originally present in the mixture be m gm. Then the mass of water in the mixture = $(150-m)$ gm.

The mass of steam condensed into the mixture = $\{217{\cdot}5-(150+55)\}$ gm. = 12·5 gm.

Since *throughout the period* of melting of ice the temperature remains the same at 0°C, the calorimeter neither gains nor loses any heat. Therefore heat lost by steam in condensing to water at 100°C and that required by the amount of water in coming down to 0°C must be equal to heat gained by m gm. of ice in the mixture at 0°C to be converted to water at the same temperature.

Now heat lost by steam in condensing to water at 100°C is = $(12{\cdot}5\times540)$ cal. and heat lost by 12·5 gm. of water at 100°C to come to 0°C is = $(12{\cdot}5\times100)$ cal,

∴ total quantity of heat lost = $(12{\cdot}5\times540+12{\cdot}5\times100)$ cal. = 8000 cal.

Again heat gained by m gm. of ice in melting = $(m\times80)$ cal.

Thus from the principle that heat gained equals heat lost we get

$m\times80=8000$, whence the required mass $m=100$ gm.

2. A glass beaker of negligible thermal capacity contains 1 lb. of water and some ice. When 1 oz. of steam at 100°C has been supplied to the vessel and the contents are at a temperature of 3°C, how much ice was there to begin with ? (Latent heat of ice = 79 cal./gm. Latent heat of steam = 537 cal./gm.)

[Utkal. U.—1951]

Ans. Let the mass of ice be x gm, Since 1 lb = 453·6 gm. and 1 oz. = 28·35 gm., heat lost by 28·35 gm. of steam in condensing at 100°C is = $(28{\cdot}35\times537)$ cals. and heat lost by condensed steam to cool down to a temperature 3°C is = $\{28{\cdot}35\times(100-3)\}$ cals. = $(28{\cdot}35\times97)$ cals.

Now in melting at 0°C heat gained by ice = $(x\times79)$ cals.

and in rising to 3°C heat gained by melted ice = $(x\times3)$ cals.

Equating the heat lost to the heat gained, we get

$$453{\cdot}6\times3+x\times82=28{\cdot}35\times537+28{\cdot}35\times97,$$

whence $x=202{\cdot}7$

The original mass of ice is thus 202·7 gm.

3. Steam at 100°C is passed into a zinc vessel of mass 500 gm. containing 1 kgm. of ice and 1 kgm. of water at 0°C. The temperature of the mixture rises

to 30°C. If the mass of the calorimeter with its contents is found to increase by 232 gm., find the latent heat of steam. (Specific heat of zinc = ·092 ; latent heat of water = 80).

Ans. Let the required latent heat be L cal./gm.

Heat lost by steam in cooling to water at 100°C is $= 232\,L$ cal.

„ „ condensed steam in cooling to 30°C is $= (232 \times 70)$ „

Heat gained by the ice in melting $= (1000 \times 80)$ „

„ „ by this water and the water in the calorimeter in rising from 0°C to 30°C is $= (2000 \times 30)$ cal.

„ „ by the vessel in rising from 0°C to 30°C is $= (500 \times {\cdot}092 \times 30)$ cal.

Hence by the proved principle, $232L + 232 \times 70 = 80.000 + 60{,}000 + 1380.$

or $232L = 125140$, whence the required latent heat L is 539 cals./gm.

EXERCISES ON CHAPTER V

Reference

Arts. 40, 42 & 45 — 1. What is the unit quantity of heat? Define thermal capacity of a body. How would you find the thermal capacity of a body in practice? (C. U.—1962)

Arts. 40, 42 & 45 — 2. What is meant by unit quantity of heat? Define 'B. Th. U.' and 'calories' and indicate the numerical relation between them. Explain the term 'water equivalent' of a calorimeter and describe how it is determined.
(Gau. U.—1962 ; *Cf.* Poo. U.—1961)

Arts. 41 & 42 — 3. Explain the terms 'specific heat', 'thermal-capacity' and 'water equivalent' of a body, and state the units used in expressing them.
(V. U.—1952 : Del. H. S.—1962 ; C. U.—1953 ; Utkal U.—1969)

Art. 41 — 4. A copper calorimeter weighs 180 gm. and the specific heat of the material is ·09. Find the quantity of heat required to raise its temperature from 25°C to 65°C.

Ans. 643 cal.

Art. 44 — 5. How would you find the thermal capacity of a given calorimeter by experiment? (Mad. U.—1967)

Art. 45 — 6. Describe an experiment to determine the specific heat of a solid. Mention the sources of errors and how they are avoided. (Utkal U.—1953 , Mad. U.—1967)

Arts, 41 & 44 — 7. Define specific heat. Describe an experiment to determine the water equivalent of a copper calorimeter and the specific heat of copper. (*Cf.* V. U.—1954 ; C. U.—1963)

Art. 44 — 8. A calorimeter contains 129·4 gm. of water at 12°·2C. To this is added 65·7 gm. of water at 38°·6C, and after stirring the temperature of the mixture is found to be 21°C. Find the water equivalent of the calorimeter.

Ans. 2 gm.

Art. 44 — 9. Supposing you were given a thermometer reading only from 50°C to 100°C and some water, of which the temperature was below 20°C, describe an experiment to determine roughly the temperature of water, without using any other thermometer. (C. U.—1953)

Reference

HINTS :—By the given thermometer and up to a *known* higher temperature T°C heat a sufficient quantity of water in a vessel and pour a quantity of this water in to the calorimeter containing the cold water, so that after stirring the mixture the temperature is above 50°C, the exact value being known from the thermometer. Thus knowing the water equivalent of the calorimeter and masses of hot and cold water, the initial temperature of cold water may be found. This temperature is approximate, because some heat escapes from the mixture during the process of heat exchange.

10. Ninty-six gms. of lead shots (sp. ht = 0·03) at 99°C are poured into 148 gms. of a liquid at 29°C contained in a lead vessel weighing 300 gms. If the final temperature of mixture is 33°C, find the specific heat of the liquid. (Mad. U.—1957) Art. 46

Ans. 0·26.

11. A copper calorimeter of weight 10 gm. contains 12 gm. of water at 98°·1C. To this is added a copper coin at 15°C and the temperature of the calorimeter and its contents decreases to 2°·8C. Find the mass of the coin. (Sp. ht. of copper = ·093). Art. 45

Ans. 9·5 gm.

12. Describe an experiment to determine the specific heat of a solid which is soluble in water, explaining carefully how you would calculate your result. (Gau. U.—1963) Arts. 45 & 46

13. A piece of marble weighing 25 gm. is heated to 100°C and is immediately dropped into 61 gm. of water contained in a copper calorimeter weighing 100 gm. with the stirrer kept at 25°C. What will be the final temperature of water and the calorimeter after the mixture has been thoroughly stirred, assuming that no heat has been lost? Given the sp. hts. of marble and copper to be 0·20 and 0·09 respectively. Specific heat of water may be taken to be unity. Art. 45

Ans. 30°C.

14. The temperature of three different liquids A, B and C are 14°C, 24°C and 32°C respectively. On mixing equal masses of A and B the temperature of the mixture is 20°C and on mixing equal masses of B and C the temperature of the mixture is 31°C. Supposing equal masses of A and C were mixed, what would be the temperature of the mixture? (Pat. U.—1965) Art. 46

Ans. 29°·6C (nearly).

15. A calorimeter, whose water equivalent is 5·0 grammes, is filled with 25·0 grams of water. It takes 4 minutes to cool from 25°C to 17°C. When the same calorimeter is filled with 30·0 grammes of liquid, it takes 180 seconds to cool through the same range. Calculate the specific heat of the liquid. (Raj. U.—1963)

Ans. 0·58.

16. Describe how the specific heat of a liquid is measured by the method of cooling. State the principle of the method and write down the formula used for calculating the result. (Raj. U.—1953 ; Del.U.—1962 ; R.P.B.—1990 ; Mad.U.—1969) Art. 47

17. Define the term, specific heat. Explain why specific heat of a gas depends on the conditions under which this quantity is measured. (U. P. B.—1964) Arts. 41 & 43

Reference

Art. 48

18. Explain why a gas has two specific heats. Describe how the specific heat of a gas at constant pressure has been measured. (Nag. U.—1965 ; Del. H. S.—1971 ; R. P. B.—1964)

Art. 48

19. Why is the specific heat of a gas at constant pressure greater than that at constant volume ? (Raj. U.—1974)

Art. 48

20. Describe Regnault's method of determining the specific heat of a gas at constant pressure, bringing out (*a*) how the constancy of pressure is maintained and (*b*) how the mass of the gas passed is determined. (Raj. U.—1974)

Arts. 51 & 53

21. A quantity of water is heated in a vessel with a thermometer dipping in it. The temperature of water rises as heat is applied to it, until it reaches 100°C when water begins to boil. After this temperature of water remains constant, even though heat is continuously applied to it, till the whole amount of water is evaporated. Explain this. (V. U.—1954)

Art. 50

22. A lump of platinum weighing 100 gm. is heated in a flame until its temperature has reached that of the flame. It is then removed and dropped quickly into a calorimeter which has a water equivalent of 5 gm and contains 495 gm. of water. If the temperature of the water rises from 22°C to 30°C, find the temperature of the flame. (Sp. ht. of platinum = 0·36)

Ans. 1141°C (approximately).

Art. 50

23. How will you proceed to measure the temperature of a furnace by a calorimetric method ? Develop the working formula and discuss the sources of error in the method. (Utkal U.—1971)

Art. 52

24. What is mean by the statement that 'the latent heat' of fusion of ice is 80 ? (C. U.—1960)

Arts. 41 & 52

25. Define 'specific heat' and 'latent heat of fusion of a solid. How many units of heat are required to melt 100 gm. of tin originally at 20°C ? (Melting point of tin = 232°C : latent heat of fusion of tin = 14 cal./gm. ; sp. ht. of tin = 0·5.)

Do the values of these constants depend on the kind of thermometer used, Fahrenheit or Centigrade ? (C. U.—1954)

Ans. 2460 cal. Yes, they do.

Arts. 41, 52 & 53

26. Explain the meaning of the following statements :— (*a*) The latent heat of fusion of sulphur, which melts at 113°C. is 9 cals. per gm. (*b*) The sp. ht. of sulphur is 0·17 cal. per gm./°C. (Gau. U.—1965)

Art. 54

27. How would you determine the latent heat of fusion of ice, mentioning the precautions needed ? (C. U.—1956 ; Mad. U.—1959, '70 ; Pat. U.—1958 ; And. U.—1951)

Art. 54

28. What would be the final temperature of the mixture when 5 grammes of ice at—10°C are mixed up with 20 grammes of water 30°C ? The specific heat of ice is 0·5.

Ans. 7°C.

Art. 54

29. Find the rise in temperature when 35 gm. of liquid sulphur at its melting point is poured into a copper calori-

meter weighing 48 gm. containing 100 gm. of water at 14°C. (Sp. ht. of copper = 0·1 cal. per. gm. per °C ; sp. ht. of sulphur = 0·17 ; L. H. of sulphur = 9 cals./gm, and melting point of sulphur = 13°C) (Gau. U.—1955)

Ans. 22° 2C.

30. A copper calorimeter of mass 100 gm. and specific heat 0·1 contains 200 gm. of a liquid of sp. ht. 0·4 at temperature of 30°C. Sixteen grammes of pure ice at 0°C are dropped into the liquid. Find the resulting temperature. (Latent heat of fusion of ice = 80 cals./gm.) (Mad. U.—1966) Art. 54

Ans. 16° 03C.

31. Describe and indicate the use of Black's Ice Calorimeter. (C. U.—1960) Art. 55

32. A piece of iron weighing 15 gm. is dropped at a temperature of 113·6°C into a cavity in a block of ice, of which it melts 2·5 gm. If the latent heat of ice is 80 calories per gram, find the specific heat of iron. (C. U.—1958) Art. 55

Ans. 11.

33. Describe Bunsen's ice calorimeter and its working. How will you use it to find the density of ice ? Show the calculation clearly. (P. U.—1962 ; R. P. B.—1953, '68) Art. 55

34. The density of ice at 0°C is ·916 and that of water is 1 gm. per c.c. Weighing 600 gm., a piece of metal is raised to a temperature of 80°C. Placed in a mixture of ice and water, it melts some of the former. The volume of mixture is found to decrease by 8·4 c.c. How much of ice is melted and what is the specific heat of the metal ? Art. 55

Ans. 91 6 gm. ; ·152.

35. Twenty-five gms. of wax are heated to 60°C and poured into 100 gms. of water at 5°C in a calorimeter of water equivalent of 10 gms. Find the final temperature of the mixture. [Melting point of wax = 45°C ; sp. ht. of solid wax = 0·3 ; sp. ht. of liquid wax = 0·4 ; latent heat of fusion of wax = 40 cal./gm.] (Utkal U.—1958) Art. 55

Ans. 17° 34C.

36. Describe Bunsen's Ice Calorimeter and explain how it is used to determine specific heat of a substance. Art. 55

(Raj. U.—1952 ; Nag. U.—1952, '55 ; U. P. B.—1955 ;
E. P. U.—1951, '52 : Del. H. S.—1952 ; P. U.—1958)

37. One gram of metal of sp. ht. ·088 at 100°C is dropped into a Bunsen's ice calorimeter, in which the mass of mercury required to fill 1 cm. of the index tube of Bunsen's ice calorimeter is 0·26 gm. The thread of mercury moves through 52·5 mm. If the latent heat of fusion of ice be 80 calories/gm. and the relative density of mercury be 13·6, calculate the density of ice. (U. P. B.—1952 ; Nag. U.—1963) Art. 55

Ans. ·909.

38. Twenty grams of water at 15°C are put in the tube of a ice calorimeter and it is observed that the mercury thread moves through 29 cm. Twelve grams of a metal at 100°C are then placed in water and the mercury thread moves through 12 cm. Find the specific heat of the metal. Arts. 55

Ans. 0·1.

39. Steam at 100°C is passed into a calorimeter of mass 55 gms. containing 150 gms. of a mixture of ice and water Arts. 54 & 56

Reference

until all the ice is just melted. The mass of the calorimeter and its contents is then found to be 217 gms. How much of ice was originally present in the mixture ? [Latent heat of fusion of ice = 80 cal./gm. ; laten theat of steam = 540 cal./gm.]

Ans. 81 gm. (Utkal. U.—1957)

Arts. 55 & 56 — 40. Define latent heat of vaporisation and describe an experiment to measure the latent heat of steam.

(Utk. U.—1963 ; And U.—1962 ; V. U.—1955 ; Del. H. S.—1960 ; Pat. U.—1961, '63)

Arts. 54 & 56 — 41. Into a calorimeter containing 175 gm. of water and some ice, steam of mass 10 gm. and temperature 100°C is passed. The temperature of the contents rises to 10°C. If the water equivalent of the calorimeter is L gm., calculate the mass of ice initially present. Given latent heat of water = 80 cal./gm. ; latent heat of steam = 540 cal./gm. (And. U.—1965)

Ans. 50 gm.

Art. 56 — 42. Alcohol boils at 78°C, its latent heat of evaporation is 202 calories per gram and its mean specific heat when liquid is 0·65. *C*alculate the least quantity of water at 10°C needed to condense 100 gm. of alcohol vapour at 78°C into liquid at 15°C. (Utkal. U.—1964)

Ans. 4859 gm.

Arts. 54 & 56 — 43. A copper calorimeter weighing 100 gm. contains a mixture of 10 gm. of ice and 100 gm. of water at 0°C. Steam at 100°C is passed into the mixture, until the final temperature of the calorimeter and its content is 10°C. Determine the mass of steam which has passed into the calorimeter, given that the latent heat of steam = 540 cal./gm., latent heat of ice = 80 cal./gm. and sp. ht. of copper = 0·1.

Ans. 3·17 gm.

Art. 55 — 44. The diameter of a capillary tube of a Bunsen's ice calorimeter is 1·4 mm. When a piece of metal weighing 1·09 gm, at 100°C is dropped into the calorimeter, the mercury thread moves by 10 gm. Calculate the specific heat of the metal. The latent heat of ice is 80 cal./gm. and the density of ice is 0·9 gm./c.c. (Del. H. S.—1962)

Art. 56 — 45. A copper vessel of water equivalent 60 gm. contains 600 gm. of water at 30°C. A Bunsen burner, adjusted to supply 100 calories per second, is used to heat the vessel. Neglecting all heat losses, calculate (*a*) the time required to raise the water to the boiling point and (*b*) the time required to boil 50 gm. of water. [Latent heat of steam = 540 calories per gm.] (C.U.—1952)

Ans. (*a*) 7 min. 42 sec. ; (*b*) 12 min. 12 sec. (from start).

Art. 57 — 46. Describe Joly's Steam Calorimeter and explain how it may be used to determine the specific heat of a gas at constant volume.

(Nag. U.—1960 ; U. P. B.—1962 ; R. P. B.—1965, '69)

CHAPTER VI

CHANGE OF STATE

58. **Fusion and Solidification**—Fusion (or *melting*) is the change of a solid substance from the solid state to the liquid state. Solidification (or *freezing*) is the change of a liquid substance from the liquid state to the solid state. If a solid body be heated continuously, its temperature gradually rises until at a certain temperature it beigns to melt. If once the process of melting has started, the temperature undergoes no change, till the whole of the substance is converted into the liquid state. This temperature, which varies from one solid to another, is called the **melting point** of the substance. If heating is continued even after the whole of the solid has melted, the temperature of the liquefied mass rises in the usual way.

Take a mass of melting ice in a suitable vessel and fnsert a thermometer in it ; the temperature of the ice is found to be 0°C, To the ice apply heat by placing this vessel in another bigger vessel containing hot water ; ice beings to melt. Stir well. The thermometer, however, does not show any change of temperature until the *whole* of the ice is melted. If a quantity of paraffin wax is heated in a similar way, it is found that the paraffin begins to melt at nearly 55°C and that the temperature remains practically constant at this point until the entire mass is melted.

On the other hand, if a mass of liquid is continuosly cooled, the temperature gradually falls until at a certain temperature the liquid begins to solidify. Throughout the process of solidification the temperature, which is different for different liquids, remains constant and is called the **freezing point** or the *solidification temperature* of the liquid. If the cooling is continued even after the whole of liquid is solidified, the temperature of the resulting solid decreases in the usual way.

Super-cooling—Cooled slowly and undisturbed many liquids may be cooled below their solidifying point without solidification taking place, *e.g.*, freed from dissolved air, pure water has without being frozen been cooled to—20°C by Despretz. This phenomenon known as *super-cooling*, *surfusion* or *superfusion* and the liquid in this condition is said to be *super-cooled*. This condition is unstable, for if the liquid is slightly shaken or if a piece of a substance in a solid form is dropped into the liquid, solidification ensues at once and the temperature quickly rises to the true solidification point. For super-cooling, the liquid taken must be *pure* and free from any suspended or dissolved foreign substances.

Sublimation—There are certain solids which under ordinary circumstances pass directly into the vaporous state without passing through the intermediate liquid state and *vice versa*. Such a process is known as *sublimation*. Iodine crystals, camphor, sulphur, arsenic, benzoic acid, naphthalene are examples of these. Ice and snow also sublime slowly, even when below the freezing point.

59. **Determination of Melting point**—The melting point of a solid can be determined by either of the following methods.

(a) **Capillary Tube Method**—A small quantity of the substance in the liquid state is sucked into a thin-walled capillary tube. The lower end of the tube is then sealed. The tube C (Fig. 44) is fastened to the thermometer T, the sealed end being close to the bulb. in a glass vessel, the open end of the capillary tube being kept above the liquid. The bath is slowly heated and kept well-stirred. After some time a temperature is attained when the solid in the capillary tube just begins to liquefy ; the temperature of the bath is noted. The flame is then removed and the bath is kept well-stirred and allowed to cool. Just when the substance in the tube begins to solidify, the temperature is agsin noted. Although from a theoretical standpoint these two readings are identical, due to uncertainties of practical observation the two readings should not differ by more than one degree and their mean is taken as the melting point of the solid.

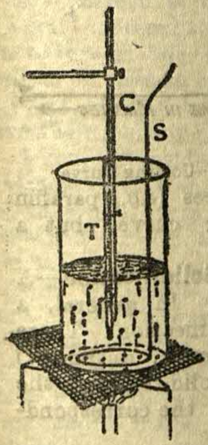


Fig. 44

This method is specially suitable when a very small quantity of the solid, which has not a high melting point, is available. (*Vide* J. Chatterjee's *Intermediate Practical Physics.)*

(b) **Cooling Curve Method**—A lump of solid is taken in a small calorimeter and is heated in a water bath or oil bath, until the whole of the solid melts and the temperature rises a few degree above the melting point. The bulb of a thermometer is now kept immersed in the liquid. The calorimeter is placed on a piece of cork inside a large vessel and is allowed to cool undisturbed. As the cooling proceedes, the temperature of the substance in the calorimeter is noted at regular intervals (say, half a minute). At a certain temperaturo, solidification sets in and the thermometer records a constant temperature and the observation is continued for some time after the whole mass gets solidified. A *curve* is now plotted with time as the abscissa and temperature as the ordinate. The general form of the cooling curve is as shown in Fig. 45. It would be noticed that during the process of solidification the portion of the curve is practically parallel to the time axis ; this indicates a *constant* level of temperature during the period. If the solid is heated again, it would be found that at a certain temperature the solid begins to melt and that the temperature remains constant, till the whole mass of the solid has melted. The temperature, which remains constant during the process of solidification as well as that of fusion, is found to be the same. This is termed the *melting point* of the particular solid or the *freezing point* of the corresponding liquid.

There are certain substances (such as, glass, pitch. wax, fat, etc.,) which are solids at ordinary temperature but which on being heated pass, before melting, through a *plastic* or **viscous** state, which lies intermediate between solid and liquid. This state may extend over a considerable range of ,temperature depending on the particular substance. Again there are certain liquids (such as, glycerine, acetic acid and some other organic acids and oils) which pass through an intermediate *viscous* state before changing into the solid state. These liquids have, therefore, no definite solidification point. In the case of a substance, which is a **mixture** of different substances (*e.g.*, paraffin wax), we get no single level step in the cooling curve, but a series of level steps,

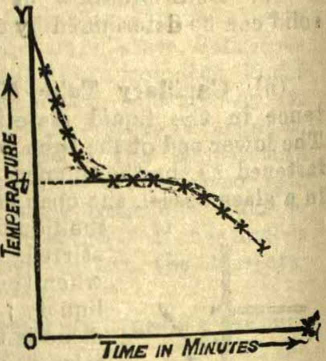


Fig. 45.—Cooling Curve

60. Change of volume in Fusion and Solidification—A substance always changes in volume on melting. In general, a substance expands on melting and contracts on solidification. There are, however, a few solids (*e.g.*, ice, cast iron, bismuth, antimony) which contract on melting. In the former case the solid sinks in the resulting liquid, while in the latter the solid floats on the corresponding liquid.

The *expansion of water on freezing* is very nearly 9%. so that 11 parts by volume of water at 0°C become 12 parts by volume of ice at the same temperature. This means that 1 c.c. of water at 0°C forms 1·09 c. c. of ice at 0°C, Ice, therefore, floats on pure water. Take a tumbler of water with a lump of ice floating on it. Note the level of water. Ice gradually melts away. The volume of water formed by the melting of ice is less by $\frac{1}{11}$th of the volume of ice ; also the volume of ice above water is $\frac{1}{11}$th of the total volume of ice taken. Hence the volume of the water formed by the melting of ice exactly occupies the space formerly occupied by the immeresed portion of ice ; so *when ice on water in a vessel melts, there is* no *change in the water level.*

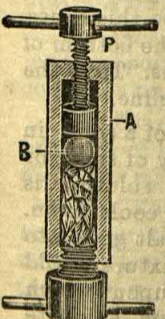
If water is contained in a closed space, its expansion of freezing takes place with a great force. A hollow ball of stout cast iron is completely filled with water, closed with a screw stopper and placed in a freezing mixture of ice and salt. After a time the water inside the ball freezes when the ball bursts asunder with a cracking sound. If completely filled with water and tightlycorked. a bottle be kept in open air on a frosty night, it would burst. Owing to the freezing of water inside, water pipes in cold countries often burst during a frost.

Since ice expands on freezing the ice that is formed, would float on the surface of water. So during frost in cold countries, rivers, lakes etc,, would have blocks of solid ice floating on water leaving enough of water space for aquatic animals to survive. Also, since ice is a bad conductor of heat it does not allow all the heat of water below it to escape and so the water does not freeze to a very great depth, even when the temperature above water is very low. Cast iron expands on solidification ; a lump of cast iron, therefore, floats on molten metal. In an iron casting, the mass of molten iron expands on solidification and is forced into intimate contact with every part of the mould, so that the castings are very sharp and well defined in outline.

61. Effect of Pressure on Melting Point—The melting point of a solid is found to depend on the pressure which it is subjected to. The nature of the effect is as follows : Substances like ice, which *contract on melting* have their melting points *lowered* by an increase *of* pressure, while substances like paraffin wax, which *expand on melting*, have their melting points *raised* by an increase of pressure.

A simple consideration explains the above facts. On melting, ice contracts in volume. External pressure tends to decrease the volume and therefore, helps the process of melting. Hence under increased pressure ice melts at a lower temperature. It has been calculated that the melting point of ice at 0°C is lowered by about 0·0073°C for increase of pressure of one atmosphere ; that is to say, the melting point of ice is lowered to −1°C by a pressure of about 137 atmospheres. Again, paraffin wax expands on melting ; external pressure helps to decrease the volume and therefore, opposes the process of melting. Hence under increased pressure its temperature gets raised so as to bring about fusion. This holds for all substances which expand on melting.

Regelation—If two pieces of ice are pressed together for a few seconds, then on releasing the pressure the pieces are found to cling together at the region of contact. The pressure lowers the melting point of ice and so the ice melts at the surface of contact. On releasing the pressure the melting point rises again and the water formed freezes binding the two pieces. The phenomenon of the melting of ice due to pressure and the subsequent resolidification, when the pressure is removed, is known as **regelation** (*re*. again : *gelare*, to freeze). This was first noticed by Faraday. *The lowering of the melting point of ice by increased pressure* can be demonstrated by the following experiments.



Fig, 45

Mousson's Apparatus—The apparatus consists of a stout iron cylinder A (Fig. 45) closed at one end with a screw plug and fitted at the other with screw plunger

P. The cylinder is partly filled with water which is frozen by placing the cylinder inside a mixture of ice and salt. A small metal ball B is then placed on the top of ice within the cylinder which is now closed by the screw plunger. The whole is then surrounded by melting ice and pressure is gradually applied by working the plunger in. When the screw at the bottom is taken out, the metal ball is found to come out first. This shows that during the application of pressure ice melts and that the ball goes down ; hence the melting point of ice is lowered by the increased pressure. When the pressure is released, the water freezes again, leaving the ball at the bottom.

Bottomleys Experiment—A large block of ice is placed on a suitable support (Fig. 46). With a heavy weight attached underneath, a loop of wire is passed over the block near about its middle. It is found that the wire makes its way slowly through the block and ultimately passes out. The block however remains unbroken, the track of the wire being marked by the section of ice,

It can be explained thus. Just under the wire the ice, being subjected to great pressure, has its melting point lowered and, therefore, it melts. The latent heat necessary for the melting of ice is taken practically from the surrounding ice. Hence the temperature of ice near the wire gets below 0°C. The wire passes down and the water flows above. The water is now free from pressure and since in contact with ice below 0°C, its temperature is 0°C, it freezes again and thus joins the two surfaces of ice together.

As the heat necessary for fusion is to be partly conducted through the wire from the upper side to the lower one, the wire transmits heat more rapidly, if it be made of a better conductor. Thus a copper wire cuts through more quickly than an iron wire of the same diameter, whereas a piece of thread cuts through very slowly. Snow balls can be formed by pressing pieces of snow. The markings of the slate of the wheels of heavy cart on ice can be similarly explained. To some extent the motion of *glaciers* may be attributed to this cause. Subjected to a very great pressure, a layer of ice at the bottom of the glacier melts and the water freezes out but resolidifies. Thus the bottom being soft, the glacier begins to move down the incline.

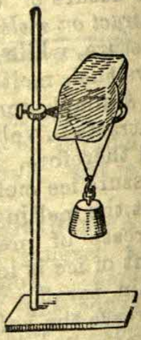


Fig. 46

62. Freezing Mixture—The process of a solution of a solid in liquid is akin to a change of state. Thus if a quantity of sugar or any salt is dissolved in water, the necessary heat for solution is taken from water and the solution ; therefore the mixture cools down. When common salt is mixed with ice, a quantity of salt goes into solution and absorbs some heat from ice. Thus the mixture would have a temperature below 0°C. This is the principle upon which *freezing mixture* acts. Thus by dissolving crystals of sodium sulphate or ammonium nitrate in a mixture of ice, a temperature of −15°C may be attained.

When the temperature of the freezing mixture falls below 0°C and some salt goes into solution, there is an internal pressure due to the dissolved salt acting within the solution. This is called the *solution pressure.* This solution pressure helps in liquefying more ice and then absorbing more heat from the mixture, an additional quantity of salt goes into the solution. This explains the lowering of temperature of a freezing mixture. Ultimately an equilibrium of temperature is attained, when the rate of supply of heat from the surrounding atmosphere is equal to the rate of absorption of heat due to the dissolved substance. This is the minimum temperature attained by the mixture.

Some Freezing mixtures

Ingredients	*Parts by weight*	*Temp. of the mixture*
Ammonium sulphate and ice	...10 ...10	−17·4°C
Powdered ice and common salt	...10 ... 1	−22°C
Calcium chloride and ice	...10 ... 2	−35°C
Carbon dioxide (solid) and ether	... 4 ... 1	−77°C

When accumulated ice melts, it absorbs the latent heat from air, whereby the temperature of the air falls and fog is formed. Again, when a liquid passes into the solid state, it gives out the latent heat. Thus during a snowfall the moisture in the atmosphere changes into snow and gives out the latent heat, whereby the air feels warmer.

63. Laws of Fusion—(1) A solid under constant pressure begins to melt at a certain temperature which is constant for the same solid and which is called the *melting point* of the solid at the given pressure. The melting point of a solid is the same as the solidification point of the corresponding liquid.

(2) The rate of fusion is proportional to the supply of heat ; but the temperature of the solid remains constant at the melting point until fusion is complete.

(3) If a substance (*e. g.,*) paraffin wax expands on melting, its melting point is raised by pressure. If a subtance (such as ice) contracts on melting, its melting point is lowered by pressure.

(4) During fusion a unit mass of each substance absorbs a definite quantity of heat known as its *latent heat of fusion,* which is constant for the particular substance.

64. Vaporisation and Liquefaction—The change of a substance from the liquid state to the vaporous or gaseous state is known as **vaporisation**, while the change of a substance from vapour to liquid is known as **liquefaction** or *condensation.*

Depending upon the circumstances under which it takes place, the process of *vaporisation* falls under the two catagories : (*i*) **evaporation and** (*ii*) **ebullition or boiling.**

Evaporation—If some water contained in a shallow dish be left at the ordinary temperature in a fairly dry room in an open vessel. the quantity of water is found to decrease gradually and disappear ultimately. Such disappearance is much more rapid in the case of methylated spirit, turpentine, ether, etc. In all these cases the liquid gradually passes into the gaseous state and as the mass of the liquid is not disturbed, the change must be taking place at the surface of the liquid. The process is known as *evaporation*. Evaporation, then, is a slow change from the liquid state to the gaseous one which takes place at the surface of the liquid and at all temperatures. From the above we gather that in an *unlimited space* (such as the open atmosphere) the process of evaporation goes on continuously until the whole of the liquid is changed into vapour. But in a closed or *limited space* it would be found that after some time the liquid shows no further decrease in volume. This shows that under definite conditions a limited space can contain only a limited quantity of vapour.

Ebullition or Boiling—Take some water in a glass flask closed with a stopper, through which an exit tube is fitted and a thermometer is inserted into the water. As the flask is heated over a Bunsen flame, the mass of the liquid gradually rises in temperature and the rate of evaporation also is found to increase. On the inside of the walls of the flask some bubbles appear which become larger with rise of temperature of the liquid, finally detach themselves from the walls of the flask and rise towards the surface. These are *bubbles* of air dissolved in the liquid. As the temperature rises further, bubbles of steam form at the bottom of the flask and while rising up, they collapse and condense with a sharp click which causes the water to 'simmer'. As the temperature of the liquid increases, bubbles reach the surface of that liquid when simmering ceases. Bubbles of steam are then found to escape vigorously throughout the mass of the liquid and the temperture of the water is found to remain steady. The water is now said to be *boiling* or in a state of *ebullition*.

Thus *ebullition* or *boiling* is the *rapid* change of a substance from the liquid state to the gaseous one for a particular liquid and this takes place *throughout the mass* of the liquid and at a *definite* temperature, under given conditions. The temperature remains constant throughout the process, provided the pressure remains unchanged. The **boiling point** of a liquid is a particular temperature, at which it changes from liquid to vapour under definite conditions and which remains constant so long as boiling goes on. Different liquids have different boiling points.

If a quantity of water, which has been previously boiled so as to drive away dissolved air, be allowed to boil, it may sometimes be found that there is no continuous bubbling. The reason is that suddenly a large bubble forms and bursts forth so as to throw the water violently upwards. The temperature of the liquid fluctuates slightly. The liquid again remains quiescent and similar boiling takes place after an interval. This is known as **boiling by bumping**. If a small piece of cotton or a fragment of glass or porcelain be introduced into the liquid, bumping ceases and continuous boiling takes place.

Distinction between Evaporation and Ebullition—The following are the distinguishing features between evaporation and ebullition—

(*i*) Evaporation is a *slow* process, but ebullition is a *rapid* one

(*ii*) Evaporation takes place at the *surface* of a liquid, while ebullition occurs *throughout its mass.*

(*iii*) Evaporation takes place at *all temperatures*, while ebullition occurs at a *definite* and *constant* temperature, called the boiling point, which depends on the nature of the liquid and the general conditions.

Similarity between Fusion and Ebullition—The following are the points of similarity between fusion and ebullition—

(*i*) The temperature remains constant throughout each process.

(*ii*) The latent heat is absorbed during each change.

(*iii*) Usually there is an increase in volume during each change.

(*iv*) Just as pure liquids may be cooled under suitable conditions below the normal freezing point without solidification taking place, so also they may be heated above their boiling points without boiling.

(*v*) The freezing point as well as the boiling point of a liquid change with pressure.

(*vi*) The freezing point of a solution is *lower* than that of the pure solvent and the boiling point of solution *higher* than that of the pure solvent the difference in each case being approximately proportional to the concentration of the solution, *e.g.*, a saturated solution of common salt freezes at about −22°C and boils at 108°C.

65. **Factors influencing Evaporation**—The rate of *evaporation* of a liquid is experimentally found to depend upon the following six factors :—

(*a*) *The nature of the liquid* —The lower is the boiling point of a liquids, the greater is the rate of evaporation.

(*b*) *The temperature of the liquid*—The higher is the temperature of the liquid, the more rapid is the evaporation. So as to prevent rapid evaporation liquid medicines containing volatile ingredients are labelled to be kept in a cool place.

(c) *Area of the exposed surface*—As a genetal rule, the larger is the area of the surface of the liquid exposed to air, the quicker is the rate of evaporation, *e.g.*, a given quantity of a volatile liquid will evaporate more rapidly when contained in a dish than in a tumbler.

(d) *The pressure on the surface*—The less is the pressure on the surface, the higher is the rate of evaporation. Thus in vacuum the evaporation is extremely rapid. In pharmaceutical works liquid extracts are rapidly concentrated and solid extracts are manufactured by the process of *evaporation in vacuo.*

(e) *The pressure of the upper air close to the liquid surface*—The less is the quantity of the vapour already present in a given volume of air, the higher is the rate of evaporation. Water evaporates more rapidly in dry air than in damp air. For this reason wet linen and muddy roads dry more quickly in winter than in rainy seasons.

(f) *The removal of air in contact with the surface of the liquids*—When the layer of air above the liquid surface containing the vapour is removed by wind, a fresh layer of air comes in the place and helps evaporation which thus proceeds rapidly and continuously. Hence wet linen dries more rapidly on a windy day than on a calm day.

Factors influencing Boiling Point—The boiling point of a liquid is influenced more or less by—

(a) *the vapour pressure on the surface of the liquid*,

(b) *the presence of any dissolved impurity*, solid, liquid or gaseous* and (c) the boiling point which depends to a small extent on the material of the vessel and the degree of cleanliness of its inner surface.

66. **Vapour Pressure**—The vapour of a liquid formed at any temperature exerts pressure like a gas. This is called the *vapour pressure* of the liquid at the corresponding temperature.

Take two barometer tubes A and B (Fig. 47), each about a metre long and completely filled with pure dry mercury. Close the open ends and invert both of them over a trough containing pure dry mercury. Clamp both the tubes vertically side by side. In both tubes the mercury stands at the same level. The height of the mercury column from the surface of mercury in the trough indicates the atmospheric pressure at the time. The space above mercury in each tube is Torricellian vacuum and contains only mercury vapour having negligible pressure.

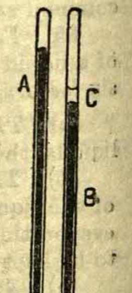


Fig. 47

Into one of the tubes say B, introduce a drop of water by means of a small bent pipette. Being lighter the watar rises to the top of the mercury column and immediately evaporates. The level of the mercury column in B comes down to some extent. This indicates that the vapour so formed exerts pressure within the closed space of B. The difference of mercury levels in A and B measures the pressure of the water vapour.

Again, introduce a drop or two of water into it. The water evaporates and the increased quantity of vapour exerts greater pressure, as shown by a further depression of the mercury column. Continue adding water till a thin layer of water at C appears on the surface of the mercury column, showing that the liquid no longer evaporates. The difference of the heights of the mercury columns determines the pressure exerted by the vapour at the time. Now introduce a few more drops of water. No longer does the mercury column get depressed. This indicates that the pressure exerted by the vapour has reched its *maximum value*.

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*It is for this reason that in the determination of the boiling point of a liquid the bulb of the thermometer should be placed *in the vapour* of the liquid, and *not* in the liquid itself. Again, the boiling point of a solution is always higher than that of the pure solvent.

This shows that under given conditions a closed space can hold only a limited quantity of vapour. In this condition the space is said to be **saturated** with the vapour. Hence if in a closed space the vapour remains in contact with its liquid, the space is definitely saturated with the vapour. The pressure exerted by saturated vapour is consequently *maximum* and is referred to as the maximum vapour tension or *saturated vapour pressure* at the given temperature.

When a space contains less than the maximum amount of vapour that it can contain at the given temperature, it is said to be **unsaturated** and the pressure now exerted is known as the *unsaturated vapour pressure*. Evidently, at a given temperature the pressure of unsaturated vapour is less than that of the saturated vapour. If with different liquids the above experiment be repeated, it would be found that *at any given temperature the saturated vapour pressures of different liquids are different from each other*:

67. Effect of Pressure on Saturated and Unsaturated Vapours—Any vapour in contact with its liquid in a closed space is always saturated. It is found that *saturated vapour does* not *obey Boyle's Law*, but that *unsaturated vapour does.*

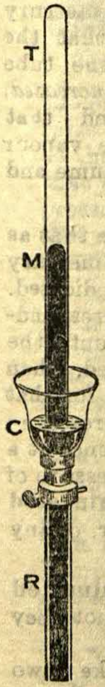


Fig. 48

(1) *Effect of Change of Volume on Saturated Vapour at Constant Temperature*—Take a clean barometer tube. Fill it completely with pure dry mercury and invert the tube over a deep cylindrical trough C (Fig. 48), also containing pure mercury. Introduce sufficient water into the tube, so that the mercury column is depressed to the maximum extent by the water vapour and a layer of water remains on the mercury column. The volume of the enclosed vapour is proportional to the length of the space above the mercury head and the difference between the heights of the mercury column and the barometric height gives the pressure of vapour.

Now push the tube gradually into the trough and observe the height of the mercury column. Note that as the space above the mercury column occupied by the vapour is *decreased*, more and more of the vapour is condensed into water, but that the height of the mercury column above the surface of the mercury in the trough remains *unaltered*. This shows that the pressure of saturated vapour does *not* change with volume. Again, taking care that its open end is always under mercury and that some water is left on the surface of mercury, raise the tube gradually. As the inside volume is *increased*, some of the water quickly evaporates and saturates the space, but the height of the mercury column above the mercury in C remains the same. This indicates that the saturated vapour pressure remains *unaltered.* This takes place until the volume of the space above mercury is increased to such an extent that the whole amount of water gets evaporated. Any further increase in the volume would make the space unsaturated and the pressure of the vapour would be inversely proportional to the volume. That this would be so follows from Boyle's Law.

The above experiment indicates that at a constant temperature the amount of vapour necessary to saturate a space is proportional to the volume of the space, but that the pressure exerted by the saturated vapour remains the same, whatever be the volume of the space. Hence at constant temperature the pressure exerted by saturated vapour is independent of the volume. Thus we come to the conclusion that **saturated vopour does not obey Boyle's Law.**

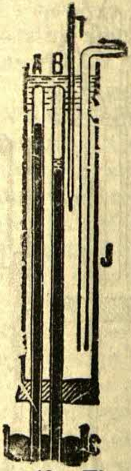
(2) *Effect of Change of Volume of Unsaturated Vapour at Constant Temperature*—Take the same apparatus as in Fig. 48. Introduce a drop or two of water into the tube and observe the depression of the mercury column as the water evaporates. Raise the tube until the water is completely evaporated ; now raise the tube a little further so as to be assured that the space above the mercury becomes completely *unsaturated.* Precaution being taken that the open end remains always under the mercury, gradually raise the tube at this stage. Note that as the volume of the vapour gets *increased,* the height of the mercury column increases. It is found that the pressure of the enclosed vapour *decreases.* Find the vapour pressure in each case and note that the product of the volume and the corresponding vapour pressure is practically constant.

Again, push the tube gradually into the trough. Observe that as the volume of the vapour is decreased, the height of the mercury column decreases. An *increase* in vapour pressure is thus indicated. Note that in this case the product of the volume and the corresponding vapour pressure is practically constant. This goes on until the diminished space becomes saturated with the vapour contained, when liquid would appear and the pressure would be the maximum for that temperature. No further decrease in volume will change the pressure ; only more and more of the vapour will be condensed. Hence at a constant temperature the product of the volume and the pressure of unsaturated vapour is approximately constant ; 'that is **unsaturated vapour approximately obeys Boyle's Law**. With the vapour of any other liquid the above facts can be verified.

68. Effect of Temperature on Saturated and Unsaturated Vapours—It would be found that saturated vapour does not obey Charles' Law, but that unsaturated vapour does.

(1) *Change of Temperature of Saturated Vapour*—Take two barometer tubes A and B filled with pure dry mercury. Invert them over a mercury trough C (Fig. 49). Jacket the tubes by a wider glass tube J, reaching almost up to the mercury surface and having its lower end closed by a cork through which passes the tube.

The jacket contains water which can be heated by introducing steam through a glass tube reaching almost down to the bottom. The temperature is noted by a thermometer T suspended within the jacket. With water at room temperature fill the jacket, The height of the mercury column from the mercury surface in the trough gives the barometric height at the time. Introduce sufficient water into one of the tubes (say, B) by means of a bent pipette, so that the mercury column is depressed to the maximum extent and a layer of water remains on the mercury column. The difference in heights of the two columns gives directly the saturated vapour pressure at the temperature of the bath. The tubes being inserted inside the same bath, a change of temperature affects both the mercury columns equally.



49. Fig

Now by dropping ice into the bath, lower its temperature. Note that as the temperature falls, more and more of the vapour is condensed and that the height of the mercury column increases. The indication is that the vapour pressure, which is always saturated, decreases as the temperature falls. The difference in the heights of the mercury columns in the two tubes gives directly the pressure of the vapour at the corresponding temperature. Again, raise the temperature of the bath by passing steam into it. Note that as the temperature rises, more and more of the water evaporates and that the mercury column gradually goes down. This indicates that the pressure of the saturated vapour increases with the rise in temperature. The experiment can be continued upto a temperature till the water just completely evaporates, after which the vapour becomes unsaturated.

It is to be remembered that so long as the temperature remains constant, the pressure of the *saturated* vapour is not affected by a change of volume. Therefore, the pressure of *saturated vapour* changes with change of temperature, but *not* according to Charles' Law; it always equals the saturated vapour pressure at the particular temperature. This shows that **saturated vapour does not obey Charles' Law.**

(2) *Change of Temperature of Unsaturated Vapour*—The apparatus, which can be conveniently used for the purpose, consists of a wide graduated tube AB (Fig. 50) having at one end a funnel tube E provided with stopcocks S_1 and S_2. The lower end of AB is connected by a thick-walled flexible rubber tubing to another glass tube DF which is open at the top. The lower parts of AB and DF and the whole of the rubber tube are filled with pure dry mercury. The tube AB is jacketted by a wide glass tube J closed at the lower end by a cork, through which the lower end of AB passes. The jacket contains water which can be heated by passing steam into it by a glass tube I, reaching almost up to the bottom of the bath; the temperature is recorded by the thermometer T. The jacket tube is held

vertically by the clamp C fixed to a suitable stand. The tube DF can be moved up and down along a vertical stand against a metre scale and can be clamped at any position.

Open S_1 and S_2. Raise DF until the mercury in AB comes out of S_1, so that the air in AB escapes fully. Close the stopcocks and lower FD, until the mercury level in AB goes down sufficiently to leave a vacuous space above the mercury surface. Evidently the difference in the mercury levels in AB and DF is the barometric height at the time. In the funnel pour some liquid, say water. Open S_1, when some amout of the liquid occupies the space between S_1 and S_2. Close S_1 and then open S_2, so that a *very small* quantity of water is discharged into the vacuous space in AB and at once gets evaporated. Observe that owing to the pressure of the unsaturated vapour the level of mercury in AB sinks. Increase the space above mercury a little more by lowering FD and make sure that the space becomes unsaturated. Note down the position of the mercury level in AB. Subtracted from the initial difference, the difference in the mercury levels in AB and DF, *i.e.*, the barometric height, determines the pressure of the unsaturated vapour at the temperature.

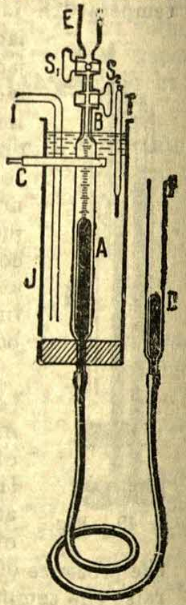


Fig. 50

Now by passing steam into the bath, gradually raise its temperature. At intervals maintain the temperature constant for some time by regulating the flow of steam. Observe that as the temperature rises, the mercury column in AB gets depressed more and more. Raise DF each time, till the mercury level in AB comes to the original position, so that the volume of the vapour is kept constant. Note that the difference in the mercury levels in AB and DF gradually decreases ; this is indicative of an increase in vapour pressure. It is found that the increase of pressure with temperature approximately follows Charles' Law. If the temperature of the jacket is decreased and the volume of vapour is kept constant by the lowering DF, the pressure of the vapour is found to decrease with a fall in temperature ; this nearly obeys Charles' Law. This goes on, till at a certain temperature the space becomes saturated with the vapour contained. If the temperature is further decreased, the pressure being always the saturated pressure for the temperature in question, falls at a quicker rate. It is found, therefore, that with the change of temperature the pressure of unsaturated vapour changes in obedience of Charles' Law. This proves that *unsaturated vapour approximately obeys Charles' Law.* With the vapour of any other liquid the above facts may be verified.

69. Distinction between Saturated and Unsaturated Vapours —When a space contains the maximum quantity of any vapour which can be contained in the volume at the given temperature. the space is said to be *saturated* with the vapour. The pressure exerted by the saturated vapour is the maximum for that temperature. The presence of some amount of the liquid in a closed space ensures that the space is saturated. If the space does not contain the maximum possible amount of vapour at the given temperature, the space is then *unsaturated* with the vapour.

At a constant temperature the pressure of saturated vapour does *not* depend on a change of volume. If the volume is *decreased*, some amount of the vapour is condensed ; but the pressure remains the same. Again if the volume is *increased*, to saturate the space some amount of the liquid at once evaporates, the pressure remaining the same as before. In the case of *unsaturated* vapour, a change in volume at constant temperature produces a change in pressure and approximetely follows Boyle's Law. If the pressure is increased, the volume decreases ; this goes on until the volume becomes saturated with the vapour, after which the pressure cannot be changed any further ; but any attempt at compressing the volume would condense more and more of the vapour. *Saturated vapour, does* not *obey Boyle's Law ; but unsaturated vapour approximately does.*

If the temperature of saturated vapour in contact with its liquid is increased, more and more of the liquid evaporates ; the pressure increased with temperature, it being always the saturated pressure at the particular temperature. If the temperature is *decreased*, some volume of the vapour is condensed, the pressure falling to the saturation pressure for that temperature. The change of pressure in either case does not, however, take place in consonance with Charles' Law. But in the case of *unsaturated* vapour the pressure changes with temperature approximately in accordance with Charles' Law. If the temperature decreases, the pressure decreases, till at a certain temperature the space becomes saturated with the vapour present. On further lowering the temperature, more and more of the vapour is condensed the pressure each time coming to the saturated pressure at the particular temperature. *Saturated vapour*, therefore, *does* not *obey Charles' Law* ; *but unsaturated vapour does approximately.*

The following table supplies the saturated vapour pressure of water with the corresponding temperature.

Saturated Vapour Pressure of Water with Temperature

Temp. (in °C)	Press. (in mm.)	Temp. (in °C)	Press. (in mm.)	Temp. (in °C)	Press. (in mm.)
4	6·10	16	13·62	28	29·94
6	7·01	18	15·46	30	31·77
8	8·04	20	17·52	32	35·59
10	9·20	22	21·02	34	40·02
12	10·53	24	22·09	36	44·90
14	11·48	26	25·65	38	50·26

70. Pressure by a Mixture of Gases and Vapours—Take the same apparatus as in Fig. 51. Keep the jacket filled with water at the room temperature. Open S_1 and S_2 and draw in a quantity of dry air into AB. Adjust DF, so that the mercury levels in AB and DF are the same. The air in AB is now at atmospheric pressure. Mark the position of the mercury in AB. Put some ether in the funnel. Through the stopcocks introduce sufficient liquid into AB so as to leave some of it unevaporated. Observe that the mercury column in AB is depressed by the pressure of the vapour. Now raise DF till the level of mercury in AB is brought back to its former position. As the volume of the air is the same as before, its pressure is equal to the atmospheric pressure. The difference of mercury levels in DF and AB is, therefore, equal to the pressure of saturated vapour at the room temperature.

Again, open S_1 and S_2. Raise DF till the air, the ether vapour and a little of the ether are driven out of AB, some only being left on the mercury surface in AB. Close S_1 and S_2 and lower DF sufficiently to get the ether evaporated. The difference of mercury levels in DF and AB remains the same, even though DF is lowered still farther, provided some ether is left in AB. This difference gives the pressure of saturated ether vapour at the room temperature, when formed in vacuum. It is found that *the pressure of the saturated ether vapour at a particular temperature, whether formed in vacuum or in presence of air, is the same.*

If in the above experiment the volume of the space over mercury in AB be increased or decreased by lowering or raising DF, the total pressure due to air and vapour will found to change. But if due to a change of volume of the air the alteration in pressure as given by Boyle's Law be taken into account, the pressure due to the vapour alone is found to remain the same, provided there is enough of the liquid to keep the space saturated. But if the space is unsaturated, then for any change in volume the total pressure is found to change according to Boyle's Law. If with water or any other liquid the experiment be repeated, similar result will be obtained. Again, if the saturated pressures of two liquids be *separately* determined in closed space, the saturated pressure, when both the liquids are *simultaneously* present in given volume, would be found to be equal to the sum of the pressures exerted by each, provided there is no chemical action between the two. Summarised in *Dalton's Law*, the results are given below :

Dalton's Law of Vapours—The laws of *Partial Vapour Pressure*, known as Dalton's Law, may be stated thus :

(1) The saturated pressure of a particular vapour in a closed space, at a given temperature depends only on this temperature and is independent of its volume or of the pressure of any other vapour or gas, provided they do not chemically react with each other.

(2) The total pressure exerted in a closed space by a mixture of gases and vapours having no chemical action upon one another is the

sum of pressures which each would separately exert, if it alone occupied the whole space at the existing temperature.

The first law applies only when the space is saturated, while the second law always holds good, no matter whether the space is saturated or not.

Example :

Two bulbs of volumes 1 litre and 2 litres are connected by a tube of narrow bore. Initially the air inside the two bulbs is at 0°C and 76 cm. pressure. The smaller bulb alone is raised to the temperature of 100°C. What is the pressure inside the system ? [And. U.—1952]

Ans. Suppose that the smaller bulb is isolated and then raised to 100°C. Then the pressure inside the bulb is $76\times(1+100\times{\cdot}00366)$ cm. Now connect this bulb to the larger bulb which to start with, is supposed to be empty. If the pressure of the two bulbs of total capacity of *3* litres be P_1 cms., then from Boyle's Law

$$76\times(1+100\times{\cdot}00366)\times1=P_1\times3,\quad\text{whence}\quad P_1=\frac{76\times1{\cdot}366}{3}$$

Next isolate the larger bulb, the pressure inside being 76 cm. and connect it to the smaller one which is now supposed to empty. Then if the pressure of the system be P_2 cm., it follows from Boyle's Law that

$$P_2\times3=76\times2,\quad\text{whence}\quad P_2=\frac{76\times2}{3}.$$

Dalton's Law of Pressure, then, ensures that the total pressure of the system

$$\text{is}=P_1+P_2=\left(\frac{76\times1{\cdot}366}{3}+\frac{76\times2}{3}\right)\text{cm.}=(76\times1{\cdot}122)\text{cm.}=85{\cdot}272\text{ cm.}$$

71. Vapour and Gas : Critical Temperature—Use of the two terms *vapour* and *gas* is made with no hard-and-fast distinction. Generally speaking, *vapour* is used to mean the gaseous state of a substance which is a liquid or a solid at ordinary temperature and pressure ; instances of this are water vapour, ether vapour, alcohol vapour and vapour of iodine or of camphor. The term *gas* generally denotes a substance which under ordinary temperature and pressure remains in the gaseous state ; such substances are oxygen, nitrogen, etc. A general distinguishing feature is that a *vapour* can be easily liquefied at the ordinary temperature by increase of pressure alone, while under similar conditions a *gas* would get more and more compressed.

It is found that even a vapour cannot be liquefied by pressure, however great, if it be above a particular temperature which depends on the nature of the vapour. Hence above this temperature the vapour behaves like a gas obeying Boyle's Law. On the other hand, if cooled below a certain very low temperature, a so-called permanent gas (*e.g.*, hydrogen or oxygen) can be liquefied by suitable pressure, so that below this temperature the gas behaves as the vapour of the liquid.

Thus for every substance in the gaseous state there is a certain temperature such that the substance can be liquefied by suitable pressure alone if it be below this temperature ; but it cannot be liquefied, if it be above that temperature, however great the pressure may be. This temperature is known as the **critical temperature** of the gaseous substance. Hence *below* its critical temperature a substance in the gaseous state behaves like a vapour and above this it behaves as a *gas*. In other words, *a gas is not other than a vapour above its critical temperature.* The critical temperature of oxygen, nitrogen and hydrogen are very *low*, being respectively—$-118{\cdot}8°$C., $-147{\cdot}1°$C and $-239{\cdot}9°$C. Each gas must, therefore, be cooled below its critical temperature before it can be liquefied by the application of suitable pressure.

Critical Pressure—The pressure required to liquefy a gas at its critical temperature is called its *critical pressure.*

Critical Volume—The volume occupied by one gramme of a geseous substance at its critical temperature and critical pressure is called its *critical volume.*

Examples :

1. Dipped into mercury, a barometer tube contains some air. There is introduced a quantity of water, a part of which remains unevaporated and the height of the mercury column is now found to be 70 cm. What is the height of the column when the tube is depressed so as to reduce the volume occupied by the air to half the original volume ? (Atmospheric pressure and temperature at the time are 76 cm. and 20°C respectively. Saturated vapour pressure of water at 20°C is 17·5 mm.)

Ans. Obviously, the pressure of air and saturated vapour inside $=(76-70)$ cm. $=6$ cm.

$\therefore$ the initial pressure of air only $=(6-1{\cdot}75)$ cm. $=4{\cdot}25$ cm.

When the volume of the air is reduced to a half, the pressure of the saturated vapour remain the same ; but the new pressure of air becomes equal to $(2\times4{\cdot}25)$ cm. *i.e.*, 8·5 cm.

Let the new length of the mercury column be $=h$ cm. Accordingly,

$h+8{\cdot}5+1{\cdot}75=76$, whence the desired height h is $65{\cdot}75$ cm.

2. Five hundred c.c. of hydrogen is collected over water at 25°C when the barometric height is 762 mm. Calculate the weight of hydrogen when dried. (Density of hydrogen at N.T.P. $=0{\cdot}09$ gm. per litre ; saturation water pressure at 25°C is 23·55 mm.)

[Mad. U.—1950]

Ans. Since hydrogen is collected over water, the gas is saturated with water vapour, and the pressure exerted by the mixture is the sum of the individual pressures. Let the pressure exerted by hydrogen alone at 25°C and having a volume of 500 c.c. be p mm.

Then, by the condition of experiment, $p+23{\cdot}55=762$, whence $p=738{\cdot}45$.
Then, from Boyle's Law, if the volume be v c.c. at N.T.P.,

$$\frac{760\times v}{273}=\frac{738{\cdot}45\times500}{273+25}\text{, whence } v=445{\cdot}1 \text{ c.c.}$$

So of this volume the mass of hydrogen $=(445{\cdot}1\times{\cdot}00009)$ gm. $=0{\cdot}04$ gm. (approximately).

3. A uniform tube, 80 cm. long and 1 sq. cm. in cross-section, is filled, with mercury and it is inverted over a mercury trough with the open end 2 cm. below the mercury. The height of the mercury column is found to be 74·4 cm., the true atmospheric pressure being 76 cm. If the error, that has crept in, is due to the pressure of (unsaturated) water vapour, what will be the volume of the latter at the atmospheric pressure ?

Ans. The length of the space above mercury $= \{80-(2+74\cdot4)\}$ cm. $=3\cdot6$ cm.

$\therefore$ the volume of the water vapour $=(3\cdot6\times1)$ c.c. $=3\cdot6$ c.c. and the pressure $=(76-74\cdot4)$, *i.e.*, 1·6 cm. of mercury.

If the volume of the vapour at atmospheric pressure be v c.c., Boyle's Law states.

$76\times v=1\cdot6\times3\cdot6.$ whence the required volume v is 0·7 c.c.

4. A mass of air is saturated with with water vapour at a temperature of 100°C; on raising the temperature to 200°C without changes of volume the pressure is raised to two atmospheres. Find the pressure at 0°C of this volume of the dry air alone. [Pat. U.—1926]

Ans. Let the respective pressures of dry air at 0°C and 100°C be P_0 cm. and P cm. of mercury.

Now at 100°C the pressure of the saturated water vapour = 76 cm.

$\therefore$,, total pressure of the (saturated) moist air $=(P+76)$ cm.

As the vapour is unsaturated at 200°C, the standard equation $\frac{P}{T}=\frac{P'}{T'}$ gives

$$\frac{P+76}{100+273}=\frac{2\times76}{200+273}, \text{ whence } P=43\cdot734.$$

$$\text{Again, } \frac{P_0}{273}=\frac{43\cdot734}{100+273}, \text{ or } P_0=\frac{43\cdot734\times273}{373}=32\cdot01.$$

Thus the pressure sought for is 32·01 cm.

5. At one end a capillary tube of uniform bore is sealed and a column of air is enclosed in the tube by a pellet of water. The enclosed air column measures 10 cm. at a temperature of 17°C, and 16 cm. at 67°C. Assuming that the vapour pressure of water at 17°C is 15 mm., find the vapour pressure at 67°C. (Barometric height at the time = 760 mm.)

Ans. Denote the vapour pressure of water at 67°C by x mm.

Owing to the presence of the pellet of water the column of air is always saturated with water vapour. Now we know that the saturated vapour pressure is not affected by change of volume and the total pressure inside is therefore, always equal to the atmospheric pressure.

Since due to dry air only the initial pressure $=(760-15)$ mm. $=745$ mm. and the final pressure $=(760-x)$ mm., we know from the relation $\frac{PV}{T}=\frac{P'V'}{T'}$ that

$$\frac{(760-x)\times16}{67+273}=\frac{745\times10}{17+273}, \text{ whence } x=214\cdot1.$$

Thus the desired pressure is 214·1 mm.

72. Cold caused by Evaporation—Whenever a liquid evaporates at any temperature, some heat is necessary to be supplied as latent heat for such a change of state. If the heat is not supplied from external source it is absorbed from the liquid itself or the surrounding air and other objects with which the liquid and the containing vessel may be in contact. During the process these are, therefore, cooled, and the more rapid the evaporation, the greater the cooling produced.

The following experiments and observations testify to the above statement. :—On the palm pour a few drops of ether or alcohol. As the liquid evaporates, it absorbs heat from the palm which cools thereby. In some minor surgical operations local anaesthesia is produced by an ether spray. The bulb of a thermometer is wrapped with a piece of muslin and a few drops of ether are poured over the muslin. With evaporation of ether the thermometer records a drop in temperature. Tea is sometimes poured out in a saucer before sipping : tea in the saucer, being exposed over a large surface, evaporates more quickly and gets cooled. If for some time a person remains with wet cloths on, he is likely to catch cold, for while water evaporates, it absorbs heat from his body.

Kept in unglazed earthen pitchers, water in hot climate becomes cooler than the water kept in glass or metal vessels. The reason is that the walls of earthen pitchers being porous, water always oozes out and evaporation takes place all over the vessel, whereas in a metal vessel it may take place directly from the water surface only through the mouth. Quicker evaporation, therefore, produces greater cooling of water in the porous vessel. But when the atmosphere is saturated (as in rainy season), very little evaporation occurs and we do not find any appreciable difference in the temperatures of the water in the two vessels. When streets are watered, the water evaporates, taking the latent heat from the surrounding air, which is thus cooled down.

(i) The perspiration coming out of the pores of our skin gets evaporated *slowly* ; the vapour remains as a sort of enveloping layer over the skin. (ii) The wind of a fan removes this layer of vapour ; fresh layer quickly takes its place and this produces *rapid* evaporation. This causes removal of heat from the skin and produces cold. (iii) When the forehead of a person is sponged with eau-de-cologne, he feels refreshed, for the liquid, which contains a large proportion of alcohol, evaporates quickly and takes away much of the local heat. (iv) The same purpose is partially effected by supplying a piece of wet linen on the forehead and then fanning briskly. Hung near the doors and windows of a room, a wet *khus khus* screen or a moist and coarse canvas screen keeps it cool. As the water evaporates, the latent heat is taken from the air in the room, which thereby gets cooled. (v) Put some water on a block of wood and in the water place a beaker containing some ether. Blow air through the ether which gets quickly evaporated, Rapid cooling is produced, whereby the water on the wood may ultimately freeze.

Leslie's Experiment—Under the receiver of an air pump are placed a small shallow metallic dish containing some water and a vessel containing strong sulphuric acid. As the pump works, the pressure inside falls ; also some of the vapour formed is removed by the pump and the rest is absorbed by sulphuric acid. This produces a very rapid evaporation, whereby temperature of the water falls down quickly so that a thin layer of ice appears on the surface of the water. Contained under pressure in a vessel, liquid carbon dioxide is allowed to escape through a fine jet inside a closed wooden box ; the rapid evaporation of the liquid, that comes out cools it to such an extent that it solidifies.

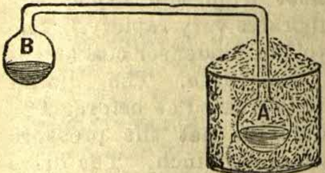


Fig. 52

Wollaston's Cryophorus—It consists of a long bent glass tube which has a bulb at each end (Fig. 52) and it contains a little water and water vapour, but *no* air. To start with all the water is transferred into one of the bulbs (say B) and the bulb A is kept inside a freezing mixture. As the vapour in A condenses, the pressure inside falls and *rapid* evaporation of the water in B takes place. The water in B is thus gradually cooled.

On holding any bulb with plam it is slightly warmed and even at the room temperature the liquid shows boiling under diminished pressure. The lowering of temperature caused by rapid evaporation of ammonia is utilised in *Cold Storage* wherein meat and other perishable foodstuffs are stored in warm climates. In refrigerating machines the same principle is utilised.

73. **Ice Machine**—This is used for the manufacture of ice on a large scale. The essential parts of the machine are (Fig. 53) :—

(1) A *tank* T filled with a strong solution of brine or of calcium chloride,

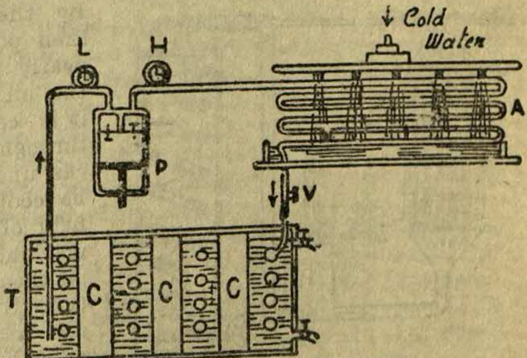
(2) *Tube Coils*—Liquefied ammonia or carbon dioxide is contained in *coils* of copper tube immersed horizontally within the solution. The circles in this tank indicate transverse section of the coils, These coils are called the *evaporator coils*.

(3) *Ice-cans* (C, C)—These contain water to be frozen.

(4) A *pump* (P)—This is a combined exhaust and compression pump and is worked electrically. It is connected to one side *directly* with the evaporator coils.

(5) A *condensing coil* (A)—The pump P is connected to the other side with the condensing coil A and the *regulating valve* V.

(6) *Pressure gauges* L and H—The former is to measure low pressure and the latter for high.



Fig, 53—A common Ice-Machine

Action—When the pump P works as a compression pump, it forces gaseous ammonia under a high pressure of about 155 pounds per square inch into the condenser coil A which is kept cool by a constant flow of water over its surface. The ammonia thus gets liquefied and is allowed to pass slowly through the valve V into the evaporator coils. The pump P now works as an exhaust pump, and asborbing the latent heat from the brine, the liquid ammonia in the evaporator coils evaporates very rapidly. The evaporated gas is then compressed again into the condenser coil and is again liquefied under the influence of cold bath. The liquid ammonia passes into the evaporator coil and is treated as before. The evaporated ammonia is pumped out so quickly that the pressure inside the coil does not exceed 34 pounds per square inch. The brine is thus continuously cooled and is brought down to the temperature of about -16°F, so that the water in the cans C, C gets frozen into blocks of ice.

74. **Cold caused by Expansion of a gas**—When a compressed gas is allowed to expand through a narrow orifice, the gas has to do work against the molecular attraction. The energy necessary for this is taken from the gas which is thereby cooled. This iscalled the **Joule-Thomson Effect**. If the carbon dioxide stored under great pressure, is allowed to escape through a narrow opening into a vessel open to the atmosphere, intense cooling is produced whereby the carbonic acid may be solidified. The principle of cooling by expansion is utilised in the modern method of liquefying air and other gases.

Liquefaction of Air—(*Linde's Method*)—The essential parts of the machine are (Fig 54) :—

(1) a powerful *pump* P acting as a compressor. (2) a *cooler* R which in a cylindrical vessel containing cold water in which pipes conveying the compressed air are placed and (3) *double-walled coil* S provided with an inner tube *S*, is placed surrounded by a non-conducting material within a box and has at its lower end a small *needle valve* V.

Action—The air is at first freed completely from CO_2 by being slowly drawn through a series of line towers (not shown in Fig. 54). By the pump P this air is then pumped to a pressure of nearly 200 atmospheres. As the air is thus again heated, it is cooled by being passed through the cooler R. The gas under high pressure may be cooled to—4°C with the help of ammonia refrigerating machine. The cold compressed air then passes down the inner tube S of the double-walled coil and escapes through the valve V. As the air suddenly expands, its pressure falls to about 16 atmospheres and its temperature records a considerable fall. This cooled air returns through the space between the

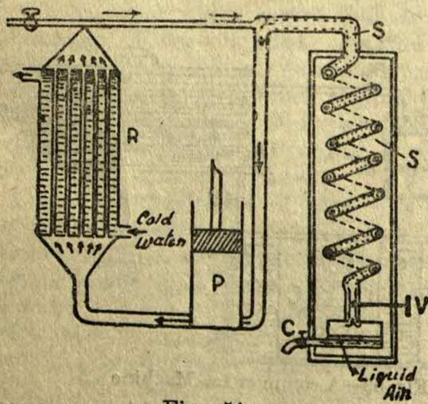


Fig. 54

two tubes and thus cools the air moving through S towards the valve V. Hence as the pump is worked, the temperature of liquefaction is reached. The critical temperature for oxygen is—118·81°C. The liquid air collects in a vessel attached below the valve and can be drawn off through the stop-cock C. The liquid air is stored in *Dewar vessel* which is a double-walled glass vessel the intervening space being made vacuum. Under a pressure of one atmosphere the boiling point of liquid air is —183°C. By making it boil under reduced pressure, a temperature of—217°C (56°K) can be attained.

***75. Production of Low Temperature**—By using a modified apparatus working on the same principle, hydrogen was liquefied by Dewar in 1891 and then by Travers. The gas was previously cooled to—200°C, and was subjected to a pressure of 290 atmospheres. The critical temperature of hydrogen is—239·9°C and liquid hydrogen bolis at —262·7°C under a pressure of 1 atmosphere. By making it boil under a reduced pressure, the liquid can be frozen to a white solid.

Cooling the gas down to—269°C, *i.e.*(4°K), Kemerlingh Onnes liquefied helium in 1908, the critical temperature of the gas being 5·26°K, and its boiling point under a pressure of one atmosphere being 4·26°K. Onnes could not solidify helium, although in 1911 he reduced the temperature to 0·99°K under a pressure of 0·013 mm. In 1926 Dr. Keesom succeeded in partially solidifying helium by subjecting it to an enormous pressure of 130 atmospheres and two years later by Giauque in 1927 it was pointed out that temperature can be lowered by demagnetising magnetised oxygen. In the case of gadolinium sulphate the magnitude of the fall was independently calculated by Debye in 1926. Gaiuque reached to temperature of 32 °K by using this substance. Afterwards De Hass, Krammers and others succeeded in recording a temperature of 0·05°K,

76. Boiling Point of a liquid—A barometer about a metre in length is completely filled with pure dry mercury and is inverted over a mercury trough (Fig. 55) By means of a bent pipette some quantity of wateris introduced into the tube. The tube is then jacketted completely with a wider glass tube with ends closed by corks, as shown in the figure. Steam is now introduced into the jacket tube by small tube at its upper end and is allowed to eacape through a tube below. With rise of temperature the pressure of the water vapour in the tube increases and the mercury column is gradually depressed. The difference between the initial height and the height of the mercury column at any instant reads the pressure of the water vapour at the corresponding temperature. Ultimately a steady state is reached. If there be sufficient water present, the level of mercury inside the tube is found to be the same as that in the trough. At this stage the water vapour inside is at the temperature of boiling water and the *pressure of the vapour is equal to the atmospheric pressure.* So we infer that the vapour pressure of water at the boiling point is equal to the atmospheric pressure. Hence the **boiling point of a liquid** *may be defined as the temperature at which the saturated vapour pressure of the liquid is equal to the pressure above its surface.*

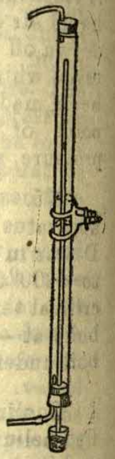


Fig. 55

Hence so long as the vapour pressure is less than the pressure on the surface, the liquid would continue to evaporate and when the two pressures are equal, the liquid would boil. The *standard* boiling point of a liquid is referred to under *normal* atmospheric pressure. Thus the assertion that the boiling point of water is 100°C means that at 100°C the pressure of water vapour is equal to the normal atmospheric pressure, *i.e.*, a pressure of 76 cm, of mercury.

77. **Effect of Pressure on the Boiling Point**—The very definition of boiling point suggests that a liquid would boil at a lower temperature if the pressure on its surface is decreased, and at a higher temperature if this pressure is increased.

(a) **Boiling under Diminished Pressure**—This may be demonstrated by the following experiment. (i) Boil some water in a beaker and place it quickly under the receiver of an air pump. Wait till the boiling ceases and then work the pump vigorously. The water is found to be boiling again. The explanation is that the vapour pressure at the decreased temperature is less ; so when by working the pump the pressure on the surface of the liquid is reduced to a value equal to the vapour pressure at the time, boiling recommences.

(ii) **Franklin's Experiment**—Take a strong round-bottomed flask and fill it to about a half with water. Boil water for some time so as to expel all the air by the steam. Remove the burner and immediately cork up the flask tightly. Support it inverted on the ring of a retort stand (Fig. 56). Wait till the boiling ceases. The space above the water is now filled with saturated vapour at a fairly high temperature, which produces the pressure on the water surface. Now pour a stream of cold water on the flask ; the water within the flask is seen to boil again, although the temperature has fallen below the normal boiling point. So long as the water in the flask remains sufficiently warm, the addition of fresh water starts the boiling again. The process can be repeated several times, the boiling taking place each time at a lower temperature.

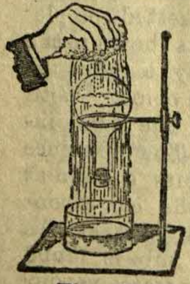


Fig. 56

Explanation—On cooling, some of the vapour contained within the flask is condensed, The pressure on the surface of the water is thereby reduced. Although much below the normal boiling point, the temperature of the water stands at about the boiling point corresponding to the reduced pressure. So boiling appears and continues until the space above is saturated with vapour, and at this lower temperature the vapour pressue becomes equal to the saturated pressure. The pouring of more water causes further condensation which starts the boiling again.

(b) **Boiling under Increased Pressure**—This may be demonstrated by :

Papin's Digester—This instrument consists of a strong metal vessel which is suitably closed by a lid and fitted with a pressure gauge and safety valve. The pressure gauge indicates the pressure of the steam within the vessel while the valve controls the pressure of the steam. The valve automatically opens when the pressure exceeds a certain limit and allows the excess steam to escape. A thermometer is inserted through the lid.

When the vessel is heated, the vapour accumulates inside and the pressure due to this vapour on the surface of water increases. Water, therefore, does not boil till a temperture is reached at which the maximum vapour pressure is equal to the pressure on the surface, It is found that water boils at about 120°C when the pressure on its surface is increased to 2 atmospheres, and at 200°C under a pressure of 15 atmospheres,

78. Regnault's Experiments—To find the *saturated vapour pressure of water at various temperatures* (from 50°C to about 230°C) Regnault devised an apparatus whereby the *variation of the boiling point of a liquid with pressure*, greater as well as less than one atmosphere, could be determined. Liquid is known to boil when its saturated vapour pressure is equal to the super-incumbent pressure. Hence by knowing the variation of the saturated vapour pressure with temperature, the variation of the boiling point of liquid with the pressure on its surface can be found.

The liquid is contained in an air-tight copper boiler B (Fig. 57) the upper part of which is connected through a Liebig's condenser C with a large round-bottomed flask E. The flask is kept immersed in a bath of constant temperature. This pressure of the air within it can be increased or decreased by a pump and can be measured by an open U-tube mercury manometer M. A sensitive thermometer is introduced through the lid of the boiler B so as to read the temperature at which the liquid boils.

By aid of the exhaust pump the pressure of the enclosed air is at first reduced to a certain value. Then as the liquid within the boiler is heated, the vapour formed, in trying to escape through the condenser, comes in contact with cold walls, condenses into liquid and comes back into B, After some time the liquid is found to boil and the temperature is noted. The pressure of the vapour at this temperature, which is equal to that of the air in E, is obtained from the manometer. By means of a condensing pump the pressure inside is then increased to a certain value above one atmosphere. The temperature of the liquid is found to rise and become steady when the liquid boils at some higher temperature. The pressure is *increased* in successive steps; the corresponding boiling point is noted in each case and is found to *increase*. As the saturated vapour pressure of a liquid at its boiling point is equal to the super-incumbent pressure, this method gives the boiling point of the liquid under various pressures. The method also gives the saturated vapour pressure of the liquid as given by the manometer at various temperatures. By this method Regnault was able to find the saturated vapour pressure of water of from 10°C to 230°C. the value at the latter temperature being 27·5 atmospheres. Thus by knowing the boiling point of water, the corresponding vapour pressure is known from Regnault's table.

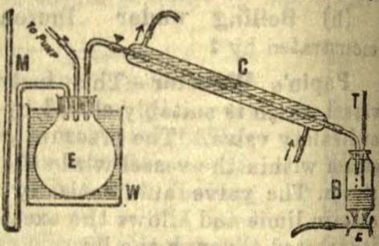


Fig. 57

Evidently any variation in the atmospheric pressure changes the boiling point of the liquid. Thus at sea-level, where the atmospheric pressure is maximum, the boiling point also attains the highest value. At higher altitudes the atmospheric pressure decreases and so the boiling point comes down. The boiling point of water at the top of a mountain is, therefore, less than 100°C. Thus, at Darjeeling (6800 feet above the sea-level) water boils at about 94°C. At Quito, which is the highest city in the world (9520 feet above the sea-level) water boils at 90°C. At very high altitudes, therefore, it would be impossible to cook food properly in open vessel. This difficulty can be avoided by taking closed vessels in which the vapour, that is formed automatically, increases the pressure on the surface of the water and hence the boiling point rises. The boiling point of water is found to decrease by 1°C as the pressure falls by about 26·1 mm. Hence in determining the upper fixed point of a thermometer the atmospheric pressure at the time is to be carefully noted and a correction, if necessary, is to be made for the variation of the boiling point of water.

The following Table supplies the variation of Boiling Point of Water with Pressure.—

Boiling Point of Water at Specified Pressures

Pressure (in mm.)	Boiling point (in °C)	Pressure (in mm,)	Boiling point (in °C)
750	99·63	764	100·15
752	99·70	766	100·22
756	99·85	768	100·29
758	99·93	770	100·37
760	100·00	772	100·44
762	100·07	774	100·51

79. Measurement of Saturated Vapour Pressure of Water—The maximum vapour pressure of water at various temperatures can be determined in the following way :

(a) *For ordinary temperature* (between 0°C and 100°C): by *Regnault's Method*—Two barometer tubes are completely filled with pure mercury and are inverted over a mercury trough. They are jacketted (as shown in Fig. 50) by a wider glass tube, reching almost up to the mercury surface in the trough. Into one tube sufficient water is introduced so that a thin layer of it is allowed to remain on the surface of the mercury, while the other tube serves as the barometer. The water in the jacket is heated by passing steam into it. The temperature is noted by a thermometer, while the pressure of the saturated vapour is obtained directly from the difference of the mercury levels in the two tubes.

(b) *For low temperature* (below 0°C) : by *Gay-Lussac's Apparatus*—The tube, in which water is to be introduced, is bent over at the upper end. The bent portion, which generally ends in a bulb, contains the liquid and in order to lower the temperature to the required extent, it is surrounded by a freezing mixture. The other tube reads the atmospheric pressure and the difference of mercury levels determines the vapour pressure directly.

(c) *For higher temperature* (above 190°C)—For this purpose Regnault's apparatus (as shown in Fig, 57) is used and the usual procedure is followed.

Ramsay and Young's Experiment—For studying low vapour pressure of liquids Ramsay and Young modified Regnault's apparatus. Their apparatus consists of a wide glass tube A provided with a side tube B which is connected to a large glass bulb C (Fig 58). This bulb can be immersed in a freezing mixture and acts as an air reservoir. The outlet pipe from C is connected to a T-piece D—one end E leading to an air pump, and the other to a manometer M. The thermometer T and a tap funnel F meant to introduce the liquid under test are inserted air-tight into the tube A. To the bulb of the thermometer is fastened some cotton wool.

The tube is placed in a water bath (not shown in the figure) and the temperature is kept sufficiently high. Through the tap funnel and into the tube is introduced the liquid which moistens the cotton wool. Due to the temperature the liquid evaporates and soon the space around the temperature bulb is saturated with the vapour. The temperature now shows a steady value, which corresponds to the *boiling point temperature* of the liquid at a pressure maintained by the pump. This pressure is, however, recorded by the manometer. By altering with the help of

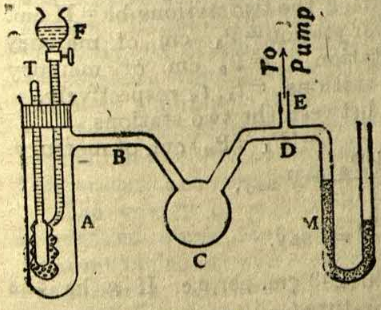


Fig. 58

the pump the pressure within the apparatus and taking the corresponding thermometer readings, series of measurements of the boiling points, corresponding to various pressures, can be obtained. So as to maintain a constancy of pressure during observation, the most essential point is to make the whole apparatus air-tight.

80. Laws of Ebullition—The following are the observed laws under which a liquids boils ;

(*a*) A liquid boils at the temperature at which its maximum vapour pressure is equal to the pressure on the surface.

(*b*) Under a particular pressure every liquid has a definite boiling point. The boiling point rises or fallsaccording as the super-incumbent pressure increases or diminishes.

(*c*) If the pressure remains unchanged, the temperature, at which a liquid begins to boil, remains stationary until the whole of the liquid is converted into vapour.

(*d*) To convert unit mass of a liquid into vapour without any change of temperature. a definite quantity of heat, known as the *latent heat of vaporisation* of the liquid, is required.

81. Determination of Height by Boiling Points (Hypsometry)—When a liquid boils, its vapour pressure is equal to the superincumbent pressure. The boiling point of water is conveniently determined at any place by means of a *hypsometer*. The corresponding saturated vapour pressure is obtained from Regnault's table and hence the atmospheric pressure at the place is found. The difference between the atmosperic pressures at two stations can be thus determined ; but this difference is equal to the weight of vertical column of air of unit cross-section and of height equal to the difference of altitudes between the two stations The pressure and the temperature of the air column are not uniform throughout but may be taken to be the mean of the pressures and the temperatures at the two stations, for they do not differ much.

Let the dfference of altitudes between the two stations be = H cm.
,, atmosphetic pressure at the lower station = P_1 cm. of mercury
atmosphetic pressure at upper station = P_2 cm. of mercury
and the temperarures at the two stations = t_1, t_2 respectively.

Then the difference of pressures between the two stations = $(P_1 - P_2)$ cm. of mercury.

$$\therefore \text{ the mean pressure} = \frac{P_1+P_2}{2} = P \text{ say,}$$

$$\text{Also the mean temperature} = \frac{t_1+t_2}{2} = t \text{ say.}$$

The density of air at N.T.P. = 0·001293 gm. per c.c. If ρ be the mean density at pressure P and temperature t,

$$\frac{P}{\rho\times(t+273)} = \frac{76}{0{\cdot}001293\times 273}, \text{ whence } \rho = \frac{0{\cdot}001293\times 273}{76}\times\frac{P}{t+273}$$

∴ the weight of H c.c. of air of mean density ρ

$$= \frac{0{\cdot}001293\times 273\times 981}{76}\times\frac{HP}{t+273}.$$

As this is equal to the weight of a column of mercury of unit cross-section and of height $(P_1 - P_2)$ cm, we readily obtain

$$(P_1 - P_2) \times 13{\cdot}6 \times 981 = \frac{0{\cdot}001293 \times 273 \times 981}{76} \times \frac{HP}{t+273}.$$

whence in centimetres $H = \dfrac{13{\cdot}6 \times 76}{273 \times 0{\cdot}001293} \times \dfrac{(P_1 - P_2)(t+273)}{P}$.

*82. **Molecular States of Matter**—According to the *molecular theory of matter*, a vapour is considered to contain gronps of small aggregation of molecules. These groups move about like single molecule but with relatively low velocities. With lowering of temperature the number of these groups per unit volume increases and ultimately they may coalesce into a liquid drop.

In a *liquid* there are groups which are composed of a larger number of molecules. A molecules in the liquid state, therefore, moves with a velocity smaller than that of a molecule of its vapour at the same temperature. In a *solid* the heavy groups lose even the small linear velocity they had in the liquid state and consequently have fixed positions relative to other groups of molecules. They can only oscillate over a small distance about their normal positions of rest. When the groups are arranged in a regular array, the form of the solid is *crystalline*, and where irregular, it is *amorphous*.

Effect of Heat—When a solid is heated, the molecular groups take up the heat energy and vibrate more violently about their mean positions. The heat supplies the consequent increase in the kinetic energy of the molecules.

With rise in temperature the average energy of vibration of the groups goes on increasing, till at a certain temperature some of the groups begin to detach themselves from the adjacent groups, break up and form into liquid type molecular groups. Fusion now sets in and any further supply of heat energy is utilised in breaking the groups. Hence during fusion the temperature remains constant. If the body be crystalline, the groups begin to break up at a *definite* temperature which gives a *definite* melting point of such a solid; but in an amorphous state the loosely-held molecules are detached at a temperature lower than that of the more regular or more closely packed ones and hence in such a substance the melting begins at one temperature and completed at some little higher temperature.

The energy of motion of some of the groups may occasionally increase so much that they may overcome the attraction of the adjacent molecules and escape from the liquid surface; this explains **evaporation**. As the molecules with the highest kinetic energy leave the liquid, the mean kinetic energy of the remaining molecules decreases to some extent and so the temperature of the liquid somewhat falls. This explains the *cooling produced by evaporation*.

Evaporation increases with rise of temperature. With further supply of heat the gas moleceles increase in energy so much that they escape as bubbles from the liquid. **Boiling** begins at this stage. The temperature, at which this occurs, depends on the nature and aggregation of the gas molecules. Hence for a particular liquid under assigned conditions boiling occurs at a *definite* temperature called the **boiling point.**

Any further supply of heat energy is utilised in breaking up the liquid groups into similar gas molecules ; the temperature, therefore, remains constant at the boiling point so long as boiling continues. The kinetic energy of the gaseous molecules is far greater than the kinetic energy of the molecules in the liquid groups at the boiling point ; this excess of energy, supplied by the heat absorbed while boiling is going on, appears at the **latent heat of vaporisation.**

If evaporation takes place in a *closed space*, there would be constant interchange of molecules between the vapour and the liquid. Some gas molecules at that stage leave the liquid surface and move about in the space above and while moving towards the liquid, some of these may be caught back into it. The rate of escape of the molecules depends on the temperature, while the number of molecules per unit volume in the space above the liquid depends on pressure.

If at first the space contains little or no vapour, the number of molecules leaving the liquid must exceed the number of molecules re-entering the liquid per unit time. But the latter increases relatively as the quantity of vapour in the space increases, until finally as many molecules re-enter the liquid per second as leave the liquid. At this stage number of vapour molecule per unit volume of the space remains the same. The space is then said to be *saturated* with the vapour ; the pressure of the vapour is now the **saturated vapour pressure** at the temperature, which remains constant so long as the temperature remains the same. A change of volume causes a corresponding change in the number of the molecules ; but the pressure opposes the escape of the gas molecules from the liquid. The kinetic energy of escpe being supplied in the form of heat, *more* heat energy is now necessary. This explains the *rise of the boiling point under increased pressure.*

It stands to reason, therefore, that a liquid contains a small portion of the free gas molecules, whereas a gas contains a smaller number of liquid type group molecules. The **critical temperature** may be defined as the temperature at which a liquid and its vapour can simultaneously exist in all proportions.

EXERCISES ON CHAPTER VI

Reference

Art. 61 — 1. Two bits of the ice being pressed together from one mass. Explain. (V. U.—1955 ; C. U.—1953, '60)

Art. 61 — 2. Write a short note on regelation. (V, U.—1954)

Arts. 61 & 72 — 3. How can you demonstrate that pressure changes the melting of a body and that evaporation produces cold ? (Utkal. U.—1962)

Reference

4. Write short notes on freezing mixtures and ice-making machine. Arts. 62 & 72

5. State the laws of fusion of a substance, (Mad. U.—1959, '71) Art. 63

6. Distinguish between boiling and evaporation. What condition determines whether a liquid will boil or evaporate? (Gau. U.—1965 ; C. U.—1950 ; P. U.—1961 ; Dac, U.—1957 ; Pat. U.—1951, '64. '72) Art. 64

7. Define boiling point of liquid, Describe carefully how the maximum tension of aqueous vapour is determined for temperatures below and above the normal boiling point. (Gau. U.—1967 ; Del. U —1968) Arts. 64 & 7

8. What are the points of distinction between evaporation and ebullition? Define vapour tension of a liquid. Arst. 64, 66 & 68

A glass cylinder is provided with an air-tight piston and contains inside some saturated water vapour at 20°C, State and explain what will happen when (*a*) the temperature is raised to 30°C, the piston remaining in position ; (*b*) the temperature is lowered to 10°C, the volume remaining unaltered ; (*c*) the piston is raised up, the temperature being kept constant ; and (*d*) the piston is lowered down, the temperature being unaltered. (Utkal. U.—1969)

9. Write a note on the behaviour of vapour towards Boyle's and Charle's laws of gases. (Raj. U.—1963) Arts. 66 & 68

10. State and explain the various factors that influence the rate of evaporation of water. (Mad. U.—1970) Art. 65

11. Explain saturation vapour pressure of a liquid. (Gau. U.—1972 ; Mad. U.—1969) Art. 66

12. Saturated with water vapour at 30°C, a quantity of air exerts a pressure of 74 cm. of mercury when its volume is 200 c.c., and a pressure of 116·6cm. when its volume is 125 c.c., the temperature remaining constant. Calculate the pressure exerted by the vapour. Arts, 67 & 68

Ans. 30 mm.

13, Over water a gas is collected in 100 c.c. tube and measures 72·8 c.c. ; the temperature and the pressure of the atmosphere at the time are 25°C and 74·39 cm. respectively. Calculate the volume of the dry gas at N. T. P. (The vapour pressure of water at 25°C is 23·48 mm. and the density of mercury is 13·6 gm. per c.c.) Arts. 66 & 69

Ans. 61·51 c.c.

14. Distinguish between saturated and unsaturated vapours. (Del. H. S.—1964 ; Pat. U.—1961. '74 ; C. U.—1961, Gau. U.—1952. Utkal U.—1964 ; Nag. U.—1964 ; E. P. U.—1950, '72 ; P. U.—1962) Art. 69

15. Enunciate and explain Dalton's law of partial pressure. (Pat. U.—1970) Art. 70

16. (*a*) Why is it unwise to sit in a draught with wet clothes on? (C. U —1947. *Cf.* '50) Art. 72

(*b*) Explain why earthen warepots keep water cooler than metal pots. (V. U.—1955)

17. Explain why the reading of a thermometer is altered when wrapping a wet rag round its bulb. (C. U.—1948) Art. 72

18. Write a note on Wollaston's cryophorus. (Pat. U.—1965) Art. 72

Reference

Art. 73 19. Explain the principle, construction and working of an ice plant. (Raj. U.—1963 ; E. P, U.—1972)

Arts. 76 & 78 20. Define boiling point of a liquid. Describe suitable experiments to show that water can be made to boil at temperatures greater or less than 100°C.

At Darjeeling the barometric height is on average 23 inches. If there is a change of ·04°C in the boiling point of water for a change of 1 mm. of Hg. at what temperature will water boil there ? (Dac. U.—1962)

Ans. 92·97°C.

Art. 77 21. Write a short note on Papin's digester. (E. P. U.—1963 ; Pat. U.—1965)

Art. 76 22. Describe an experiment to show that the vapour pressure of a liquid exposed to air at its boiling point is equal to the atmospheric pressure. (Gau. U.—1968)

Arts. 76 & 78 23. Define boiling point of a liquid. Describe suitable experiments to show that water can be made to boil at temperature greater or less than 100°C.

(P. U.—1962 ; C. U.—1951, '56 ; Pat. U.—1962)

Art. 77 24. A flask is half-filled with water and heated till the water boils. After a short time when the upper part of the flask is filled with steam, it is well-corked and inverted. Cold water is now poured over the flask. What will happen ? Why ?

(C. U.—1958 ; V. U.—1954)

Arts. 78 & 80 25. State the laws of boiling. How is it that things cannot be cooked properly on high montains ? How can water be made to boil at any temperature above 100°C ?

(Pat. U.—1965 : Del.U .—1963)

Art. 79. 26, Describe Ramsay and Young's apparatus and explain how it is used for measuring saturation vapour pressures at various temperature. (Nag. U.—1964)

Art. 76 27. Describe an accurate method of determining the pressure of saturated vapour at different temperatures.

The pressure of the saturated vapour of water at 50°C is 9·2 cm. of mercury and it is found that under a pressure of 9·2 cm. of mercury water boils at 50°C. Explain fully the relation expressed in the statesment. (Utkal. U.—1964)

Art. 79 28. How would you determine the maximum pressure of aqueous vapour between 0°C and 100°C ?

(Utkal U.—1964 ; Pat. U.—1967 ; C. U.—1957 ; Dac. U.—1951 ;
Gau. U.—1952 ; Mad. U.—1959)

Art. 79 29. Describe how you will measure the saturated vapour pressure of water at the room temperature. (C. U.—1959)

Art. 81 30. Explain how you are able to determine (approximately) the height of a mountain by finding the boiling points of water at its top and bottom. (C. U.—1955 | Pat. U,—1963)

31. What is the fundamental difference between a solid and a liquid ? Paraffin wax is said to be a soft solid, and sealing wax a viscous fluid. In : what does the distinction beiween the two consist ? (Gau. U.—1965)

CHAPTER VII

HYGROMETRY

83. Water Vapour in the Atmosphere—As evaporation continuously goes on from all water surfaces (such as those of seas, rivers, lakes, etc), there is always some aqueous vapour present in the atmosphere. The exterior of a tumbler containing iced water gets moistafter some time, This is due to the condensation of water vapour of the atmosphere on the tumbler. The formation of dew, mist, cloud, rain, etc., also indicates the presence of water vapour in the atmosphere. The study and measurement of the quantity of aqueous vapour in the atmosphere is called **hygrometry.**

84. Dew Point—Air is not ordinarily saturated with water vapour. If moist air in open space is gradually cooled, the pressure of the moist air at the region falls bellow that of the neighbouring atmosphere which is at the ordinary temperture. Hence moist air from surrounding places comes up to the space in order to equalise the pressure. As the cooling proceeds, a temperature is ultimately reached, at which the moisture accumulated at the region is *just sufficient* to saturate the space, pressure remaining unchanged. The temperature is called the *dew point*. If the temperature is slightly lowered below the dew point, some of the vapour is condensed and is deposited as *dew*. Hence *the dew point is the temperature at which a given mass of air becomes just saturated with the quantity of aqueous vapour actually present in it*. Also it follows that the pressure of the aqueons vapour actually present in air at the ordinary temperature is equal to the saturated vapour pressure at the *dew point* at that time. The statement 'The dew point of the atmosphere is 23°C at some time", means that the amount of water vapour present in a given volume of air at that time would just saturate the air at 23°C.

85. Relative Humidity (or **Hygrometric State of the Air**)—Humidity gives us information about the amount of aqueous vapour present in the atmosphere at some particular time.

Absolute humidity is defined as the quantity of aqueous vapour actually present in a given valume of the atmospheric air. It is usually expressed in grammes per cubic metre of air.

Relative humidity is defined as the ratio of the mass of aqueous vapour actually present in a given volume of air to the mass of water vapour required to saturate the volume at the observed temperature. So relative humidity gives the degree of saturation of the atmosphere as regards moisture. It is, therefore, termed the **hygrometric state** or *fraction of saturation* of the air.

Hence **relative humidity**

$$=\frac{\text{mass of aqueous vapour actually present in a given volume of air at } t^0\text{C}}{\text{mass of aqueous vapour required to saturate the same volume at } t^0\text{C.}}$$

But since unsaturated vapour present in a given volume of air obeys Boyle's Law, the pressure exerted by the vapour is nearly proportional to the mass of the vapour present. So the ratio between the two masses of vaponr may be taken to equal the ratio of the corresponding vapour pressures. Hence relative humidity may be alternatively defined as the ratio of the pressure of the aqueous vapour actually present in the atmoshpere to the saturated vapour pressure at the temperature of the air. That is to say,

Relative Humidity

$$=\frac{\text{pressure of aqueous vapour actually present in air at } t^\circ\text{C}}{\text{pressure of aqueous vapour required to saturate the air at } t^\circ\text{C.}}$$

$$=\frac{\text{saturated vapour pressure at the dew point at the time}}{\text{saturated vapour pressure at the temperature of the air}}$$

Relative humidity is, however, generally expressed as the *percentage saturation* of air. In this case it is expressed as a ratio, viz.,

$$=\frac{\text{saturated vapour pressure at dew point}}{\text{saturated vapour pressure at air temperature}}\times 100 \text{ per cent,}$$

Let the saturated vapour at the dew point be f and that at the temperature of air be F, we thus write :

$$\text{Relative humidity}=\left(100\times\frac{f}{E}\right)\text{ per cent.}$$

The feeling of dampness or dryness of air depends on the relative humidity. It does not, therefore, depend on the absolute quantity of water vapour *actually present* or on the quantity necessary to saturate the air under given conditions ; but what it depends on is the *ratio* of the former quantity to the latter. Thus on a misty winter day the quantity of vapour in a given volume of air may be less than that on a hot summer day when the air appears to be dry. This is so because the amount of aqueous vapour present on the latter occasion is a *smaller fraction* of amount necessary to saturate the space.

Again, the lower is the value of the relative humidity. the more rapid is the evaporation ; hence the dampness or dryness of the air is judged by *the rate of evaporation at the time,* A very moist day in summer feels much more uncomfortable than a dry day. Since in the former case the atmosphere is nearly saturated with water vapour producing a stuffy sensation. When the relative humidity is low, the atmosphere consumes the water vapour more rapidly. It is for this reason that in our country a damp cloth dries more rapidly in winter than in summer.

To forecast weather relative humidity is determined from day to day at meteorological stations. When the relative humidity is high we may expect rain. For safe navigation a knowledge of the actual atmospheric condition and a forecast for immediate future are essential now-a-days to pilots of aeroplanes. Information about relative humidity is necessary, for the Public Health Department, as germs of certrin diseases multiply well in moist atmosphere. For the growth of plants, supply of moisture ia to be regulated in hot houses. The knowledge is also of importance for textile industries, In spinning rooms elaborate arrangements are made to maintain artificially a uniform degree of dampness. At the cold storage of meat and fruit, in the holds of ship and also in artificial seasoning of timbers a successful control of humidity is needed.

Hygrometers (*Gr. hygros*, wet ; *metron*. a measure) are instruments for determining the humidity of the air at any particular time and place. According to the principles upon which they work hygrometers are divided into four classes :—

(1) *Dew point Hygrometer*, (2) *Chemical Hygrometer*.
(3) *Wet and Dry Bulb Hygrometer*, and (4) *Hair Hygrometer*.

86. **Dew point Hygrometers** (*Condensation Hygrometer*)—One of these instruments directly determines the dew point. The ratio of the saturated vapour pressure at the dew point to that at the atmospheric temperature gives the relative humidity.

Daniell's Hygrometer—It consists of two glass bulb A and B (Fig. 59 which is connected by a glass tube bent twice at right angles and mounted on a vertical stand so that the bulbs project downwards. The instrument contains only ether and ether vapour, all the air having been removed before the apparatus is finally sealed In order to observe clearly the deposition of any dew on the surface of the bulb A, either it is blackened from inside or else a gilt band is fastened over the surface near the middle. Mounted inside A, there is placed a sensitive thermometer T. The thermometer T_1 mounted on the stem of the instrument gives temperature of the outside.

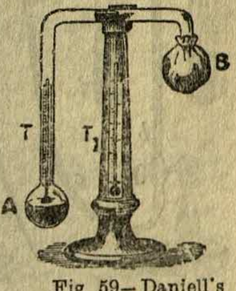


Fig 59—Daniell's Hygrometer

In using this instrument all the ether is at first transferred into the bulb A by tilting the apparatus and a piece of muslin or linen is wrapped round B. Ether is then poured drop by drop on the linen. As the ether evaporates, it takes the necessary heat from the bulb B and cools it rapidly. Consequently the ether vapour in B is condensed ; this causes a rapid evaporation of the ether in A. The temperature of this ether, and hence that of the bulb A, gradually decreases so long as the linen is kept moist with ether. Also the mass of the air surrounding the bulb A falls in temperature and ultimately a thin film of dew forms on the bulb ; this makes the surface of A appear dull, The temperature, at which the dew just appears, is read from T. The spilling of ether is discontinued and the linen is allowed to dry up. The bulb A warms up and the temperature, at which the dew just disappears, is noted. The mean of these two temperatures is taken as the dew point at the time. Observation should be made either by a telescope or through an intervening plate of glass, in order that the heat of the body of the observer or his breath might not distrurb the deposit of dew on the instrument.

∴ the saturation pressure at the dew point $=(7{\cdot}5+{\cdot}03)=7{\cdot}53$ mm,

Thus the required R. H. $=\left(\frac{7{\cdot}53}{13{\cdot}5}\times 100\right)\%=57{\cdot}7\%$.

***88. Chemical Hygrometer** (*Absorption Hygrometer*)—The mass of water vapour present in a given volume of air can be directly determined by this hygrometer and so the absolute humidity can be found by this process. The pressure of the vapour is calculated and the *relative humidity* also is determined.

The apparatus consists of a large aspirator (Fig. 61) filled with water and connected with a system of U-tubes through the 'trap' bottle. The U-tubes contain either lumps of freshly fused calcium chloride or else pieces of pumice stone soaked in strong sulphuric acid, whereas the trap bottle contains strong sulphuric acid. Short pieces of India rubber tubing make all the connections air-tight. Together with their contents the drying U-tubes are carefully weighed before the experiment is started.

The aspirator is opened whereupon water escapes and air is sucked through the system of U-tubes. The moisture of the air is absorbed by the drying tubes, while any moisture rising from the water in the aspirator is absorbed in the 'trap' bottle. The experiment being carried on for some time, the aspirator is closed and the drying tubes are disconnected and weighed again. The increase in the weight gives the mass of aqueous vapour present in the volume of air drawn in ; this equals the volume of water that has flown out. But the air becomes saturated with moisture when it comes in contact with water within the aspirator. The temperatures of the air outside and inside the aspirator are noted.

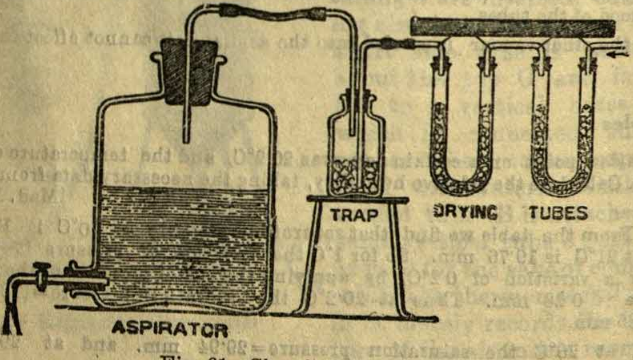


Fig. 61—Chemical Hygrometer

Let the mass of the aqueous vapour absorbed be $=m$ gm.,
" volume of the air drawn in $=V$ c. c.,
" temperature of air inside $=t^{\circ}$C,
" pressure of aqueous vapour $=f$ mm. of mercury
and " atmospheric pressure at the time $=H$ " " "

Since the density of aqueous vapour relative to air is 0·622 and the density of dry air at N.T.P. is 0·001293 gm. per c.c., we find that : the density of aq. vapour at N.T.P. $=(0{\cdot}622\times0{\cdot}001293)$ gm. per c.c.

Let the density of aqueous vapour at pressure f and temperature t be $=d$ gm. per c. c.

$$\text{From Boyle's Law, } \frac{f}{d(t+273)}=\frac{760}{0{\cdot}001293\times0{\cdot}622\times273}$$

$$\text{whence } d=\frac{f\times0{\cdot}001293\times0{\cdot}622\times273}{760(t+273)}. \qquad (\ldots 88.1)$$

As $d=m/\mathrm{V}$, it can be obtained from the experiment. Hence f, the pressure of the vapour actually present, is known from eqn. (88·1) and is given by

$$f=\frac{760}{0{\cdot}001293\times0{\cdot}622\times273}(t+273)d \text{ mm.}=3460d(t+273) \text{ mm.}$$

If as found from Regnault's Tables, the saturated vapour pressure at t°C be F then :

$$\text{Relative Humidity}=\frac{f}{F}=\frac{760}{0{\cdot}001293\times0{\cdot}622\times273}\times\frac{m(t+273)}{\mathrm{VF}}$$

$$=\frac{3460m(t+273)}{\mathrm{VF}}. \qquad \ldots(88{\cdot}2)$$

To obtain the relative humidity as *percentage*, the above expression is to be multiplied by 100. To find the absolute humidity take the ratio of m and T/1000000. Thus the ratio gives the amount of water (in gm.) present in a cubic metre of air.

It is to be noted that the temperature of the air outside may be different from that of air inside the aspirator. Also the atmospheric air is generally unsaturated, while that within the aspirator gets saturated. So the pressure of the vapour, and hence that of the dry air, inside or outside, would not be the same. But as the differences mentioned are very small, they are not generally taken account of.

Example :

A cubic metre of air of 25°C was drawn in through a chemical hygrometer and 11·22 gm. of water was deposited in the tubes. What is the relative humidity of the air ? The maximum pressure of water vapour at 25°C is 23·52 mm. The mass of 1 litre of dry air at N. T. P. is 1·293 gm. The density of water vapour relative to dry air is 0·625.) [Pat. U.—1955]

Ans. Since in moist air the volume of water vapour in the same as that of dry air, we can assume that a cubic metre of water vapour at 25°C has a mass of 11·22 gm. So at 25°C the density ρ of water vapour is given by

$$\rho=\frac{11{\cdot}22}{100^3}\text{gm./c.c.}=11{\cdot}22\times10^{-6}\text{ gm./c.c.}$$

At 25°C this water vapour has a saturation pressure of 23·52 mm. Again the mass of 1 c.c. of dry air at N. T. P =·001293 gm. Hence the mass of 1 c.c. of water vapour at N. T. P. =(·001293×·625) gm. which is its density. To find the actual pressure f of water vapour at 25°C we apply Charles' and Boyle's Laws in the following equation :

$$\frac{f}{11\cdot22\times10^{-6}\times(25+273)}=\frac{760}{\cdot001293\times\cdot625\times273},$$

whence $f=\dfrac{760\times11\cdot22\times10^{-6}\times298}{\cdot001293\times\cdot625\times273}$.

Now at 25°C the saturation pressure F of water vapour = 23·52 mm.

whence the R. H. $=100\times\dfrac{f}{F}=100\times\dfrac{760\times11\cdot22\times10^{-6}\times298}{23\cdot52\times\cdot001293\times\cdot625\times273}$

Below we proceed to calculate by Log-Table :

log 760 $=2\cdot8808$	log ·001293 $=\bar{3}\cdot1116$	
log 11·22 $=1\cdot0500$	log ·625 $=\bar{1}\cdot7259$	$2\cdot4050$
		$0\cdot7152$
log 10^{-6} $=\bar{6}\cdot0000$	log 273 $=2\cdot4362$	$1\cdot6898$
log 228 $=2\cdot4742$	log 23·52 $=1\cdot3715$	Antilog $1\cdot6898$
	$0\cdot7152$	$=48\cdot9$
log 100 $=2\cdot0000$		
$2\cdot4050$		

Hence the R. H. = 48·9%

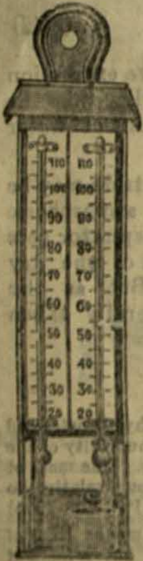


Fig. 62

89. The Wet and Dry Bulb Hygrometer—(*Psychrometer* or *Manson's Hygrometer*)—It consists of two exactly similar mercury thermometers (Fig. 62), mounted vertically side by side on the same board with their bulbs freely exposed to air. One thermometer gives the temperature of the air ; the bulb of the other is covered with muslin, which is kept continuously moistened with water by means of a cotton wick dipped into water which is kept in a small attached vessel.

As water evaporates from the muslin, the temperature indicated by the wet bulb thermometer is lower than that of the dry bulb one. The difference in the readings depends on the hygrometric state of the air. For example, on a very dry day evaporation would be very rapid and so the difference in the readings increases, whereas on a damp day evaporation would be slow and the difference decreases. If the atmosphere is already saturated, no evaporation takes place and the two thermometers record the same reading.

The relative humidity can be determined by obtaining the dew point with help of Glaisher's factor.

Let temperatures recorded by the dry and the wet bulb thermometers be t_1 and t_2 respectively.

The temperature of the dew point at the time $=t$.

Then from Glaisher's formula we find $t_1-t=F(t_1-t_2)$ ...(89.1)

where F is the Glaisher's factor.

The dew point is thus found. The saturated pressures at t and t_1, being read off from Regnault's table, the relative humidity is calculated.

Dry Bulb and Wet Bulb Thermometers

Dry Bulb Temp (in °C)	Glaisher's Factor	Dry Bulb Temp. (in °C)	Glaisher's Factor	Dry Bulb Temp (in °C)	Glaisher's Factor
4	7 82	11	2·02	24	1·72
5	7·28	12	1·99	26	1·69
6	6 62	14	1 94	28	1·67
7	5 77	16	1·87	30	1·64
8	4·92	18	1·83	32	1·63
9	4·04	20	1 79	34	1·61
10	2 26	22	1 75	36	1·59

90. Hair Hygrometer—The apparatus consists of a human hair fixed at a point A of the instrument and stretched over three small pulleys B, C and D (Fig. 63). Then the hair is wound a *few turns* over a revolving small drum E and is finally fastened to the end of a light spring SF. A pointer P fixed to the drum E is movable over the graduated dial.

With rise in atmospheric humidity the hair has a property of increasing continuously in length. Consequently when the hair elongates, the spring draws it further downwards. As a result the drum with its attached pointer rotates over the scale graduated in units of relative humidity. In practice, to obtain the required tension a strand of 20 to 30 fibres of hair is put together. Regarding the behaviour of human hair in humidity the following pieces of information are available from Griffith's investigations.

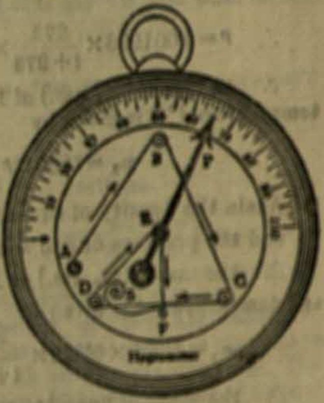


Fig. 63

With lapse of time the readings of a hair hygrometer tend to give higher readings. This is probably due to the fact that being always in tension due to the spring the hair gets a permanent set. The hair should be, however guarded against any undue strain. From time to time the calibration of the instrument should be checked. A hair hygrometer has the advantage of being a direct reading instrument. It can be used at a temperature below the freezing point of water, where neither a dry bulb nor wet bulb thermometer can work.

The above Table supplies the temperature of the dry bulb with the corresponding value of Glaisher's factor.—

Example :

At some hour the readings of a Wet bulb and a Dry bulb hygrometer are 52°F and 58°F. Find the humidity.

Ans. Dry bulb temperature $t_1 = 58°\text{F} = 14{\cdot}4°\text{C}$, and Wet bulb temperature $t_2 = 52°\text{F} = 11{\cdot}1°\text{C}$. Hence from equ. (89·1), $14{\cdot}4 - t = \text{G} \times 3{\cdot}3$. Now for 14°C, $\text{G} = 1{\cdot}92$ and for 16°C, $\text{G} = 1{\cdot}87$. Hence from a proportionate calculation it follows that G at 14·4°C is $= 1{\cdot}91$. Thus $14{\cdot}4 - t = 1{\cdot}91 \times 3{\cdot}3$ or $14{\cdot}4 - t = 6{\cdot}3$. Hence the dew point $t = 8{\cdot}1°\text{C}$. Now from Regnault's Table of Saturated Vapour, the maximum vapour pressure at 8·1°C is 8·67 mm. and that at 14·4°C is 13·1 mm.

Thus the relative humidity $= \left(100 \times \frac{8{\cdot}67}{13{\cdot}1}\right)\% = 66{\cdot}1\%$.

91. Mass of the Moist Air—To find the mass of a given volume of *moist* air find (1) the mass of *dry* air of the same volume at the given temperature and at pressure due to air only and (2) the mass of an equal volume of aqueous vapour at the temperature and at pressure due to the vapour only. Their sum gives the required mass.

Let the temperature of the atmosphere at the time be $= t°\text{C}$,
the pressure of the atmospheric air $= \text{P}$ mm. of mercury
and the pressure of aqueous vapour actually present $= f$ mm. „
From Dalton's Law, pressure due to air only $= (\text{P} - f)$ „ „
Now the density of dry air at N. T. P. $= 0{\cdot}001293$ gm.

Let the density of dry air at temperature t and pressure $(\text{P} - f)$ be ρ gm. per. c.c.

From Charle's and Boyle's Laws, $\dfrac{\text{P} - f}{\rho(t + 273)} = \dfrac{760}{0{\cdot}001293 \times 273}$

$$\therefore \quad \rho = 0{\cdot}001293 \times \frac{273}{t + 273} \times \frac{\text{P} - f}{760}.$$

∴ the mass (m_1 gm.) of 1 litre of dry air at pressure $(\text{P} - f)$ and temperature $t°\text{C}$ is given by

$$m_1 = 1000 \times \rho = 1{\cdot}293 \times \frac{270}{t + 273} \times \frac{\text{P} - f}{760}$$

Again the density of aqueous vapour with respect to air $= 0{\cdot}622$ and the pressure due to aqueous vapour alone $= f$.

∴ the mass (m_2 gm.) of 1 litre of aqueous vapour at pressure f and temperature t is given by

$$m_2 = 1{\cdot}293 \times 0{\cdot}622 \times \frac{273}{t + 273} \times \frac{f}{760}.$$

∴ the mass of one litre moist air at the time $= m_1 + m_2$

$$= 1{\cdot}293 \times \frac{273}{t + 273} \times \frac{\text{P} - f}{760} + 1{\cdot}203 \times 0{\cdot}622 \times \frac{273}{t + 273} \times \frac{f}{760}$$

$$= 1{\cdot}293 \times \frac{273}{t + 273} \times \frac{1}{760} \times (\text{P} - f + 0{\cdot}622 \times f)$$

$$= 1{\cdot}293 \times \frac{273}{t + 273} \times \frac{\text{P} - 0{\cdot}378 \times f}{760},$$ expressed in grammes.

Example :

1. On a certain day the temperature of air is 23°C and the relative humidity is 60%. What fraction of the mass of the water vapour in the air would condense, if the temperature decreases to 10°C ? (Saturated pressure of water vapour at 23°C is 21·1 mm. and at 10°C it is 9·2 mm.).

Ans. Let the mass of water vapour present in given volume be m gm.

Now relative humidity $= \dfrac{\text{mass of water vapour present } (m)}{\text{mass of vapour reqd. to saturate}} = \dfrac{60}{100} = \dfrac{3}{5}$.

∴ the mass of aqueous vapour required to saturate at 23°C is $= m \times \dfrac{5}{8}$.

Again, $\dfrac{\text{mass of water vapour required to saturate at } 10^\circ\text{C}}{\text{mass of water vapour required to saturate at } 23^\circ\text{C}}$

$= \dfrac{\text{saturated pressure at } 10^\circ\text{C}}{\text{saturated pressure at } 23^\circ\text{C}} = \dfrac{9{\cdot}2}{21{\cdot}1}$.

∴ the mass of aqueous vapour required to saturate at 10°C is $= \dfrac{9{\cdot}2}{21{\cdot}1} \times \dfrac{5}{3} m$

$= 0{\cdot}73 \times m$.

∴ the required fraction of condensation $= 1 - {\cdot}73 = 0{\cdot}27 = \dfrac{27}{100}$.

2. Find the mass of a litre of moist air at 30°C when the barometer reads 758·6 mm. and the dew point is 16·1°C. At 16·1°C the maximum pressure of water vapour = 13·6 mm. of mercury,

Ans. Pressure due to water vapour only = 13·6 mm. of mercury.

∴ Pressure due to dry air alone = (758·6 − 13·6) mm. = 745 mm. „

Now mass of 1 litre of dry air at 30°C and 758·6 mm

= mass of 1 litre of dry air at 30°C and at pressure 745 mm.

+ mass of 1 litre of water vapour at 30°C and at pressure 13·6 mm.

Now the volume of 1 litre of dry air at 30°C and 745 mm. when reduced to N. T. P.

$$= \left(1 + \frac{273}{30+273} \times \frac{745}{760}\right) \text{ litres.}$$

Also when reduced to N. T. P. the volume of 1 litre of aqueous vapour at 30°C and 13·6 mm.

$$= \left(1 \times \frac{273}{30+373} + \frac{13{\cdot}6}{760}\right) \text{ litres.}$$

∴ the mass of dry air $= \left(1{\cdot}293 \times \dfrac{273}{303} \times \dfrac{745}{760}\right)$ gm. $= 1{\cdot}1419$ gm.

and the mass of aqueous vapour contained $= \left(1{\cdot}293 \times {\cdot}622 \times \dfrac{273}{303} \times \dfrac{13{\cdot}6}{760}\right)$ gm,

$= 0{\cdot}0129$ gm.

∴ the mass of 1 litre of moist air = (1·1419 + 0·0129) gm. = 1·1548 gm.

92 Condensation of water Vapour in the Atmosphere— The water vapour given off from various water surfaces rises up in the atmosphere and remains mixed up with air in a gaseous form. The different conditions, under which the condensation of water vapour takes place, give rise to the formation of dew, fog, mist cloud, rain, snow, frost or hail. When we exhale in winter, the warm water vapour gets condensed in the colder atmosphere and presents a *misty* appearance.

Dew—During day time the water vapour present in the atmosphere remains unsaturated due to higher temperature and hence the presence of water vapour is not directly noticeable. During night the warmth due to the sun is no longer present and the surface of the earth as well as other objects fall in temperature. The layers of air *very near* the surface of earth cool and ultimately become saturated with the water vapour contained in it. If the cooling proceeds further, the air can no longer hold the same amount of water vapour, a portion of which is, therefore, deposited as *dew* on the surface of the cold bodies.

Under the following conditions the deposit of dew is more copious. (1) There must be a *clear sky*, as this promotes radiation from the earth and objects lying on it; due to this the surface of bodies quickly cool and so dew is rapidly formed on them. (2) There must be *absence of wind*, as in this case a particular layer of air may remain in contact with the cold objects for a sufficient time to be cooled below the dew point. (3) The bodies, on which dew is formed are to be (i) *good radiators*, as they would cool rapidly, and (ii) *placed close to the earth*. If the object is at a higher level, the air on getting slightly cooled due to contact sinks down and a warmer layer from above comes to its place. The result is that no layer of air, which comes in contact with the body, is sufficiently cooled to deposit its vapour as dew. For this reason dew is formed mostly on blades of grass and scarcely on leaves of trees.

Fog : Mist—If the temperature falls to such an extent that not only the objects on the surface of the earth but also the whole mass of air at or near its surface is cooled below the saturation point, condensation takes place throughout a large mass of air in the *lower* regions on suspended dust and smoke particles and thereby gives rise to *fog* or *mist*. In hot countries it is usually formed during a still, cloudless winter night after a calm, warm day. A very dense mist is ordinarily called *fog*. Water is transparent: but owing to diffused reflections from condensed globules of water a patch of fog appears opaque.

Fog may be formed on land as well as on water surfaces Land fog is formed when a mass of warm air becomes sufficiently cooled on coming in contact with the cold surface of the earth during morning, evening or night. In this case the fog gradually extends upwards and the condensation takes place on smoke and hygroscopic dust particles serving as nuclei for condensation. When the surface of the earth is cooled by radiation at night, the air in contact is cooled and some of the condensed vapour is deposited as dew, while some remain hanging in the air. In autumn or winter, fog is often seen on water surfaces. Here the water and also the layer of air just in contact with its surface are warmer than the air above. Water vapour is then given off in larger quantities and when air in a nearly saturated condition cools owing to contact with the cold air above, fog is formed.

In large towns dense fog is due to the condensation of water vapour on smoke and dust particles, although smoke does not cause the fog. On some occasions the mist becomes very dense late at night and continues till the following morning. It is found that the day is warmer on such occasions. Owing to the warmth of the day greater evaporation takes place and deposits large mass of water vapour in the atmosphere. When the temperature falls by radiation, condensation takes place readily and a dense fog is formed. As the air warms up by the heat of the sun, it rises above the dew point ; the condensed water particles, therefore, evaporate and the fog slowly disappears.

Cloud—Clouds are mist formed *high up* in air. Charged with water vapour, large mass of warm air rises to higher levels and becomes cooled. The cooling may be brought about by contact with cold air above or on account of the expansion due to lowering of the pressure in the upper regions. If the temperature falls below the saturation point, the excess of moisture immediately condenses on floating dust particles as minute drops of water and becomes visible as **clouds.** Clouds may either remain almost stationary or be moving about with wind. In either case, however, the constituent particles are in constant motion among themselves. A cloud remains suspended in air, because while tending to fall down, the small particles enter into a warmer region and quickly evaporate. Again, as the mass of water vapour is in a state of going higher up due to buoyancy, it expands due to lowering of pressure. The expansion lowers the temperature of vapour to the condensation point so as to form cloud over again. Thus there is a constant vertical up-and-down motion of water vapour in the region of the clouds.

Forms of Clouds—Varying with the conditions under which they are formed, clouds have a variety of forms and structure. The following classifications were made, and names given, by Luke Howard, a London Chemist in 1803.

(1) **Cirrus**—These are fibrous or feather-like clouds of uniformly white colour occupying the highest region of at least 25,000 feet in the atmosphere. Owing to the very low temperature at the high altitudes, these are composed of particles of ice formed by the freezing of the constituent water drops.

(2) **Cumulus**—These consist in extensive rounded masses, convex above and flat below, and resemble heaps of cotton wool. When columns of warm, moist air rise to cold higher regions, they get condensed at the upper extremities, so that clouds are formed on the top of these columns at heights ranging between 5000 feet and 10,000 feet above the earth's surface.

(3) **Stratus**—These are made up of extended horizontal sheets of grey colour, hanging low in the atmosphere at a height of about 2,000 feet. Characteristic of fine weather, these are formed due to condensation taking place at the regions of contact of layers of air at different temperatures.

(4) **Nimbus**—These are black rain clouds appearing like formless masses and with rugged edge at a height of about 1000 to 1500 feet.

Rain—Whenever two fine droplets of water in a cloud come into contact, they coalesce to form a larger drop and fall down as *rain.* As a drop falls, some of the vapour in different layers of air condenses on the cold drop; the drop thus gradually grows in size as it falls down. In size and velocity of fall the rain drops vary.

Rain Gauge—The amount of rainfall in a locality is measured by a simple instrument called a *rain gauge*. The form now commonly in use and known as 'Simon's Rain-gauge' consists in its simplest form, of a funnel provided with a circular brass rim, exactly five inches in diameter and fitted to a cylindrical body which is joined to the base. The base is kept fixed to the foundation and the height of the rim of the funnel is one foot above the ground. The rain falling into the funnel collects into a bottle placed inside the cylinder and the quantity deposited during a period of observation is measured in a glass cylinder graduated to hundredths of an inch. An inch of rain that the water collects, would fill the depth of an inch within the cylinder with its base equal to the rim of the funnel. The annual rainfall at a locality is expressed in inches. A fall of one inch of rain on a square yard of surface represents 4·69 gallons. The average annual rainfull in Bombay is nearly 45 inches and that at Cherrapunji is 100 inches. At the latter place we have the greatest amount of rainfall per annum.

Sleet—If the falling rain freezes before it reaches the ground, it is called *sleet*.

Hoar-frost—If the temperature of the surface of the earth and of adjacent objects fall below 0°C before the dew point is reached, the water vapour in contact with surfaces directy condenses into icy crystals which are deposited as fast as they are formed. This deposit is called *frost* or **hoar-frost**. Thus frost is not frozen dew but is formed directly by the freezing of the water vapour. Frost is injurious to plants.

Snow—When condensation takes place at a height where the temperature is below 0°C, the water vapour directly condenses into crystals of snow around the suspended nuclei. Being an aggregated form of *snow flakes*, the crystals fall to the ground as *snow*. These crystals have a variety of forms and structures.

Hail—Due to violent air current accompanying thunder-storms the condensed moisture in the upper region is carried up and down several times through regions of snow and rain. Owing to this *half stones* are formed with alternate layers of white snow and clear ice. They may be various shapes and sizes and are more or less transparent.

EXERCISES ON CHAPTER VII

Reference
Art. 84

1. Why does a glass tumbler cloud over' on the outside when ice-cold water is poured into it?
(C. U.—1949. '60; Dac. U.-1949; Pat.—U.—1963)

Arts.
84 & 85

2. Define 'Relative Humidity' and 'Dew point' and explain how the determination of the dew point enables you to calculate the relative humidity at a particular place.
(C. U.—1963; Mad. U.—1970; R. P, B.—1971; Pat. U.—1963, '65; Utkal. U.—1969; V. U.—1953)

Reference

Art. 83

3. The average temperature of a summer day is much higher than that of a winter day; but still a damp cloth dries more quickly on a winter day. Explain why. (All. U.—1969)

Art. 85

4. The temperature of air in a closed space is observed to be 15°C, and the dew point is 8°C. If the temperature falls to 10°C, how will the dew point be affected? (Pressure of aqueous vapour in mm. of mercury at 7°C is 7·47 and at 8°C is 8·02).
(Utkal U.—1964; Pat. U.—1971; Gau. U.—1969)

Ans. Dew point = 7·75°C.

Art. 85

5. A hot day in Puri causes greater discomfort than an equally hot day in Delhi. Why? (C. U.—1958)

Art. 85

6. Define relative humidity of air and state how it can be determined by knowing the dew point and using a table of saturated pressure of aqueous vapour.
(Pat. U.—1962; U. P. B.—1968)

Art. 85

7. The temperature of air on a certain day 23°C and the relative humidity is 55%. What fraction, of the mass of the water vapour in the air would condense, if the temperature decrease to 10°C? (At 23°C the saturated vapour pressure is 21·1 mm.; at 10°C, it is 9·2 mm.).

Ans. 0·21.

Art. 86 & 87

8. Describe a dew point hygrometer and explain the method of working it. What use is made of this instrument in weather forecasting? (C. U.—1963; Mad. U.—1963).

Arts. 85 & 86

9. Explain what you mean by humidity. Describe any method of determing it.

On a certain day the temperature of the air at a place is 17°C and the dew point is 12°C. Find the relative humidity. (Maximum vapour pressure of water at 12°C is 1·046 cm. of Hg. and at 17°C is 1·442 cm.) (Dac. U.—1963)

Ans. 72%.

Arts. 85 & 86

10. State what is meant by the term 'relative humidity' and describe how its value may be determined.
(Nag. U.—1968; U. P. B.—1962; Andhra U.—1970)

Art. 87

11. What is meant by Regnault's Table of saturated pressure of aqueous vapour? Show how this table will enable you to determine the relative humidity of air, if the dew point is known. (C. U.—1964)

12. The table below gives the maximum pressure of water vapour in mm. of mercury :—

4°...6·1	10°... 9·1
7°...7·0	12°... 9·1
8°...8·0	14°...11·9

Arts. 79 & 86

Explain how these data have been obtained experimentally and find the actual pressure of water vapour present in a room at 14°, when the dew point is found to be 5°. What is the relative humidity of the air in the room? (Dac. U.—1961)

Ans. 6·4 mm.; 63%.

Art. 86

13 One day when the temperature of the air was 16·5°C., the dew point was found to be 12°C. Find the relative humidity. (Maximum vapour pressure of water at 12°C is 1·046 cm. of Hg., at 16°C is 1·364 cm. of Hg. and at 17°C is 1·442 cm. of Hg.)

Ans. 74·5% (approximately).

Art. 87

14. Describe Regnault's Hygrometer and explain how you use it to determine relative humidity.
(Nag. U.—1964; U. P. B.—1962)

Reference

Arts. 85, 87 & 89 — 15. Define 'Relative Humidity'. Describe Regnault's Hygrometer and the Wet and Dry bulb Hygrometer and point out their relative advantages. (Mad. U.—1968)

16. Define relative humidity.

Art. 89 — Describe a wet and dry bulb hygrometer. How would you determine the relative humidity with its help ?
(Del. U.—1963 ; C U.—1958)

Art. 89 — 17. Explain the use of a wet bulb and a dry bulb thermometer. (Mad. U.—1969)

Art. 91 — 18. A cubic metre of air, the relative humidity of which is 80%, is dried at atmospheric pressure. Find the change in volume, if the atmospheric pressure is 765 mm. and the temperature is 30°C (Max. pressure of water vapour at 30°C is 32 mm.)

Ans. 34×10^3 c.c.

Art. 91 — 19. Calculate what fraction of the mass of the water vapour in air would condense, if the temperature of the air fell from 20°C to 5°C and if originally the humidity was 60%. (Saturation vapour pressure at 20°C is 17·5 mm. and at 5°C is 6·5 mm.).
(Mad. U.—1968)

Ans. 3 81.

Art. 92 — 20. Explain how dew is formed mostly on blades of grass and not on the leaves of trees.
(Utkal. U.—1964 : Pat. U.—1965)

Art. 92 — 21. Cloudless nights are better for the formation of dew than cloudy ones. Explain. (U. P. B.—1970. ; Dac. U.—1959)

Art. 92 — 22. Why does a morning fog generally disappear before noon ? (C. U.—1960)

Art. 92 — 23. Explain the formation of clouds.
(C. U.—1962 ; U. P. B.—1967)

Arts. 86 to 90 — 24. Write a short essay on hygrometer.
(U. P. B.—1958 ; B. H. U.—1961)

Art. 92 — 25. Write a brief illustrative account of the nature and mode of formation of clouds, fog and dew. Indicate the conditions that are favourable for their formation. (Utkal. U.—1962)

Art. 92 — 26. Write a short note on Fog. (V. U.—1954)

CHAPTER VIII

TRANSMISSION OF HEAT

93. Modes of Transmission.—There are three distinct processes by which heat may be transferred from one place to another. These are *Conduction*, *Convection* and *Radiation*.

(1) **Conduction** *is the process in which heat is transmitted from the hotter part of a body to the colder* one or *from one body to a colder body in contact with it without any transference of material particles.* For this process the agency of some material medium is necessary. Heat is transmitted by conduction through solids. If one end of a metal rod is held in a flame, the other end soon feels warm ; the end gradually gets hotter and ultimately becomes too hot to be held in hand. Consider the rod to be composed of layers of material particles. Here heat is first communicated to the particles of the rod in contact with the fire ; such particles transfer heat to the adjacent particles and these to the next layer of particles and so on by way of *vibratory motion.* In this way heat passes on from one end of the rod to the other.

(2) **Convection** *is the process in which heat is transferred from one point to another by actual movement of heated material particles from a region of higher temperature to another of a lower temperature.* Liquids and gases are generally heated by the process of convection. Take a flask partly filled with water (Fig. 64) and drop a few crystals of potassium permanganate into the water ; heat the flask at the bottom. A coloured stream of water is found to rise up from the bottom to the middle of the flask and then to diffuse back down the sides. The water at the bottom is heated first and being ligher, it rises up the middle, while from the top colder water comes down the sides to take its place. Two currents are thus set up in the mass of liquid,—an *upward* current through the middle and a *downward* current along the sides. These currents tend to heat the entire mass of the liquid indirectly and are, therefore, termed *convection currents*. Again, if we hold our hand above a flame, we feel hot. Being heated, the air in contact with the flame becomes lighter and rises up, while colder air from the sides rushes in to occupy the space vacated by the heated air. The cycle of operation is repeated and we get a continuous current of heated air.

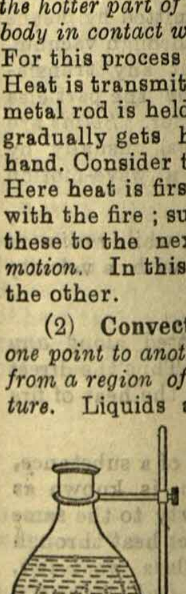


Fig. 64

Radiation *is the process in which heat can pass from one point to another without the intervention of any material medium.* It is by this process heat reaches us from the sun, although the vast space between the sun and the atmosphere of the earth is vacuous. Radiation is thus a distinct process. If we stand before fire, we feel hot. Heat reaches our body not by convection, for due to convection heated air rises upwards, while cold air rushes from the sides *towards* the fire. Nor has the heat reached by conduction, for air is a bad conductor and the effect of the cold stream of air moving upwards would neutralise the outward transference of heat by conduction. In such cases, therefore. heat comes by *radiation*.

94. Distinction between Conduction, Convection and Radiation—(1) In *conduction* and *convection*, heat is transmitted by means of *material particles*, while in *radiation* heat can pass through vacuum. In *conduction* heat passes from layer to layer without any transference of the particles of the body, while in *convection* the particles carrying heat actually move from one part to the other. *Solids* are heated by *conduction*, and *fluids* (liquids and gases) usually by *convection*, while the process of *radiation* involves a wave-motion through empty space.

(2) *Conduction* and *convection* are *slow* processes, while *radiation* is an *extremely quick* process ; heat radiation travels with a velocity equal to that of light.

(3) In *conduction* and *convection*, *heat* is transmitted along *any* path, while in *radiation* heat travels *in straight lines*. Thus the direct heat of the sun can be obstructed by an umbrella and the heat of fire can be shut out by an intervening screen.

95. Comparison of Conductivities—The property of a substance, by virtue of which it can transmit heat by conduction, is known as **conductivity**. Not all substances possess this property to the same degree. Substances, *e.g.*, metals, which readily conduct heat through them, are called *good conductors*, while those, *e.g.*, glass. wool felt, gases, *etc.*, which do not conduct so readily, are termed *bad conductors*. The conductivity of different solids can be roughly compared by the following experiment.

Ingen-Hausz' Experiment—The apparatus consists of a metal trough T (Fig. 65) provided with a number of holes at its side. A number of rods *a*, *b*. *c*,...are fixed exactly into these holes. The rods are of the same dimensions but of different materials (such as brass, iron, copper, wood and glass). The rods extend slightly to the same distance into the trough and are all coated outside with paraffin wax. The trough is filled with water kept boiling by some heater. So one end of each rod is raised to the temperature of boiling water. While heat is conducted, temperature rises gradually to the melting point of paraffin wax as indicated by the melting of wax over the rods.

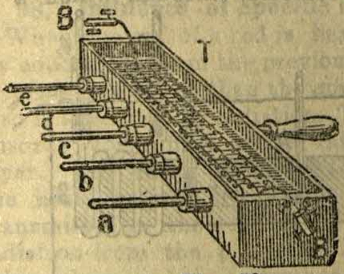


Fig. 56

When the steady state has been attained, it is found that wax has melted over different distances along different rods, the distance being large in case of copper and very small in the case of wood or glass. This shows that the conducting powers are different for different substances. Theoretical considerations clearly show that *the conductivity of a particular material is proportional to the square of the length over which wax has melted on the rod of the meterial, when the stationary temperature has been attained.* This is a very ready method of comparing the thermal conductivites of different materials.

96. Thermal Conductivity (*Coefficient of Conductivity*)—The *thermal conductivity* of material is the *quantity of heat that flows in one second through a unit area of a plate of the material of unit thickness, the difference of temperatures between the faces being unity and the flow of heat being normal to the faces.*

Take a plate of some material with its pair of opposite parallel faces at different temperatures. No heat from any sides being assumed to be lost, all the heat will flow from the face at a higher temperature θ_1 to the opposite face at a lower temperature θ_2. The total quantity of heat Q flowing through the plate is then found to be directly proportional to A the area of the plate, and to the time t in seconds but inversely proportional to d, the thickness of the plate. Thus,

$$Q \propto \frac{(\theta_1 - \theta_2)\,At}{d}$$

$$Q = k.\ \frac{A(\theta_1 - \theta_2)t}{d}, \qquad \ldots(96.1)$$

where k is a constant which depends solely on the nature of the material of the plate.

If A=1 sq. cm $\theta_1 - \theta_2 = 1^\circ$, $t=1$ sec, and $d=1$ cm., Q becomes equal to k. The constant k is called the thermal conductivity of the material of the plate. This constant is a vector quantity.

The quantity $\frac{\theta_1 - \theta_2}{d}$ is called the *temperature gradient* or fall of temperature per unit length. Hence the *thermal conductivity* of a substance may also be defined as the quantity of heat flowing in unit time through unit area of any section of the substance, the temperature gradient normal to the section being 1° per unit length.

97. Determination of Thermal Conductivity—The thermal conductivity of a *good conductor* can be readily determined by means of Searle's apparatus (Fig. 66). AB is a bar of the material, the thermal conductivity of which is to be found. So as to minimise loss of heat from its sides, the bar is covered with felt. The end A is placed in a chamber, through which steam is passed, while round the other end a coil C of thin copper tubing is soldered, through which a steady flow of cold water is maintained. The temperatures θ_1 and θ_2 of the water at entrance and exit respectively are indicated by two thermometers. Two other thermometers are placed at a known distant apart into mercury contained in two grooves on AB.

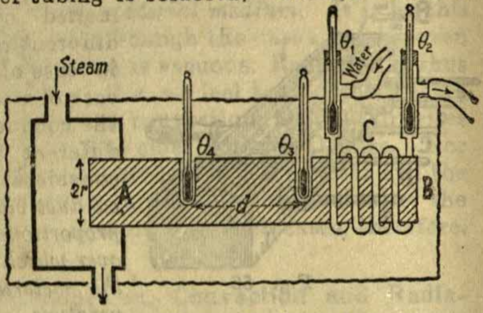


Fig. 66 = Seral's Apparatus

By regulating the flow of water a *steady state* of temperature is reached.

Loss by radiation being neglected, the quantity of heat now flowing through any section is the same, so that the quantity of heat flowing into the bar at the hot end is equal to the quantity flowing out at the other. The quantity of water passing out in a given time is collected, the temperatures at the entrance and the exit being noted. Thus the quantity of heat absorbed by the water is found and this also gives the quantity of heat flowing through the bar during the time.

Let the cross-section of the bar be $= S$ sq. cm.,
,, length of the portion under observation $= d$ cm.,
,, difference of the temperature at the two points $= \theta_4 - \theta_3$,
,, time during which experiment is continued $= t$ sec.,
,, quantity of water collected $= m$ gm.,
and ,, required thermal coductivity $= k$.

Temperatures of water at the entrance and exit $= \theta_1$ and θ_2, say.

Then the quantity of heat flowing through the bar during the time is $m(\theta_1 - \theta_2)$. Also this quantity of heat $= \dfrac{kS(\theta_4 - \theta_3)\,t}{d}$ ....(97·1)

On equating, we can calculate k. The unit of k in C. G. S is calorie per cm. per°C per sec. (For other methods of measuring conductivities, vide volume III).

The following Table supplies the thermal conductivities of various substances :—

Thermal Conductivities

Substance	Conductivity. (C. G. S,)	Substance	Conductivity. (C. G. S.)	Substance	Conductivity. (C. G. S.)
Aluminium	·49	Silver	·97	Ebonite	4×10^{-4}
Copper	·92	Brass	·26	Felt	9×10^{-5}
Gold	·70	Glass	2×10^{-3}	Flannel	$2{\cdot}3\times10^{-4}$
Iron	·14	Fireclay	4×10^{-3}	Procelain	$2{\cdot}5\times10^{-4}$
Mercury	·	Asbestos	6×10^{-4}	Rubber	$4{\cdot}5\times10^{-4}$

98. Influence of Specific Heat on rate of rise of Temperature—When one end of a rod is heated, its temperature gradually rises. By conduction from the previous heated layer each portion of the rod receives *more* heat than the quantity it *transmits* to the next layer ; also it *loses* heat by radiation. Thus in this state each layer of the rod absorbs some heat which raises the temperature of the particular layer. The rod is now at the **variable state.** But after some time the heat received by any layer becomes *equal* to the quantity transmitted to the next layer together with that which is lost by radiation from the particular layer. No further absortion of heat takes place and so no further rise of temperature occurs. This is called the **stationary state.**

In the *variable state* the rate of rise of temperature at any section of the rod, that is to say, the quickness with which that section of the rod is raised in temperature, depends not only on the thermal conductivity but also on the specific heat of the material of ihe rod. In the *stationary state* there is no further absorption of heat and so the flow of heat depends only on the thermal conductivity.

Take a unit volume of the rod in the variable state of temperature.
Let the density (*i. e.*, mass per unit volume) of the material be $=\rho$,
" specific heat of the material $=s$,
" thermal conductivity $=k$,
" rise of the temperature of the substance per second $=t$°C,

Thus k = the quantity of heat reaching unit volume per second $=\rho st$

$$\therefore\quad t = \text{rate of rise of temp.} = \frac{k}{\rho s} = \frac{\text{thermal conductivity}}{\text{thermal capacity per unit volume}}$$

In other words, the rise of temperature of a unit volume of the rod is *directly* proportional to the thermal conductivity (hence to the quantity of heat reaching the volume) and *inversely* proportional to the thermal capacity per unit volume, *i. e.* (specific heat × density). The fraction $k/\rho s$ is constant and determines the *rate of rise of temperature* of a given rod. This constant has been termed **diffusivity** by Kelvin and **thermometric conductivity** by Maxwell.

For a body of higher thermal capacity the rate of rise of temperature is slower. Hence during the *variable* state of a substance of low specific heat, the temperature may rise quickly to the steady point, although the conductivity may be less. For example, the thermal conductivity of iron is double that of lead, while the specific heat of iron is about four times that of lead. If two rods of iron and lead of the same dimension be coated with paraffin wax and exposed at one end to the same temperature (as in Fig. 65), wax is found to melt first on the lead rod ; but ultimately when the steady state is reached, the wax would melt upto a larger distance on the iron rod.

Examples :

1. An iron boiler, 1·25 cm. thick, contains boiling water at atmospheric pressure. The heated surface is 2·5 sq. metres in area and the temperature of the

underside is 120°C. If the thermal conductivity of iron be 0·2 and the latent heat of steam 536 calories per gm, find the mass of water evaporated per hour.

Ans. Evidently the water in the boiler is at 100°C.

Now 2·5 sq. metres $=(2{\cdot}5\times10^4)$ sq. cm.

Let the quantity of heat conducted per hour be Q calories.

$$\therefore\quad Q=\frac{0{\cdot}2\times2{\cdot}5\times10^4\times(120-100)\times60\times60}{1{\cdot}25}\text{, cal., by eqn. (99, 1),}$$

$$=288\times10^6\text{ cal.}$$

$\therefore$ the required quantity of evaporated water $=\dfrac{288\times10^6}{536}$ gm. $=537{\cdot}3$ kgm.

2. The effective wall area of a room is 100 square metres. Find the rate of flow of heat through the walls of thickness 60 cm., when the temperature of the interior is 30°C and that of exterior is 0°C. (Thermal conductivity of the material of the wall $=0{\cdot}002$ C. G. S.) [Mad. U.—1961]

Ans. Area of the walls $=100$ sq. m. $=10^6$ sq. cm. ; thickness $=60$ cm. ; difference of temperatures $=30$°C. ; thermal conductivity $=0{\cdot}002$.

Hence the desired quantity of heat $Q=\dfrac{0{\cdot}002\times10^6\times30}{60}$ cal/sec.

$$=1000\text{ cal./sec.}$$

3. A hollow cube of metal with inner sides 10 cm. and of thickness 1 cm. is filled with melting ice and kept surrounded by water at 100°C. How much ice will melt in an hour? (The conductivity of metal $=0{\cdot}5$ and latent heat of fusion of ice $=80$).

Ans. The cube has six faces, each of area 100 sq. cm.

$\therefore$ the total area through which heat enters $=600$ sq. cm.

$\therefore$ the quantity of heat conducted per hour

$$=\frac{0{\cdot}5\times600\times100\times60\times60}{1}\text{ cal.}=108\times10^6\text{ cal.}$$

$\therefore$ the required amount of ice melting per hour $=\dfrac{108\times10^6}{80}$ gm.

$$=1{\cdot}35\times10^6\text{ gm.}$$

4. Find the difference in temperatures between the two sides of a boiler plate 20 mm. thick and of conductivity 0·2 C. G. S. unit when transmitting heat at the rate of 600 kilocalories per square metre per minute. [Raj. U.—1963]

Ans. Here $Q=(600\times1000)$ calories : conducting area $A=(100\times100)$ sq. cm. ; time $t=60$ sec. ; thickness $d=2$ cm. ; conductivity $k=0{\cdot}2$ C. G. S.

Hence $\dfrac{2\times100\times100\times(\theta_1-\theta_2)\times60}{2}=600\times1000$, whence $\theta_1-\theta_2=10$.

$\therefore$ the required difference of temperatures $=10$°C.

5. A current of dry steam is maintained through a glass steam trap surrounded by melting ice on all sides. Calculate the thickness of the glass walls and the steam condensed per minute, if the total area of the glass trap is 100 sq. cm., ice melted per minute is 75 gm. and thermal conductivity of glass is ·001. [Nag. U.—1953]

Ans. Here $Q=$ heat absorbed by 75 gm. of ice in melting $=(75\times80$ cal.) taking latent heat of ice to be 80 cal./gm. ; $t=60$ sec. ; conductive area $A=100$ sq. cm. difference of temperatures $\theta_1-\theta_2=100$°C ; conductivity k of glass $=\cdot001$.

Hence we get

$\dfrac{{\cdot}001\times100\times100\times60}{d}=75\times80$, whence $d=0{\cdot}1$.

The thickness sought for is thus 0·1 cm.

99. Conductivity of Liquids—With the exception of mercury and metals in the molten state, liquids are in general bad conductors of heat. In measuring the conductivity of a liquid, difficulty arises on account of (1) the convection current set up and (2) the conduction of heat through the walls of the containing vessels.

The thermal conductivity of water can be demonstrated in the following way. Take a thin-walled test tube (Fig. 67) nearly filled with cold water Into the water sink a piece of ice suitably weighed. Hold the test tube in an inclined way and heat the water near the top in small flame. It will be found that the water at the top may be boiling, while the ice does not appreciably melt. By heating the tube at the top convection current is avoided and the low conductivity of water is clearly shown. Due to this low thermal conductivity of water, seasonal variation of atmospheric temperature does not sensibly affect the temperature of water at the bottom of the deep sea, since such water preserves a fairly uniform temperature all through the year.

The conductivity of liquids can be *compared* by Despretz's Method. The apparatus consists of a tall cylinder fitted with a number of equidistant holes arranged vertically. Each hole is provided with a water-tight cork, through which the bulb of thermometer can be inserted within the cylinder. The cylinder is nearly filled with liquid and cylindrical copper vessel is fitted on to the top. By circulating hot water at about 100°C into the upper vessel the liquid in the cylinder is heated by conduction. Very slowly heat travels downwards, showing the low *conductivity* of the liquid. After a sufficiently long interval a steady state is attained when the temperature indicated by each thermometer is noted The liquid is removed and the experiment is repeated with another liquid. If the distance from the bottom of the copper vessel to points showing the same rise of temperature be l_1 and l_2 respectively in the case of liquids of thermal conductivites k_1 and k_2. theory shows that

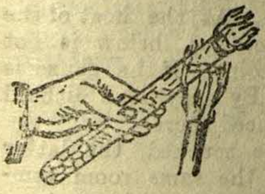


Fig. 67

$$\frac{k_1}{k_2}=\frac{l_1^2}{l_2^2},$$

(For other methods of measuring conductivity of liquids, *vide* Vol III of the Book)

100. Conductivity of Gases—The conductivity of gases is extremely low, the gases (except hydrogen and helium) being worst conductors of heat. Owing to the effects of convection and radiation the measurement of conductivity of gases is rendered very difficult.

The *conductivity* of gas was measured by Kundt and Warburg by observing the rate of cooling of a heated thermometer bulb of approximately known thermal capacity. The bulb was enclosed in a spherical vessel which contained the experimental gas and whose outer surface was kept at 0°C by a water bath. By reducing the pressure to less than 150 mm. the effect of convection was rendered negligibly small, while the conductivity remained unaffected. By observing the rate of cooling of the thermometer when the enclosure was made as perfect a vacuum as possible the effect due to radiation only was found. The difference between the two observations gives the quantity of heat transmitted by conduction only. Knowing the radius of the enclosure, the difference of temperatures and the time for the fall, the thermal conductivity k was calculated. As determined by more careful experiments the value of k for air is 0·00056 C. G.S. (For experimental method of measuring conductivity of gases, *vide* Volume III of the Book).

101. Good and Bad Conductors—If a piece of wire gauze is placed upon the flame of a gas burner (Fig. 68), the flame appears only *below* the gauze but does not get through it. So rapid is the conduction of heat away from the flame by wire gauze that the gas above the gauze does not get sufficient heat to reach the *ignition point* and so the flame does not extend above it. If just over the gauze a lighted match-stick is now applied, a flame is produced, showing that the gas has been all along passing through the meshes. If the gauze were held for a sufficiently long time so as to be heated above the ignition point of the gas, the flame would extend above it.

Again, a piece of wire gauze is held a little above the burner the gas is turned and lighted *above* the gauze. Flame is produced above, but does not extend below, the gauze. As the heat of the flame is rapidly conducted by the gauze, gas below is not sufficiently heated so as to reach the ignition point. If the gauze is moved up or down the flame also moves. By appyling a lighted match-stick below the gauze, flame can be produced there.

If a piece of wood and a sheet of iron be touched, the latter feels colder than the former, although both are the *same* room temperature. Iron being a better conductor takes away heat from the hand more rapidly than wood does; iron, therefore, cools the hand more rapidly and so it appears colder. If the hand is colder than the common temperature of the two objects, to the touch the iron sheet will appear warmer than the piece of wood for heat will pass into the hand more rapidly from the metal than from the wood. Again, if a piece of wood and a piece of metal have been lying in the sun, the latter will appear uncomfortably hotter to the touch. For the same reason, a marble floor appears colder than a cemented floor, and a picce of oil cloth colder than a linen sheet.

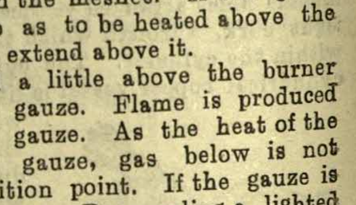


Fig. 68

A glass rod can be melted in a flame by holding it at a point very close to the flame while in metal rods even a distant point becomes too hot to touch. To prevent it from becoming incoveniently hot, handles of soldering '*iron*' and of metal kettles, tea-pots, cookers, etc., are provided with handles of non-conducting substances.

Water can be boiled in a vessel of *thin* paper, for the temperature of the water does not rise above 100°C and the paper being very thin, heat is conducted rapidly through it, so that it does not get sufficiently heated so as to be charred. Steam pipes exposed to air are covered with loosely packed asbestos, so that very little heat is lost. Fireproof safes have asbestos lining and are made of double sheets of iron with intervening space filled with wood-ash saw-dust, etc.

The *low conductivity of air* is observed in various cases. To keep us warm we use woollen clothes, blankets, etc, in winter. These prevent the escape of heat of the body. Two reasons are there; the one is that wool is a bad conductor of heat, and the other is that the air also, which is enclosed within the interstitches of the loosely woven fabrics, is a bad conductor. These clothes are generally terms 'warm' although they are not actually warmer than other objects in the room. It is also to be noted that the mass of air should be at rest; otherwise heat will be lost by convection. If this were not so, the air surrounding our bare body would have been enough to keep us warm and there would not have been any necessity for 'warm' clothes.

Although the conductivities of wool and cotton do not differ much, the former is preferably used in the manufacture of the so called 'warm clothings'. For, wool, being nerely *loose*, holds a thicker layer of air for the same weight. A new quilt is 'warmer' than an old one, for in the latter much air has been pressed out. In cold countries, owing to the enclosed layer of air, rooms are often provided with 'double' windows to keep the room 'warm'. If ice is to be kept long, it is wrapped in a blanket or placed within saw dust, both of which, being poor conductors, prevent the heat of the atmosphere from reaching the ice.

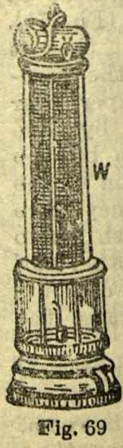


Fig. 69

Davy's Safety Lamp—The heat conductivity of a metal is utilised in the construction of the Davy's Safety lamp. This is an ordinary lamp, the flame of which is completely surrounded by a layer or two of thick iron or copper wire-gauze (Fig. 69); the air necessary for combustion passes in through the gauze. If the lamp is taken inside a mine, in which marsh gas is present, the gas enters the lamp and burns *inside* with a dull blue flame. The heat of the flame being quickly conducted by the gauze, the gas *outside* is not readily heated to the ignition point. The *appearance of the blue flame* indicates the presence of the explosive gas in the mine and acts as a warning to the miners. In the improved pattern, for stronger light the flame is surrounded with glass having wire-gauzes both above and below.

102. Convection of Liquids—The convection of heat in a liquid can be demonstrated in the following way. Arrange the apparatus as in Fig. 70. B is a flask fitted with a cork, and C a wide-mouthed vessel open at the top and closed with a cork at the lower end. The glass tube passes through both the corks. One of these D, which is straight, passes just into B and extends at the other end up into C, while the other tube E. bent as shown in the figure, reaches almost down to the bottom of B and extends to the top just into C, Covering the upper end of the tube D and taking care that no air is left in B, fill the whole apparatus with water. Colour the water in C with some dye and clamp the apparatus suitably. From below heat B with a Bunsen burner. The water in B becomes heated and rises up along D to C, while the coloured colder water from C flows down E into B to take the place of hot water. Thus due to the circulation of water which is set is up, ordinary and coloured water becomes mixed grachually. The above experiment demonstrates the principle of the *hot water heating system* in buildings.

103. Convection of Gases—When a quantity of any gas is heated, it rises up and the colder gas from all sides rushes to occupy the space, thus setting up convention currents. It is a matter of common experience that whenever a house is on fire, the wind blows from all sides towards the place and thus fans the flame. Stand a lighted candle on a basin and pour some water in it ; put an ordinary lamp chimney over the candle, so that the bottom of the chimney is under water. The flame gradually becomes weaker. As the ingress of the air from below is cut off, the flame does not get fresh supply of oxygen which is essential for combustion and so finally it goes out. If. however. a slight opening is made for air by raising the chimney even slightly above the water, the flame continues to burn.

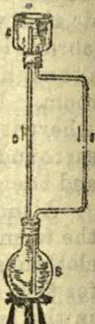


Fig. 70

Now take a piece of T-shaped card board and insert it at the top of the chimney so as to divide the upper part into two portions (Fig. 71). Light the candle again and observe that it continues to burn. Near one side of the cardboard hold the piece of thick paper over the chimney ; the smoke goes down by one side of the card-board and rises up by the other, merely indicating the direction of the air currents. So in one half of the chimney there is an inward draught supplying the fresh air to support the flame, while through the other half the heated air taking the products of combustion goes out. This experiment illustrates the principle of *ventilation* of rooms and in some coal-mines.

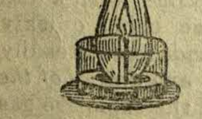


Fig. 71

Ventilation—It is the process by which the warm and impure air in a room goes out, while fresh air from outside occupies its place. Ventilation is brought about by convection currents set up in the room. Presence of inmates makes the air in a room warm and being thus lighter, it rises up and escapes through a series of holes near the cornice, known as *ventilator*. From outside fresh air is drawn in through the opening below, so that a continuous circulation of air is set up and keeps the room filled with fresh air. If there is a fire-place in a room, the draught in the chimney also helps ventilation. The heated air and smoke go up the chimney, while fresh air from the outside comes into the room.

Chimneys of lamps or furnaces are designed on the principle of *convection*. The heated air together with smoke and products of combustion go up the chimney, while fresh air enters throuqh the hole at the bottom of the burner or the fireplace. A convection current is thus set up and maintains the combustion of the flame or fire.

If the chimney be removed, *i*) the regular draught of air is no longer maintained; the carbon dioxide is not, therefore, properly removed, nor does a flame get a regular supply of oxygen and (*ii*) air coming from all sides lowers the temperature of the flame. In either case the combustion becomes incomplete and the flame becomes smoky. It can be shown that with greater intensity of the draught (i) the height of the chimney is greater and the difference of temperature of air inside and outside the chimney is larger. Hence to set up a draught, a chimney is sometimes to be warmed initially.

Gas-filled Lamps—The gas-filled electric lamps contain a small quantity of some inert gas such as argon. As the gas is heated, it rises up, carrying the heat away to the upper part of the bulb, so that without any risk of melting, the filament can be raised to a higher temperature. Further due to the convection current the tiny particles thrown off by the filament are carried to the top of the bulb. So in the case of these gas-filled lamps the blackening caused by the deposit of these particles occurs mainly at the upper part and *not* over the entire inside surface. Such lamps, therefore, have longer life.

104. Natural Phenomena based on Convection—Upon convection of air depend the following natural phenomena.

Winds—Winds are commonly due to convection currents set up in the atmosphere due to local variation of temperature. Air at a particular place gets heated somehow and rises upwards; colder air from the surrounding places proceeds towards the region and produces winds. Again, due to greater evaporation a region at a higher temperature contains larger amount of water vapour. As water vapour is lighter than air, the moisture would go up and produce a convection current. Winds are, therefore, due to convection current in the atmosphere produced due to a local decrease in density of air which originates from rise of temperature or increased evaporation or both. When the fall of atmospheric pressure at a place becomes very great, cold air from all sides rushes to the place with considerable speed and produces *storms*.

Land and Sea Breezes—Land absorbs heat from the sun more rapidly than water. Also owing to its lower specific heat, land is soon raised to a higher temperature than water. Thus during day-time land becoms more heated than the sea. The air in contact with land becomes heated and, being lighter it rises up, while the colder air over the sea blows *towards the land* and generates what is called a *sea breeze*. Again during night, the land cools more rapidly than the sea and so gradually attains a lower temperature than the latter. Being thus at a higher temperature, the air in contact with sea rises up, while the colder air over the land blows *towards the sea*, and produces what is called a *land breeze*. Scarcely beyond twenty five miles from the shore does the effect of such breeze extend.

Trade winds—The surface of the earth near the equator becomes greatly heated by the sun. The air in contact gets heated and rises up, and colder air from the north and south is drawn towards the equator. If the earth would have been at rest, this would have produced a north wind in the northern hemisphere and a south wind in the southern hemisphere. But owing to the rotation of the earth from west to east, there appears at the lower levels a north easterly wind in the northern hemisphere relative to the earth and a south-easterly wind in the southern hemisphere. These are known respectively as the *North East Trade Wind* and the *South East Trade Wind.*

Monsoons—These are *periodic* winds blowing from large continents towards the ocean and *vice-versa*. During the hot seasons the regions in Central Asia become more strongly heated than the waters of the Indian Ocean. This is due to the difference in the effects of the sun's heat on land and water. Currents of hot air rise upwards from the heated regions, while streams of air charged with moisture blow from the Indian Ocean on the south towards these regions. On the way the moist air becomes very warm and takes up plenty of moisture. When it meets a barrier of high mountains, it rushes upwards to great height and is thus cooled both by expansion and by elevation to colder regions. Hence the moisture condenses and drops as rain. Thus the moist air from the Indian Ocean being obstructed by the Assam hills causes profuse rain in Assam and Bengal. In fact, the world's greatest rain-fall occurs at Cherrapunji in Assam. Owing to the rotation of the earth from west to east this current of moist air seems to blow from the south-west and is called the *south-west* or *rainy monsoon*. During winter the rays of the sun fall vertically over the waters of the Indian Ocean, which, therefore, becomes more heated. Hence the colder air from the regions of Central Asia appears to blow across India from the north-east and is called *north-east* or *dry monsoon.*

Ocean currents—These are caused partly by convection and partly by winds blowing along the surface. The cold water in the polar regions sink down and flows towards the equator as a deep *under-current*, while the surface waters at the equator on being heated become lighter and flow towards the poles as *surface current*. The **gulf stream** is a huge surface current which starting from the Gulf of Mexico flows along the east coast of the United States, crosses the Atlantic and runs along the west coast of Europe.

105. Nature of Radiation—We know that transmission by radiation can pass through a vacuum and also through a material medium. Formerly the concept gained ground that the whole space of the universe is filled with a continuous and elastic medium called *ether*. The vibrations of atoms and molecules of a heated body set *up temperature* waves in the surrounding ether, which travel out in all directions with an enormous velocity of about 186,000 miles per second and carry the heat energy from the source to the receiver. When these waves fall on a material body, the transmitted velocity is reduced and the energy is partly absorbed in it producing a rise in the temperature of the body. It is to be noted that at the source the energy manifests itself as heat and on reaching the receiver it becomes evident again as heat: but when it passes through vacuum, it exists not as heat but as *energy of wave-motion in ether*. The latest view regarding the propagation of heat energy is that the radiation consists of transverse waves of periodic electric and magnetic intensities moving through space.

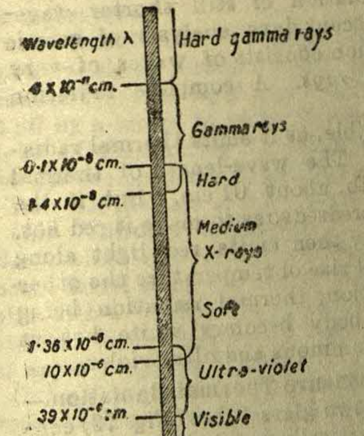


Fig. 72

On reaching us the radiation from the sun excites the sensation of heat and light, both of which are, therefore, supposed to be electro-magnetic waves. It has been found that gamma rays, X-rays, visible light, thermal radiation and wireless waves are all of the same physical nature. being transverse electro-magnetic waves. (*Vide* ELECTRICITY. Chapter XIV). The difference lies in the frequency of the waves and consequently in the wave-length. Thus waves, whose lengths are between 7800×10^{-8} cm. (red) and 3900×10^{-8} cm. (violet), produce the sensation of vision. Radiation caused by these waves is shown as **luminous radiation**. Again, when the wave-length is greater than 7800×10^{-8} cm., the radiation does not affect the sensation of vision but produces only the sensation of heat; such radiation is termed infra red or **thermal radiation**. Beyond thermal radiation, then, is the region of Hertzian waves and the radio waves. At the extreme end there are slow oscillations of alternators. The radiations can also produce chemical reactions. the effect being most marked when the wave-length is less than 3900×10^{-8} cm. beyond the violet end of the visible spectrum ; such radiation is termed **ultra-violet radiation**. Beyond ultra-violet are obtained X-rays and radiation of still shorter wave-lengths constituting gamma rays. In recent days analysis of cosmic rays reveals that a part of this radiation consists of waves of very short wave-lengths called *hard gamma rays*. A complete radiation spectrum is given in Fig. 72.

Radiation from a hot body is not visible, as it emits thermal radiation only and no luminous radiation. The wave-length of thermal radiation extends from 80×10^{-6} cm. to about ·01 cm. But if the temperature of the body is gradually increased so as to make it red hot, it becomes visible, inasmuch as the body then emits red light along with thermal radiation. With a gradual rise of temperature the other colours appear in an increasing proportion, thermal radiation being always present. Ultimately when the body becomes white hot, it emits all kinds of radiations—thermal, luminous and ultra-violet.

106. Instruments to detect and measure Thermal Radiation—

(1) *Ether Thermoscope*—It consists of two glass tubes (Fig. 73) connected by a bent glass tube and is held vertically in suitable stand. The instrument contains only some quantity of coloured ether and ether vapour, the whole of air having been expelled before the instrument is sealed. One of the bulbs is coated with black paint. When thermal radiation falls on this bulb, it is absorbed ; so the temperature of the bulb and consequently that of the contained vapour, rise. The pressure of vapour on the ether within the bulb, therefore, increases and so the level of the ether in the other branch rises.

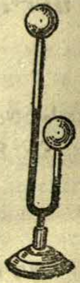


Fig. 73

(2) *Differential Air Thermometer*—Tnis was first used by Leslie. It consists of two equal glass bulbs containing air and connected by a glass tube bent twice at right angles. The tube contains syme non-volatile coloured liquid, *e.g.*, sulphuric acid. The quantity of air in the bulbs is so adjusted that when the two bulbs are at the same temperature, the liquid stands at the same level in both the limbs. If one bulb is raised to a higher temperature than the other, the air in it expands and so the level of the liquid in this stem is depressed, while that in the other rises. Small difference of temperatures is thus indicated by this instrument. In addition to these there are a few electrical instruments, *e.g.*, thermopile, bolometer, radiomicrometer, etc. which can measure a very small difference in temperatures.

107. Thermal Radiation and Light Compared—

The similarity between thermal radiation and light is established from the fact that they obey identical laws and have common properties. The following are the principal points of similarity.

(i) **Transmission through Vacuum**—*Thermal radiation like light can travel through vacuum*. From the sun and through empty space heat as well as light reach us. Within an evacuated jar a thermometer with a blackened bulb is placed. If the whole of it is now placed in the sun or before a fire, the thermometer shows a rise in temperature.

(ii) **Rectilinear Propagation**—*Thermal radiation travels very approximetely in straight lines* ; so we may use the term *heat rays* in a similar sense as *light rays*. The heat coming from a flame can be cut off by a screen in much the same way as the light coming from it can be. Subsequently it will be seen that both heat and light waves have got the property of bending their paths along the edges of opaque bodies. In the case of heat waves the bending is more prominent.

(iii) **Uniform Emission**—*In a homogeneous medium thermal radiation like light is emitted uniformly in all directions.* An incandescent metal sphere is placed in a dark room. If a delicate thermometer with a blackened bulb is placed on any side at the same distance from the sphere, it gives the same reading. From all sides the sphere also appears equally bright.

(iv) **Velocity of Propagation**—*Thermal radiation and light travel through space with a constant velocity* (186,000 miles per second). During a total solar eclipse of the sun, the stoppage of light is simultaneously accompanied by a fall of temperature.

(v) **Inverse Square Law**—*Like light thermal radiation follows the law of inverse square.* Hence the intensity of the radiation at a point varies inversely as the square of the distance of the point from the source. Take a large flat-sided tin vessel S (Fig. 74) with one face coated with lamp-black. Fill it with boiling water. Place a thermopile fitted with the cone collector at tin front of the surface. Observe the deflection of the galvanometer. Now remove the thermopile to T′ at *twice* the former distance. The deflection of the galvanometer remains unchanged.

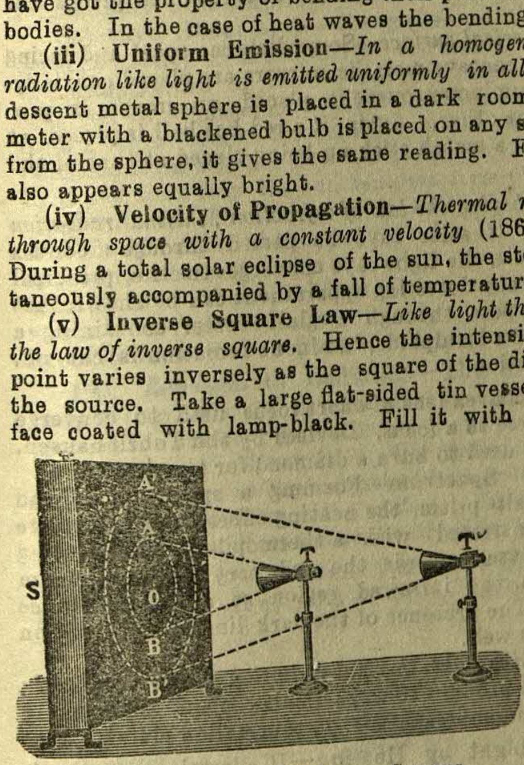


Fig. 74—Proof of the Law Inverse Squares

The area of the blackened surface, from which radiation now falls on the thermopile, is found to have increased four times ; but as the deflections remain the same, the amount of radiation received on the pile must have to remain the same. Hence the intensity of radiation on the thermopile has been reduced to a quarter. In other words, the intensity of radiation varies inversely as the square of the distance.

(vi) **Reflection**—*Obeying the laws of reflection like light, thermal radiation is reflected from polished surface.*

(a) *Reflection at a plane surface*—A polished thin plate is supported vertically. With their axes in the same horizontal plane two tubes are placed in front of the plate. A heated ball is placed near the end of one of the tubes and a thermopile is placed near that of the other, direct radiation from the ball to the thermopile being prevented by suitable screens. By rotating the tube the position for maximum effect of the thermopile is obtained. It would be found that the tubes are *equally* inclined on opposite sides of the normal to the reflector. If one of the tube is slightly displaced, the effect at once grows less marked.

(b) *Reflection at spherical surface*—Some distance apart and facing each other, two large concave metallic mirrors are placed coaxially. If at the focus of one a luminous source is placed, an image is produced at the focus of the other. Again, if a red hot ball is placed at the focus of the former, a match stick on being placed at that of the latter catches fire.

(vii) **Refraction**—*Obeying the law of refraction, thermal radiation like light is refracted.* If solar rays be passed through a convex lens, preferably of rock-salt, the light rays converge to the principal focus of the lens. With any detecting apparatus it can be shown that the heat rays be made to pass through a glass vessel containing a solution of iodide in carbon bisulphide, luminous radiation is cut off, but not the thermal radiation.

A **burning glass** is a convex lens of short focus used to converge the heat rays from a source to a focus. Devised by the Tohrinhausen, a large burning glass was used to burn a diamond for the first time.

(viii) **Continuity of Spectrum**—Forming a spectrum of the solar light with a rock-salt prism, the heating effect along the entire spectrum can be demonstrated with a thermopile. The heating effect is found to increase towards the red part of the spectrum and becomes maximum in the infra-red region at a little distance beyond the visible red, The presence of the dark line in this region has been demonstrated as well.

(ix) **Polarisation**—*Like light, thermal radiation can be polarised*; that is, the vibrations are brought to a particular plane after being transmitted through certain substances (*e. g.*, tourmaline plate).

(x) **Production of Light by Heating**—If electric current is passed through a piece of platinum wire, it first sends out thermal radiation and finally at about 1600°C it gives out white light. Thus at a rather low temperature the body gives out the long heat rays and at a sufficiently higher temperature it emits light rays along with the heat rays.

We thus find that nature of the thermal radiation as well as of luminous radiation is the same; the difference lies only in the wave-length. *Radiant heat is* then *invisible light.*

108. Absorption, Transmission and Reflection of Radiation—When thermal radiation falls on a body, it is generally spilt up into three parts ; (1) A portion is *reflected* at the surface obeying the laws of reflection. (2) A portion is *irregulary reflected* or diffused at the surface so as to be scattered in various directions. (3) A portion is *refracted* into the body. Of this last portion a part may be *transmitted* through the body and the remainder may be *absorbed* within the material. The portion absorbed raises the temperature of the body. Different bodies possess different powers of reflection, transmission or absorption.

Selective Absorption—The power of absorption of a substance depends greatly on the nature of the incident radiation and also on the temperature of the source emitting the radiation. Glass, for example, is transperent to light and also to thermal radiation coming from a source of high temperature, but is practically opaque to thermal radiation from a source at a low temperature (below red heat). The light and heat from the sun can pass through moist atmosphere to a great extent. The surface of the earth absorbs the heat, gets thereby heated and sends out thermal radiation in its turn ; but this radiation is greatly obstructed by the moist air. Again, rock-salt is very transparent to thermal radiation, while quartz behaves similarly towards ultra-violet radiation only. Opaque to luminous radiation, solution of iodine in carbon bisulphide readily transmits thermal radiation.

109. Emission of Radiation—The heat energy radiated by a body is transformed into wave-motion. As the transformation takes place at the surface, the *radiating power* of a body is likely to depend on the nature of the surface. We define **thermal emissivity** of a radiating surface as the amount of heat energy emitted per second by one square centimetre of the surface. Emissivity of different surfaces, even at the same temperature, is found to be different.

Emissivity of different surfaces—Place a Leslie's cube on a suitable stand. It consists of a thin metal cube (Fig. 75) with a hole in the upper lid, through which a thermometer can be inserted. The faces of cube are of different nature. Thus one of the faces may be coated with lampblack, another painted white, a third roughened and the fourth highly polished. Fill the cube with boiling water and keep each of the faces turned for the same time towards a thermopile in series with a galvanometer and placed a fixed distant apart. Note the deflection of the galvanometer each time. Evidently the deflection is proportional to the emissive power of the particular surface.

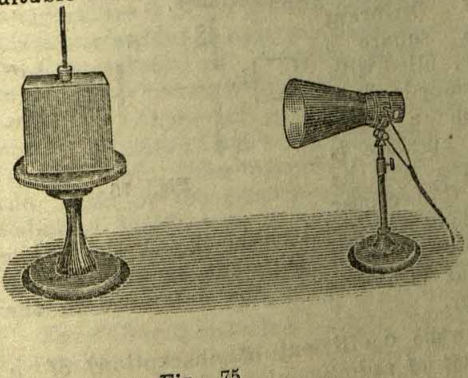


Fig. 75

It is found that for polished surface the deflection is very small; it becomes greater for the rough surface, increases further for the coated surface and is maximum for the lamp-black surface. At a temperature of 100°C, the radiating power of white lead is the same as that of lamp-back; but at higher temperature lamp-black surface is the best radiator. Hot liquids kept in a blackened vessel cool more rapidly than in a polished one. The emissive power of a lamp-black surface is, therefore, usually taken as the standard with which that of any other surfaces is compared. The radiating powers of different substances, even when coated with the same polish, are different.

The emissive (or *radiation*) **power** (or the *coefficient of emission*) *of the surface is the ratio of the amount radiation emitted in a given time by the surface to the amount emitted in the same time by an equal area of a perfectly black surface at the same temperature relative to that of the surrounding.*

Let the radiation from a surface in a certain time be $= e$,
and radiation from an equal area of a lamp-black
surface in the same time $=$ E.

Then the coefficient of emission $\alpha = e/\mathrm{E}$.

Example :

A metal sphere of thermal capacity 6·5 cal. per degree centigrade is observed cool at the rate of 0·5 degree per minute, when its temperature is 50°C above that of the surroundings. If the sphere is 3 cm. in diameter, calculate the thermal emissivity of the surface of the sphere. [Bom. U.—1954]

Ans. The quantity of heat emitted per second $= \frac{6\cdot5 \times \cdot5}{60}$ cal.

Also the area of the sphere $= (4 \times 3\cdot14 \times 1\cdot5^2)$ sq. cm.

Hence the thermal emissivity $= \frac{6\cdot5 \times \cdot5}{60 \times 4 \times 3\cdot14 \times 1\cdot5^2}$

$= \cdot0019$ cal. per sec. per sq. cm.

110. Absorption of Radiation—Absorption of radiation by the surface of a body varies with the nature of the surfaces.

Absorbing Powers of Different Surfaces—Take a suitable square L (Fig. 7) say, a Leslie's cube, filled with boiling water. Take a thermopile T in series with a galvanometer and a perforated screen S, so that the aperture is on the same line as the source and the thermopile. Place a thin disc C of copper with the face towards the pile coated with lamp-black and the other coated with the substance under scrutiny. Different surfaces absorb different amounts of heat as is shown by the difference in the deflection of the galvanometer.

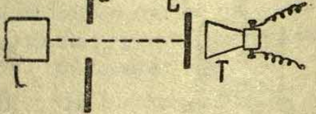


Fig. 76

The absorbing power (or the **coefficient of absorption**) *of a surface is the ratio of the amount of radiation absorbed in a given time by the surface to the total amount of radiation incident on the surface in the same time.* The absorbing and emissive powers of a surface are found to bear the same ratio.

Relation between the Emissive and Absorbing powers of a Surface—This relation is verified by Ritche's experiment. The apparatus devised by Ritche consists of two equal cylindrical metal vessels B and C (Fig. 77) filled with air and connected by a glass tube which is bent twice at right angles and contains some coloured liquid. A large cylindrical vessel A is supported between B and C. The face of A towards B is coated with lamp-black, while the other face is polished. Turned towards A, the face of B is polished, and that of C is coated with lamp-black.

The vessel A is filled with boiling water. The level of liquid in the stem on the two sides is found to remain the same; this shows that B and C are at the same temperature. Or, in other words, the amounts of energy received by them are equal. Here the polished face B absorbs a fraction of the amount emitted by the black face of A, while the black face of C absorbs wholly the fraction radiated by the polished face of A. As the liquid column does not move, the amounts of heat energy received by B and C are equal. Stated in a direct form, *the emissive power is equal to the absorbing power.*

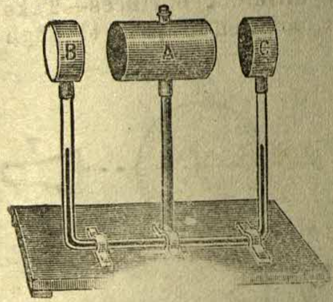


Fig. 77

Hence *good radiators are good absorbers* and poor radiators are poor absorbers. Thus lamp-black surface absorbs the greatest portion of the incident radiation and emits more than any other surface. Further, the absorbing power depends on the temperature of the surface and also on the wave length of the incident radiation. A body, which absorbs the *whole* of the incident radiation, is termed a **perfectly black body.** The radiation emitted by such a body at any temperature is called the **full radiation** for the temperature. A lamp-black surface is the nearest approach to a perfectly black body.

Comparison of Relative Heat Transparency of different materials—The radiation from a suitable source is allowed to fall directly on a thermopile and the deflection of galvanometer in series is noted. A plate of some material is now interposed and the deflection is noted again. The ratio of the latter deflection to the former gives the ratio of the portion transmitted by the plate to the total radiation incident on it. By taking plates of different materials but all of the *same thickness*, the percentage transmitted by each is obtained and can be compared. In the case of a liquid the narrow vessels of the glass containing the liquid may be similarly used.

For different materials (solids, liquids and gases) the percentage transmitted is found to vary widely. Thus for a plate 0·25 cm. in thickness, rock-salt transmits 93% of the incident radiation and glass about 28%, while a solution of alum allows only about 2%. The transparency of rock-salt is the greatest and remains the same for radiation from a source at different temperatures. For other substances *the transparency increases with the temperature of the source.*

The **transmitting power** *of a plate is the ratio of the amount of radiation transmitted to the amount directly incident on it.* The fraction of the incident radiation transmitted by a plate, 1 cm. thick, is called the *coefficient of transmissin* of the material.

111. Reflection of Radiation—The portion of the incident radiation, which is reflected by a surface, depends on the nature of the surface and also on the angle of incidence.

The **reflecting power** *of a surface is measured by the ratio of the amount of radiation (regularly reflected by the surface) to the amount of radiation directly incident on it.*

Comparison of reflecting powers of different surfaces—Take a suitable source H (Fig, 78). Allow the radiation to pass through an aperture in the screen S and to fall directly on a thermopile T connected with a galvanometer and placed a suitable distance apart. Note the deflection of the galvanometer. Now allow the radiation to fall on the experimental plate P. Now shift the thermopile from P to T′ at an equal distance from P to receive the reflected radiation, direct radiation to T′ being prevented by the screen S′. Note the deflection again. The ratio of this deflection to the former one gives the *reflecting power* of the plate for the particular angle of incidence. It is found that a lamp-black surface as the poorest reflector, while a polished metal is the best. *Good absorbers are thus poor reflectors.*

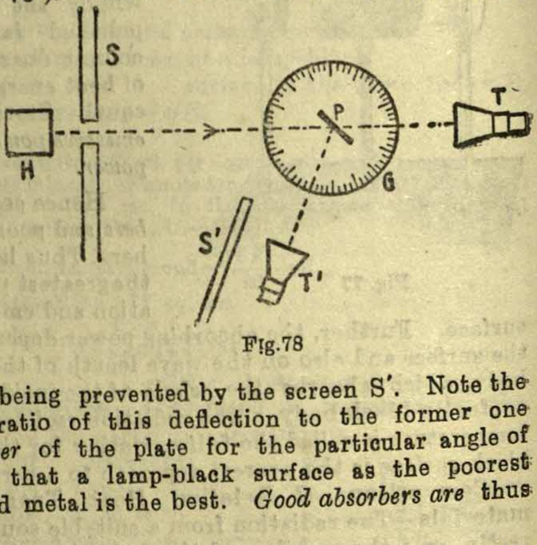


Fig.78

***112 Radiometer**—This fine apparatus was first designed by Sir William Crookes. It consists of a glass bulb B evacuated so as to have a low gas pressure inside (about 1 mm, of mercury). Four thin aluminium or platinum vanes V are mounted on a light framework and are capable of revolving with the least possible friction. One surface of each vane is blackened, while the other is polished, and all the black surfaces are turned the same way. If the radiometer is exposed to heat or strong light on one side (such as burning of a match stick within a foot from it), the vanes continue revolving as if the dark sides of the vanes are pushed by radiating energy.

The principle of action of the instrument is explained by considering the molecular motions of the gas within the bulb, When the

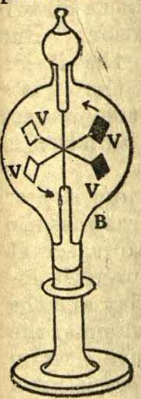


Fig. 79

pressure of gas inside is low, there would be a much smaller number of molecules within the bulb and such molecules would get a better freedom of motion. When heat radiations fall on the vanes, the black surfaces absorb and radiate heat much more than a polished surface. Consequently, the black surfaces are more heated and gas molecules, which collide with a black surface, have their kinetic energy much raised. So they rebound from this surface with a higher velocity than the polished surface. Each rebound of a molecule gives a tiny tap on the vane. These taps take place on both sides of a vane ; but the tap on the black side are more vigorous than that on the other side. The number of these taps per second on a vane is equivalent to the force contributed by the molecules. Since this force is greater on the black side, all these forces together constitute a couple which rotates the frame. If, however, the evacuation be proceeded to the extreme point so that molecular action is negligible, it is likely that in presence of heat or light radiation the vanes might rotate in the opposite direction. This is due to the pressure of heat or light radiation acting more on the polished side than on the dark. The radiation pressure is extremely feeble and is outweighed by the radiometric action at low pressure.

113. Practical observations on Thermal Radiation—We are now in a position to make the general observation that good radiators are good absorbers, while good transmitters or good reflectors are poor absorbers and poor radiators. Polished metals are the best reflectors and are, therefore used, in electric radiators. Firemen's helmets are highly polished. Also any vessel meant to keep liquid hot for a long time is highly polished on the outside ; the outside of a calorimeter is polished and the steam pipes of a locomotive are kept brightly polished. On the other hand, hot water pipes for heating a room may be blackened on the outside in order that greater radiation of heat may be obtained from them. Hot liquids cool down sooner in a black stone vessel than in a metal cup with polished surface. Polished shoes and hats are cooler than dull ones.

Again, the absorbing power of a black body is more than that of a white one. Cooking vessels should preferably be black and rough on the outside. In common unglazed earthen vessel a liquid gets comparatively heated. Exposed to the sun, a thermometer with a blackened bulb, gives a higher reading than an ordinary one. White linen is more suitable in summer as it absorbs very little of the sun's rays, while black clothing is preferred in winter, for it absorbs almost the whole of the incident radiation and thus becomes warm. A building painted white is warmer in winter and cooler in summer than one painted dark.

Radiation from a source by selective absorption is utilised in a green

house. The thermal radiation from the sun passes through the glass covering of the green-house. The ground and the plants inside absorb the radiation, get heated and send out thermal radiation. But gless being a bad conductor of heat confines it and keeps the inside warm. On the same principle depends the action of the *glass fire-screen*. The portion of the thermal radiation possessing the greatest heating effect being absorbed, only a part of the thermal radiation is transmitted along with the luminous radiation.

Since to a great extent moist air absorbs thermal radiation, it prevents the earth from becoming much heated by sun's rays. Again as the radiation escaping from the heated surface of the earth into space at night comes from a source of comparatively low temperature, it is absorbed by the moist air. Thus too much heating during the day and the cooling by radiation at night are both diminished. Again, in summer the envelope over plants and valleys maintains them comparatively warm during night ; but deserts and mountain-tops, whereon there is little moisture, cool down rapidly. On a cloudy night it feels rather warm, because the heat radiated by the earth is obstructed and sent back by the clouds. On a clear night dew is more copious, radiation takes place freely and the surface of the earth cools rapidly. As gases are poor radiators of heat, the hot gases in furnace are made to play on the fire-bricks which are thus heated and radiate heat in their turn.

Earth's surface absorbs solar heat during day and radiates the heat in night. During hot seasons the day is longer than the night, and so more heat is absorbed than emitted. This causes accumulation of heat on the earth's surface from day to day and so the weather feels hotter. During the cold seasons the night is longer than the day ; so the ground radiates during night not only all the heat it has absorbed during the day but also a portion of the accumulated heat. The earth's surface is, therefore, cooled down from day to day and causes the weather to feel colder.

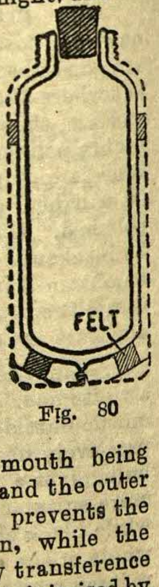


Fig. 80

114. Thermo-flask—This is used for keeping hot drinks hot and cold drinks cold for a long time. It consists of a double-walled glass vessel (Fig. 80) ; the space between the walls is exhausted of air through a fine tube at the bottom of the outer wall, which is finally sealed off. The outer surface of the inner wall and the inner surface of the outer wall are silvered. The vessel is placed on spring with a metallic or wooden case, its mouth being closed by a cork stopper. The space intervening the flask and the outer cover is sometimes packed with felt. Hence the vacuum prevents the passage of heat by conduction as well as by convection, while the silvering minimises radiation to a very great extent. Any transference of the heat by conduction through the wall of the vessel is minimised by the non-conducting packing. Thus in a thermoflask the passage of the heat to and from the interior by conduction, convection as well as by radiation is minimised. It is sometimes called a *vacuum flask*. A **Dewar flask** is similar in construction and is designed to store liquefied gases,

115. Loss of Heat by Radiation—It is experimentally found that the rate, at which a body loses heat by radiation, depends on (i) the temperature of the body, (ii) the nature and extent of its surface and (iii) the temperature of the surrounding space into which radiation takes place.

Newton's Law of Cooling—Newton first investigated the loss or gain of heat by a body placed in given surrounding and enunciated the following law: *The rate of loss of heat from a body is proportional to the difference between the temperatures of the body and its surroundings*. This law refers to bodies cooling in air: it is applicable only for small difference of temperatures and does not hold good when the temperature difference is large (exceeding about 30°C). (For further details *vide* J. Chatterjee's *Intermediate Practical Physics*, All-India Edition.)

The determination of specific heat of liquid by the method of cooling based on Newton's Law has been already described in Calorimetry. The laws of cooling or radiation have been stated by Dulong and Petit and later on, by Stefan and Boltzman. Finally from a different standpoint Planck established the law. The Stefan-Boltzmann Law lays down that the rate at which the energy is radiated by a hot black body, is proportional to the fourth power of its absolute temperature :—

$$E=\sigma T^4, \qquad \ldots \qquad \ldots \qquad (115.1)$$

where E stands for the energy emitted per square centimetre per second by a full radiator, σ is a constant ($5{\cdot}72\times10^{-5}$ ergs per sq. cm, per sec. per degree absolute^4) and T is the absolute temperature, A full radiator absorbs all the radiant energy falling on it; a lamp-black surface is a near approach to this condition. The total energy radiated is equal to the continued product of E, the surface area and the time.

According to Wien's Law, the product of the absolute temperature of a body and the wave-length, at which highest energy is radiated in a given time, is constant. Thus

$$T\lambda_{max}=B. \qquad (115.2)$$

where λ_{max}=the wave length at which maximum energy is radiated and B=a constant (0·2890 cm. degree, absolute). Thus as the temperature of the body rises, wave-length, at which maximum energy is radiated, decreases. Thus as a body is more and more heated, it glows; the first glow appears as red and on further increasing the temperature, the colour extends towords the yellow and green parts, until it becomes white hot.

Example :

A body initially at 80°C cools to 64°C in 5 minutes and to 52°C in 10 minutes. What will be the temperature of the surroundings? (G. U.—1954)

Ans. Let θ°C be the temperature of the surrounding and m and s be the mass and specific heat of the first body respectively. The loss of heat from the body in 5 minutes $= ms \times (80-64) = 16\ ms$. Hence the rate per second of the loss of heat $= 16ms/300$. Similarly for the second body the rate per second of the loss of heat $= ms(80-52)/600 = 28ms/600$.

Now the mean excess of temperature of the first body $= \frac{80+64}{2} - \theta = 72 - \theta$

and mean excess of temperature of the second body $= \frac{80+52}{2} - \theta = 66 - \theta$.

Then Newton's Law of cooling allows us to obtain the equation.

$$\frac{72-\theta}{66-\theta} = \frac{16ms/300}{28ms/600}, \text{ or, } \frac{72-\theta}{66-\theta} = \frac{8}{7}, \text{ whence } \theta = 24.$$

Thus the desired temperature is 24°C.

116. Prevost's Theory of Exchanges—If a red hot metallic ball is placed in front of a thermopile, it indicates that the ball is sending out thermal radiation. If the hot ball is now replaced by a lump of ice, it now shows that ice also is sending out some radiation. Again, if a hot body is suspended in an evacuated chamber, the walls of which are imprevious to heat so that there is no loss of heat by conduction or convection, the body will gradually cool by radiating heat to the enclosure. After some time the temperature of the body and that of the enclosure become the same and there is then no further change of temperature. It may appear that under uniform temperature, the body does not radiate, But the fact is not so, for if the temperature of the enclosure is now lowered, that of the hot body will at once begin to fall again, showing that radiation has been taking place all the time. Hence the only rational conclusion is that *every body sends out radiant energy at all times and at all temperatures*, the amount of which depends only on its own temperature and on the nature of its own surface but is not affected by the presence of surrounding bodies. Thus irrespective of its being placed near a very cold body or any other body, a red hot ball radiates the same amount of heat.

Hence we may conclude that their is always a *mutual exchange* of radiation between any body and the surrounding objects and that the rise or fall of temperature observed in a body depends upon the difference between the amount of radiation emitted by the body and absorbed by it from surrounding objects. This is known as **Prevost's Theory of Exchanges**, as it was first formulated by Prevost of Geneva in 1792. Thus when we stand before fire, we feel warm, because our body is receiving per unit of time more heat energy from the fire than it is losing by its own radiation. Again when we stand before a block of ice, we feel cold, for our body loses more heat by radiation than it receives from the ice which is at a lower temperature. When a body absorbs more radiant energy than what it itself emits, the body rises in temperature and when less, the body cools. When the quantity absorbed is equal to the quantity emitted, the temperature of the body remains constant. A body ceases to emit radiation, only when it is at the temperature of absolute zero. This theory also re-emphasises that the absorbing power of a surface is equal to its emissive power.

EXERCISES ON CHAPTER VIII

	Reference
1. Mention the ways in which a body loses heat. Give a brief explanation of each way with examples. Which will give you better protection against cold, one thick shirt or two of half the thickness, the material being the same? Explain your answer. (C. U.—1953)	Art. 93
2. What are different modes of transference of heat? Carefully distinguish between them, giving examples. (Gau. U.—1971; Andhra U.—1961: C. U.—1951, '60; Dac. U.—1960,; Pat. U.—1962; V. U.—1952; Dal. H. S.—1971,	Arts. 92 & 94
3. Distinguish between conduction and convection of heat. Illustrate their difference by examples. (C. U.—1959, '66; Dac. U.—1963; Utkal U.—1968)	Art. 94
4. Describe an experiment to compare thermal conductivities of metals in the form of rods (And U.—1970; Anna. U.—1960; Gau. U.—1965; Mad. U.—1971;	Art. 95
5. Define thermal conductivity. (U.P.B.—1964; R. P. B. —1964, '66; Nag. U.—1952, '65; E.P.U.—1961; C.U.—1952; Gau. U.—1965: Raj. U.—1973: Mad U.—1961, '72)	Art. 96
6. Describe Searle's method of finding the thermal conductivity of metals. (Gau. U.—1963; Nag. U.—1972).	Art. 97
7. Define the terms 'Thermal conductivity' and 'Thermometric conductivity.' (Gau. U.—1964; Bom. U.—1963, '69)	Arts. 96-98
8. The coefficient of conductivity of copper is 0·96. How many heat units will pass per minute across a plate of copper of surface area 1 sq.m. and one centimetre thick, when its opposite faces are kept at temperature differing by 10°C? *Ans.* 576×10^4 calories.	Art, 96
9. Define 'thermal conductivity'. Explain the statement that the thermal conductivity of glass is ·002 C. G. S. unit. (P. U.—1971; U, P. B.—1970)	Art. 96
10. How much heat is conducted in 2 hours through a pane of glass 15 sq. ft. in area $\frac{3}{16}$ inch thick, if the surface temperatures are 60°F and 10°F? (Thermal conductivity of glass is $1{\cdot}4\times10^{-3}$ cal./sec. cm. °C) (C. U.—1956) *Ans.* 8193024 calories.	Art. 96
11. The inside of a glass window 0·3 cm. thick is at a temperature of 30°C, and the outside at 40°C. Estimate the rate at which heat is entering the room through the window, if its area is 2 square metres. (U. P. B.—1964) *Ans.* 133·3 cals./sec.	Art. 96
12. A glass window pane is 1 metre long, half a metre broad and 2 mm. thick. The outside temperature is 50°C; but the inside temperature is maintained at 20°C by air conditioning. If the conductivity of the glass be ·002 calorie per cm. per sec. per degree centigrade, how much heat will be conducted through the glass per minute? (Poona U,—1964) *Ans.* 90000 cals.	Art. 96
13. Taking the conductivity of iron to be ·017 C.G S. unit what difference of temperatures should exist between the surface of an iron wall 3 cm. thick, through every square	Art. 96

Reference

metre of which heat is streaming from a furnace to the boiling water at the rate of 30,000 units per minute.

Ans. 8·8°C.

Art. 96

14. The opposite faces of a cubical block of iron of cross section 4 sq. cm. are kept in contact with steam and melting ice. Determine the quantity of ice melted at the end of 10 minutes, the conductivity of iron being 0·2 (Latent heat of ice = 80 calories). (Pat. U.—1955 ; E. P. U.—1971)

Ans. 300 gm.

Art. 86

15. Heat is conducted through a slab of two layers of different materials of thermal conductivities of 0·2 and 0·3 respectively. The thickness of each layer is 1 cm. If the temperature of two outer surfaces are 100°C and 0°C, find the temperature of the interface. (C. U.—1952)

Ans. 40°C

Art. 96

16. Steam at 100°C is passed into an iron pipe (1 metre long, 15 mm. thick) whose circumference is 10 cm. Water at 100°C collects at the rate of 100 gm. per minute. What is the temperature of the outside? (Conductivity of iron = 0·2 ; latent heat of steam = 540 cal./gm).

Ans. 93·25°C.

Art. 96

17. One end of a metal rod, 40 cm. long and 1·6 cm. in diameter, is placed in boiling water (100°C) and the other end in ice. If the thermal conductivity of the metal is 0·9 C. G. S. unit, calculate the mass of ice which melts per minute. (Latent heat of water = 80 cal. per gm.) (Mad. U.—1971)

Ans. 6·93 gm.

Art. 96

18. Find how much steam per minute is generated in a boiler made of boiler plate, 0·5 cm, thick, if the area of the walls of the fire chamber is 2 sq. metres, the mean temperatures of the plate faces are 200°C and 120°C respectively, the latent heat of steam is 532 and the conductivity of steel plate is 0·164.

Ans. 60·321 kgm.

Art. 96

19. The glass windows of a room have a total area of 3 sq. metres and the glass is 5 mm. thick. Calculate the rate of flow of heat through glass, when the inside temperature is 20°C and the outside temperature is—10°C. (U. P. B.—1960)

Ans. 9600 cals/sec.

Art. 96

20. A metal vessel, 1 sq. metre in area, whose sides are 0·5 cm. thick, is filled with melting ice and is kept surrounded by water at 100°C. How much ice will melt in an hour? Thermal condnctivity of metal is 0·02 C. G. S. unit ; latent heat of ice is 80 calories/gm. (Utkal. U.—1958)

Ans. 1800 kgm.

Art. 96

21. A suspended glass cube, each conducting face of which has an area of 144 sq. cm., is packed with ice and is closed on all sides. Thickness of the glass walls is 0·8 cm. How much ice will melt in one hour? (The temperature of the room is 20°C ; k of glass is ·0025 C. G. S ; L heat of ice = 80 cal./gm.) (Gau. U.—1964)

Ans. 2430 gm.

Reference

22. Water is being boiled in an iron vessel over an oven. Find the rate in gm./sec. at which water is evaporating, assuming that no heat is lost otherwise. The area and thickness of the bottom in contact with the oven are 250 sq. cm. and 5 mm. respectively. (Latent heat of evaporation of water=536 cal./gm., temperature of the oven=400°C, conductivity of iron='2 C. G. S. unit, boiling point of water=100°C.) (Pat. U.—1952) Art. 96

Ans. 56 gm./sec. (nearly).

23. A thin-walled copper pipe, 6 cm. in diameter, passes through a water bath at 0°C. Brine at—10°C is circulated through the pipe and a layer of ice, 5 cm. thick, has formed round it. Find approximately how long it will take for the next half millimetre layer to form. (Conductivity of ice='005 C. G. S. : latent heat of ice=80 cal./gm. and 1 c. c. of water forms 1·09 c. c. of ice at 0°C) Art. 96

Ans. 533·8 sec.

24. The effective heating surface of a boiler is 2 sq. metres and the temperature of the two sides of the steel boiler plates, 8 mm. thick, are 150°C and 180°C respectively. Find the mass of water evaporated per minute after boiling has commenced. [Latent heat of steam 525 cal./gm. , conductivity of steel=0·14] (Mad. U.—1960) Art. 96

Ans. 16·8 kgm.

25. A spherical glass of 14 cm. radius is filled with water at 100°C and immersed in melting ice. At what rate must heat be generated inside the globe, if the temperature of water in it is to be maintained constant? (Thickness of glass=1 mm., its conductivity=0·0025). (Raj. U.—1961) Arts. 96 & 97

Ans. 6154·4 cal./sec.

26. One end of a metal bar is heated. Indicate clearly the factors on which the rate of rise of temperature at any point on it depends. (Pat. U.—1955 ; Utkal U.—1961 ; R. P. B.—1952, '69 ; Raj. U.—1963) Art. 98

27. If a wire-gauze is held over the flame of a Bunsen burner, the flame cannot go up through the holes in the gauze but remains below it. Explain this. (V. U.—1954) Art. 101

28. (*a*) Explain the working of Davy's Safety Lamp. (C. U.—1958) Art. 101

(*b*) Explain why water can be boiled in a vessel made of thin paper. (V. U.—1955)

29. Explain why we get land breeze during night and sea breeze during day. (Utkal U.—1954) Art. 104

30. Explain how you would prove experimentally that heat rays like light rays obey laws of reflection and refraction. (Utkal U.—1964 ; Pat. U.—1971 : P. U.—1962) Art. 107

31. Describe and indicate the use of a Leslie's cube. (C. U.—1951) Art. 110

Reference

Arts. 109 & 110

32. Define emissive power and absorption coefficient. How can you prove that the emissive power of a surface is proportional to its absorption coefficient? (R. P. B.—1965)

Art. 109

33. A room heater is made up of 10 polished thin-walled tubes of copper each one metre long and 5 cm. in diameter. If hot water at 70°C circulates constantly through the tubes, calculate the amount of heat in calories radiated in an hour in a room where the average temperature is 15°C. [Emissivity of copper $=4\times10^{-6}$ cal./sec. per sq. cm.], (R. P. B.—1968)

Ans. 12434·4 cals

Art. 113

34. Give an account of experiments to show that good absorbers of heat are good radiators. (C. U.—1955 ; All. U.—1967)

Art. 110

35. How will you show that the absorbing powers of different substances are different? What relation does the absorbing power of a surface bear to its emissive power at the same temperature? (Utkal U.—1972)

Art. 110

36. Describe Ritchie's apparatus and its working to show that emissive power of a surface is equal to its absorptive power. (Bom. U.—1951, '64 ; Nag. U.—1956,'69 ; Gau. U.—1972)

Art. 110

37. What do you understand by the absorbing and emissive powers of a surface?

Art. 110

38. Describe an experiment to compare either the emissive powers or the absorptive powers of two surfaces. (Nag. U.—1961, '74)

Art, 114

39. Describe the principle and construction of 'Dewar vacuum flask' or a "Thermoflask".
(C. U.—1958 ; P. U.—1961 ; Gau. U.—1963 ; Del. U.—1971 ; V. U.—1953 ; And U.—1962)

40, State the laws of heat radiation and explain Newton's Laws of Cooling. (Gau. U,—1964 ; Bom. U.—1963)

Art. 115

41. State Newton's Law of Cooling and its limitations. How will you find the specific heat of a liquid by the method of cooling? (E. P. U.—1963 ; U. P. B.—1971 ; R. P. B.—1973)

Art. 116

42. Give an outline of Prevost's Theory of Exchanges. The bulbs of two identical thermometers are coated, one with lampblack and the other with silver. Compare their readings (*a*) when in a water bath in a dark room, (*b*) when in the sun, and (*c*) when exposed in a clear night. (Pat. U.—1967)

Arts. 115 & 116

43. Write notes on the Newton's Law of Cooling.
(Raj. U.—1963 ; R. P. B.—1970 ; Dac. U.—1969 ; Del. U.—1963 ; And U.—1962)

Art. 115

44. A metal sphere cools 80°C to 70°C in 5 minutes and to 62·5°C in the next 5 minutes. Calculate the temperature at the end of the next 5 minutes. (Bom. U.—1973)

Ans. 56·875°C.

Art. 116

45. Explain Prevost's Theory of Exchanges
(G. U.—1956, '62, '65 ; R. P. B.—1968 ; E. P. U.—1960, '63 ; Bom. U.—1962, '65)

CHAPTER IX

MECHANICAL EQUIVALENT OF HEAT

117. Heat and Work—In primitive ages fire was produced by rapidly spinning the blunt end of a piece of hard wood in a shallow hole made in a block of softer wood. Sparks are produced when a piece of hard metal is pressed against the rim of a revolving grindstone or when a flintstone is struck against a piece of iron. Match sticks may be ignited by rubbing. When we work tools like saws, files, drills, etc., they are found to be considerably heated. Such illustrations show that heat is produced by friction.

Again, when a body moving with a high speed strikes a target the kinetic energy possessed by a body gets converted into heat energy. Thus, when a body falling from a large height strikes the ground, it is found to rise in temperature. If a piece of iron is struck several times with a hammer, it is found to be heated. Sparks are produced when the iron-shoe put to a horse's hoofs strikes hard against a metalled road. The action of a tinder box is a very suitable illustration of production of fire by impact. At the time of pumping a bi cycle the barrel of the pump gets hot mainly due to the compression of air. All these facts go to prove that heat is produced by mechanical work. Conversely, heat energy can be converted into mechanical energy, as is illustrated by steam engines and other kind of heat engines. Such examples naturally suggest that there exists some relation between the work done and the consequent heat produced.

118. Nature of Heat—Various theories have been advanced to explain the nature of heat.

Caloric Theory—According to this theory, which was prevalent before the nineteenth century, heat was supposed to be an invisible and indestructible fluid, called *caloric*. The fluid was assumed to fill material bodies and to flow from one to an other body at a higher temperature. As caloric could be neither created nor destroyed, the total quantity of heat in a system was supposed to remain the same. So the caloric lost by one body of the system must be gained by other. The production of heat by friction or compression was attributed to the sqeezing out of the caloric.

Rumford's Experiments—While supervising the boring of cannon at Munich in 1798, Count Rumford observed that the temperature of the gun as well as of the cutting tools rose considerably during the process. It appeared probable that the generation of such a large quantity of heat was due to a change in the thermal capacity of so small a quantity of the substance owing to a mere change in form. Rumford also found experimentally that specific heat of a metal whether taken in a large mass or as chips, was the same. He further observed that there was no limit to the amount of heat that could be produced by friction. He, therefore, argued that *considered as a material substance, heat could not thus be produced without limit* and so he suggested as the alternative theory that *heat must be a kind of motion.* From independent experiments the conclusion was arrived at by Sir Humphry Davy.

Dynamic Energy Theory—It has been assumed that heat is due to a kind of motion. Of what is the motion remains the question and awaits the solution. According to *Dynamic Theory*, the molecules of a body are in a state of rapid vibratory motion. The total quantity of heat possessed by a body consists in the total amount of kinetic energy possessed by the molecules by virtue of their agitation. In solids a part of the energy may be potential energy due to the attractive forces between the molecules. When a body is heated, the rapidity of the movement of the molecules increases and so there is a gain in the kinetic energy. When the body is cooled, reverse in the case. Hence *heat consists in the kinetic energy of the constituent molecules of a body.*

119. Mechanical Equivalent of Heat—The generation of heat by mechanical work and the production of work by heat indicate an intimate relation between heat and work. Careful experiments show that there is a definite quantitative relation between the amount of work done and the amount of heat produced. In other words, a definite amount of work must be performed so as to produce a definite quantity of heat. The relation is expressed in the **First Law of Thermodynamics** which may be stated as follows, following Maxwell—

"*When work is transformed into heat or heat into work, the quantity of work is mechanically equivalent to the quantity of heat.*' If an amount of work W produces a quantity H of heat.

$$\frac{W}{H}=J. \qquad \text{or} \qquad W=JH \qquad \ldots(119.1)$$

where J is a constant provided that all the work done is utilised in generating heat and that no part of it is wasted. *This constant is the energy equivalent of the unit of heat and is termed the mechanical equivalent of heat or Joule's equivalent. Hence* **mechanical equivalent of heat** *is the amount of work done to produce unit 'quantity* (one calorie*) of heat.* It is denoted by 'J' after the name of Dr, Joule.

120. Methods for determining J—Various methods have been devised to determine the value of the mechanical equivalent of heat J, some of which are given below.

(1) **Vertical Fall Method**—A cylindrical glass tube, about 50 cm. in length and closed at each end by padded porcelain stoppers, contains a number of lead shots. The tube is held vertical, when the shots collect on the bottom stopper. On now suddenly inverting the tube, the shots fall down along the entire length of the tube and strike against the lower stopper. If the tube is quickly inverted a

number of times, the shots are found to rise in temperature slightly. This is because of the fact that the work done due to the fall of the shots is converted into heat energy which mainly goes to raise their temperature. It is here assumed that very little portion of heat energy is absorbed by the glass tube and the stoppers. So this method is an *approximate* one.

If m be mass of the lead shots and h the height through which the shots fall down each time, the loss of potential energy due to each fall is mhg. If there are n such falls. the total loss of potential energy is $mngh$. So this is the amount of work done by the shots. Again, if s be the specific heat of lead and θ the rise of temperature, heat absorbed by the system is $ms\theta$. Then

$$\text{J}=\frac{\text{work done}}{\text{heat produced}}=\frac{mngh}{ms\theta}=\frac{ngh}{s\theta} \qquad \ldots(120.1)$$

We also know that loss of potential energy of the shots manifests as the gain in kinetic energy, which is converted into heat each time by impact with the stopper. If v be the velocity of impact, the gain in kinetic energy is $\frac{1}{2}mv^2=mgh$. Thus for n impacts the total kinetic energy is $\frac{1}{2}mnv^2$. Then

$$\text{J}=\frac{\text{work done}}{\text{heat produced}}=\frac{\frac{1}{2}mnv^2}{ms\theta}=\frac{nv^2}{2s\theta} \qquad \ldots(120\ 2)$$

Examples :

1. A closed card-board tube, 1 metre long, contains some lead shots at one end and the tube is quickly inverted. so that the shots fall through the whole length of ihe tube. This operation is repeated 100 times. Find the rise of temperature, if the specific heat of lead is ·03.

Ans. If m=mass of lead shots in gm., the loss of potential energy for one inversiou of the tube=$mg\times100$ ergs. For 100 inversions, the work done=$m\times980\times100^2$ ergs, taking $g=980$ cm./sec.^{2}.

Again, heat developed=$m\times0{\cdot}03\times\theta$, where θ stands for the rise of temperature.

Hence since $\dfrac{\text{work done}}{\text{heat developed}}=\text{J}=4{\cdot}2\times10^7$, we derive

$\therefore\ 4{\cdot}2\times10^7\times m\times{\cdot}08\times\theta=m\times980\times100^2$, or, $\theta=7{\cdot}8$.

Hence the required rise in temperatwre is 7·8°C.

2. A body falls through a height of 837 metres. Assuming that all the energy of fall is convertrd into heat, find the rise of temperature of the body, if the specific heat of the body is 0·049 cals./gm. Given that $\text{J}=4{\cdot}185\times10^7$ ergs./calorie ; $g=980$ cm./sec.^{2}.

Ans. Since the K.E. of fall is equal to P. E. at the height, we have $\frac{1}{2}mv^2=mgh$=work dpne=JH, where H=heat developed. Hence, if t represents rise of temperature.

$\therefore\text{J}\quad m\times{\cdot}049\times t=mgh.$

whence $t=\dfrac{gh}{\text{J}\times{\cdot}049}=\dfrac{980\times8370}{4{\cdot}185\times10^7\times{\cdot}049}=40^\circ$

The required rise in temperature is thus 49°C.

(2) **Joule's Method**—The apparatus used by Dr. Joule consists of copper calorimeter C (Fig. 82), having four sets of lateral vanes V, V soldered to its side. The calorimeter is provided with a water-tight lid. Through a central opening in a lid is introduced a spindle S, which carries a number of paddles P, P. The paddles can just

rotate through corresponding spaces within the fixed vanes. The position of the fixed vanes and the paddles attached to the spindle is shown separately in Fig. 83. To a vertical wooden cylinder D the spindle S is attached by a removable pin G. Two cords are wound round D in the same direction in such a way that they leave the cylinder at the opposite extremities of a diameter. The other ends of the cords are stretched over pulleys P, P carrying at their ends two equal lead weights W, W. To indicate the height there are vertical scales L, L through which the weights are allowed to fall. A measured quantity of water is put into the calorimeter and its temperature is noted by a sensitive centigrade thermometer, which is introduced into the calorimeter through an aperture in the lid.

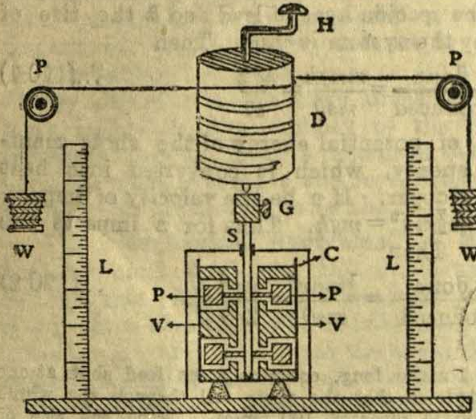


Fig. 82—Joule's Calorimeter

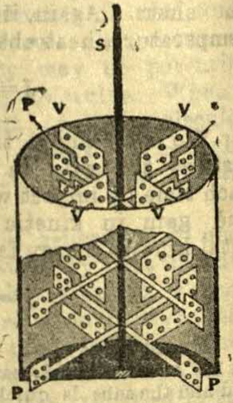


Fig. 83

By removing the pin, the cylider D is detached from the spindle and is rotated by the handle H, till the weights are wound up to a definite height. The pin is replaced and the weights are released. As the weights descend, the paddles turn and tend to communicate a churning motion to the water; but as this is partly prevented by the fixed vanes, the kinetic energy of the moving water is converted into heat. Just as the weights are about to reach the ground, the cylinder is quickly detached by taking away the pin and the weights are again wound up to the previous height; the pin is replaced and the weights are allowed to fall as before. The operations are repeated several times and the final temperature of the water in the calorimeter is carefully noted. The amount of heat necessary to raise the temperature of the calorimeter and its falling weights.

Let the mass of the each weight be $= M$ gm.,
,, height through which each weight falls $= h$ cm.,
,, number of times of fall $= n$.
Let water equivalent calorimeter and contents $= m$ gm.,
,, initial and final temperatures of water $= t_1$°C and t_2°C,

and the velocity of each weight just before reaching the ground $= v$ cm. per sec.

∴ P. E. possessed initially by the two weights $= \mathrm{M}gh$ ergs.

Also the K. E. retained by the weights just before striking the ground $= \frac{1}{2}\mathrm{M}v^3 \times 2$ ergs.

Hence energy utilised in rotating the paddles in each fall $= (2\mathrm{M}gh - \mathrm{M}v^2)$ ergs.

∴ in n falls total work done (*i.e.*, total energy utilised) $= \mathrm{W} = (2\mathrm{M}gh - \mathrm{M}v^2) \times n$ ergs.

Again, heat produced $= \mathrm{H} = m(t_2 - t_1)$ calories. ...(102,3)

Hence $\mathrm{J} = \dfrac{\mathrm{W}}{\mathrm{H}} = \dfrac{(2\mathrm{M}gh - \mathrm{M}v^2)}{m(t_2 - t_1)} n$ ergs per calorie

Precautions were taken against, and corrections were made for, the loss of heat due to *conduction* and *radiation* respectively. The loss of energy due to *friction* at the pulleys was also taken into account. Joule however, took the specific heat of water to be the same at all temperatures, the rise of temperature of water in the calorimeter was very small.

From a large number of readings Joule intially found the mean value of J to be 773 ft. lbs. of works per B. Th.U. ; that is, in order to produce the amount of heat required to raise 1 lb. of water through 1°F, the amount of work to be performed is 773 ft. lbs. By using a modified form of the apparatus Joule finally obtained the mean value at 778 ft. lbs per B, Th, U. or $4{\cdot}18 \times 10^7$ ergs per calorie.

(3) **Searle's Method** (*Friction-Cone Method*—This is a very simple and convenient laboratory method for determining the mechanical equivalent of heat. The apparatus consists of two hollow truncated cones C and C_1 of brass one fitted smoothly in the other, and the combination is attached as its upper end to a circular wooden disc D (Fig. 84). The outer cone C is fixed through an ebonite plug to a metal rod attached to a shaft, which can be rotated rapidly by means of belting b going to a hand-wheel W or a motor. The inner cone C_1 is kept in position by an ebonite ring near the top (Fig. 85). The inner cone C_1 is partly filled with a known mass of water. The bulb of a sensitive thermometer T passing through a hole in the disc is immersed in the water. A cord is wound round the circumference of the disc D, the end of which passes over the pulley P and carries a weight M at its end. The cylinder supporting C is held by sockets B_1 and B_2 in which there is a ball-bearing arrangements. The actual apparatus is shown in Fig. 84 ; a section thereof is exhibited in Fig. 85.

As the outer cone C is rotated the inner one C_1 tends to follow the same direction by frictional force but is prevented and held in position by the opposing couple generated by the weight M. By properly regulating the rotation of the outer cone the weight M can be kept suspended in a stationary position. In this case the moment of the frictional force between the cones about the axis of rotation is equal to the moment of the pull due to the weight M. The rate of revolution of the cone is given by the speed counter S.

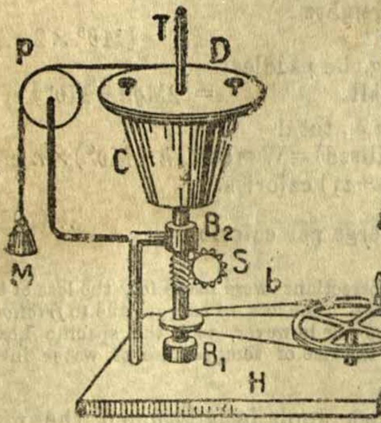


Fig. 48—Friction Cone Method

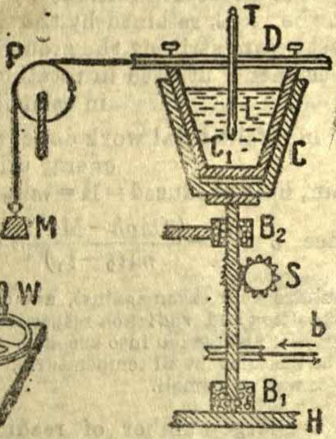


Fig. 85

Let the radius of the wooden $= r$ cm.,
" frictional force between the cones $=$ F dynes
" mean radius of the cones $= a$ cm,
" number of revolutions of the cone per sec. $= n$,
" water equivalent of the cones and their contents $= m$ gm.
and the rise of temperature (corrected for radiation) $= t°$C.
Now the moment of the frictional force about the axis of rotation $= Fa$
and moment of the pull due to M about the axis of rotation $= Mgr$.

Since for steady position of load the moments are equal $Fa = Mgr$.
Now work done in n revolutions $= 2\pi naF$ ergs $= 2\pi n\,Mgr$ ergs.
Also heat developed by the rotations $= mt$ calories.

Hence the mechanical equivalent $J = \dfrac{2\pi nMgr}{mt}$ ergs per cal...(120.4)

(4) Electrical Method—Description of this method is given in the part of VOLTAIC ELECTRICITY.

121. The values of J in different Units—The most accurate value of J is 778 ft. lbs. per B. Th. U. of heat. This means that to produce one pound Fahrenheit degree of heat 778-foot-pounds of work have to be done. Conversion into C. G. S. unit is done the following manner.

We know 1 ft. lb. $= 30{\cdot}48 \times 453{\cdot}6 \times 981$ ergs.

and 1 B. Th. U. $=$ 1 lb. degree Fah. $= 453{\cdot}9 \times \frac{5}{9}$ calories

Hence
$$J = \frac{778 \text{ ft. lbs.}}{1 \text{ lb. deg. F.}}$$
$$= \frac{778 \times 30{\cdot}48 \times 453{\cdot}6 \times 981 \times 9 \text{ ergs}}{453{\cdot}6 \times 5 \text{ calories}}$$

$=4{\cdot}186\times10^7$ ergs per calorie
$=4{\cdot}186$ Joule's per caloric

This value of J is approximately taken to be $4{\cdot}2\times10^7$ ergs per calorie.

Examples :

1. With what velocity must a lead bullet at 50°C strike against an obstacle in order that the heat produced by the arrest of the motion might be just sufficient to melt it, provided all the heat produced within the bullet is utilised. (Sp. ht. of lead = ·031 , melting point of lead = 335°C ; latent heat of fusion of lead = 5·37 ; $J=4{\cdot}18\times p0^7$ ergs per calories.)

Ans. Suppose the mass of the bullet is m gm. the required velocity is v cm. per sec.

Then the K. E. $=\frac{1}{2}mv^2$ ergs.

This energy is converted into heat which raises the temperature of the bullet from 50°C to 335°C and then melts it.

∴ the heat produced $=[m\times0{\cdot}031\times(335-50)+m\times5{\cdot}37]$ cal.
$=m\times14{\cdot}205$ cal.

As W=JH, we must have $\frac{1}{2}mv^2=4{\cdot}18\times10^7\times m\times14{\cdot}205$ ergs.,

whence $v=\sqrt{2\times4{\cdot}18\times14{\cdot}235\times10^7}=34{\cdot}46\times10^3$. cm. per sec.

2. A piece of steel of mass 1200 gm. is struck by a hammer weighing 960 kgm. dropped from a height of 1·6 metres. Assuming 75% of the heat generated to be retained by the steel piece whose specific heat is 0·12, calculate its rise in temperature, [Mad. U.—1960]

Ans. Mass of hammer = 960 kgm. = 96×10^4 gm. ; height it falls through = 1·6 metres = 160 cm. Assuming $g=980$ C. G. S., the P. E. of the hammer = $96\times10^4\times980\times160$ ergs. Just before striking the piece of steel this amount is to be wholly converted into K. E.

Now heat produced = W/J. Taking, as usual, $J=4{\cdot}2\times10^7$ ergs/calorie, we find

$$H=\frac{96\times10^4\times980\times160}{4{\cdot}2\times10^7}=3584 \text{ calories.}$$

Of this amount of heat only 75% is retained by the steel piece. Thus the quantity of heat retained by the steel piece is $(\frac{3}{4}\times3584)$ cal., = 2688 cals. If the rise of temperature of the steel piece be t°C, then,

$$t^\circ=\frac{2688}{1200\times0{\cdot}12}=18{\cdot}6^\circ$$

3. If the mechanical equivalent of heat be $4{\cdot}2\times10^7$ ergs/cal. from what height must a mass of water be dropped in order to raise its temperature by 0·1°C, assuming that all the kinetic energy gained by the mass of water is used up in raising the temperature ? ($g=980$ cm./sec.^{2}) [Pat. U.—1952]

Ans. Let the height be h cm. Since the K. E. of fall is equall to the P. E. at the neight.

$\frac{1}{2}mv^2=mgh$, where m = mass of water (in gm.).

Now W=JH, or, $mgh=JH=J\times m\times0{\cdot}1$,
or, $980\times h=1{\cdot}2\times10^7\times0{\cdot}1$, whence $h=4286$.

Thus the height asked for is 4286 cm. (nearly).

4. A copper calorimeter weighing 120 gm. (of sp. heat 0·1 and containing 3901 gm. of oil (of sp. heat 0·5) is stirred by a rotating paddle. If the couple driving the paddle is 10^8 dynes cm. and if the rise of temperature is 8°C after 1000 revolutions, calculate J. [And. U.—1952]

Ans. The work done by a couple = moment of couple × angle turned through. Therefore in turning through 1000 revolutions the amount of work done by 10^8 dynes cm. is $=(10^8\times2\pi\times1000)$ ergs $=2\pi\times10^{11}$ ergs = W, say.

Now heat absorbed by the calorimeter and contents $=\{(120\times 0{\cdot}1+3901\times 0{\cdot}5)\times 8\}$ cal. $=15700$ cal. $=$ H, say.

Then $J=\frac{W}{H}=\frac{2\times 3{\cdot}14\times 10^{11}}{15700}$ ergs/cal. $=4{\cdot}0\times 10^7$ ergs per calorie.

5. The radius of the top disc of a friction cone apparatus is 20 cm. and a weight of 100 gms, remains suspended, when the speed of rotation is 300 revolutions per minute. Calculate (*i*) the moment of couple due to the frictional forces about the central axis, and (*ii*) the rate of development of energy in watts. ($J=4{\cdot}2$ joules per calorie ; $g=980$ C. G. S.) [R. P. B.—1951]

Ans. Moment of couple about the central axis $=Mgr=(100\times 980\times 20)$ dyne. cm. $=196\times 10^4$ dyne-cm. Also 300 rev. minute $=$ 5 rev. second.

So work done per second $=2\pi nMgr=2\times 3{\cdot}14\times 5\times 100\times 980\times 20$ ergs.

Hence the rate of energy $=\frac{2\times 3{\cdot}14\times 5\times 100\times 980\times 20}{10^7}$ joules per sec. $=6{\cdot}154$ joules per sec. $=6{\cdot}154$ watts.

*122. Difference of two Specific Heats of a Gas

If a quantity of a gas is heated at a constant volume, no external work is done by the gas and so the heat supplied is wholly utilised in raising the temperture of the gas, while the pressure increases. But if the volume is allowed to expand so as to keep the pressure of the gas constant, an additional amount of heat is necessary to supply the energy equivalent to the external work done. Hence, the specific heat of a gas at constant pressure C_p *exceeds* the specific heat at constant volume C_v by the amount of heat equivalent to the external work done, when one gramme of the gas is heated through 1°C at constant pressure.

Consider one gramme of a gas at pressure P dynes per sq. cm. in a cylinder fitted with a piston of cross-sectional area A sq. cm. Then the force on the piston is P A dynes.

Now heat the gas at constant pressure through 1°C and move the piston outwards through a distance *d* cm.

The external work done during expansion $=PAd$ ergs.

Now for a rise of 1°C the increase in the volume $=Ad$.

Let the volume of 1 gm. of the gas at 0°C and under a pressure P be V_0. Then $Ad=V_0/273$.

$\therefore$ for a rise of 1°C the external work done $=PAd=\frac{PV_0}{273}$ ergs

From Charles' Law $PV_0=RT_0$, where T_0 is the absolute temperature corresponding to 0°C, and is, therefore, equal to 273°A, so that

$$\frac{PV_0}{273}=\frac{P_0V_0}{T_0}=R.$$

Hence for a rise of 1°C the external work done $=$ R ergs.

This is equivalent to the difference between the specific heats of a gas. In other words,

$$C_p-C_v=\frac{R}{J} \qquad ...(122.1)$$

Mayer's Method of Determining J—To calculate J for hydrogen Mayer used the above expression before is was experimentally determined by Dr. Joule.

For hydrogen $C_v = 3{\cdot}409$ calories per gm. per°C
and $C_v = 2{\cdot}411$ calories per gm. per°C

Now the volume V_o of 1 gm. of hydrogen at N. T, P. $=11160$ c.c.

Also the normal pressure $P_o = (76\times 13{\cdot}6\times 981)$ dynes per sq. cm.
$= 1{\cdot}014\times 10^6$ dynes per sq. cm.

Hence for 1 gm. of hydrogen, $R = \dfrac{1{\cdot}014\times 10^6\times 11160}{273}$ ergs per °C
$= 4{\cdot}145\times 10^7$ ergs per°C,

Then from equation. (122,1) we get $= \dfrac{4{\cdot}145\times 10^7}{J} = 3{\cdot}406 - 2{\cdot}411$

Whence $J = 4{\cdot}152\times 10^7$ ergs per calorie.

Examples :

1. How much work is done against atmospheric pressure, when 1 gramme of water at 0°C is converted into steam ? Express your result in calories.

Ans. The atmospheric pressure, at which water is converted into steam at 100°C, is a pressure of 76 cm. mercury.

$\therefore$ the pressure $=(76\times 13{\cdot}6\times 981)$ dynes per sq. cm.

When 1 gm. of water is changed into steam at 100°C, the increase in volume is 1672 c.c.

Hence the required work done $=(76\times 13{\cdot}6\times 981\times 1672)$ ergs.

$$= \frac{76\times 13{\cdot}6\times 981\times 1672}{4{\cdot}2\times 10^7}\text{ cal.} = 40{\cdot}37 \text{ cal.}$$

2. The specific heat at constant volume of air is 0·1684 and at constant pressure it is 0·2375. Calculate the value of J. (Density of air at N. T. P= ·001293 gm. per c. c.)

Ans. The normal atmospheric pressure $P_o = (76\times 13{\cdot}6\times 981)$ dynes per sq. cm.

Now the volume V_o of 1 gm. of air at 0°C is 773·3 c.c.

Thus for 1 gm. of air, $R = \dfrac{P_o V_o}{273} = \dfrac{76\times 13{\cdot}6\times 981\times 773{\cdot}3}{273}$

Hence we have $\dfrac{76\times 13{\cdot}6\times 981\times 773{\cdot}3}{273\times J} = 0{\cdot}2375 - 0{\cdot}1684$, by eqn. (122,p)

whence $J = 4{\cdot}16\times 10^7$ ergs/calorie.

EXERCISES ON CHAPTER IX

Reference

1. Describe an experiment to establish the connection between heat and work and deduce from them the idea of mechanical equivalent of heat. (R. P. B.—1962 ; Pat. U.—1970 ; Dac. U.—1971 ; Gau. U.—1955) Art. 117

2. State the First Law of Thermodynamics. Explain what is meant by mechanical equivalent of heat. (Gau. U.—1961, '73 ; Bom. U.—1972) Art. 119

3. From what height must a block of ice drop in order that it may be melted completely ? It is assumed that 50% of the energy of fall is retained by ice. (Assume $g=980$; $L=80$ and $J=4{\cdot}2$ joules/calorie.) Art. 120

Ans. $6{\cdot}86\times 10^6$ cm.

Reference

Art. 120 4. How long will it take an electrical heating rod of 420 watts to heat 100 c.c. of water by 10°C, if no heat is lost? ($J=4{\cdot}2\times10^7$ ergs/calorie). (B. H. U.—1968)

Ans. 10 sec.

Art. 120 5. Calculate the velocity of a lead bullet whose temperature on striking an unyielding target rises by 200°C, assuming that the whole of the heat generated by the impact remains in the lead. (Specific heat of lead is 0·03.) (C. U.—1947, '61)

Ans. $22{\cdot}45\times10^3$ cm. per sec.

Arts. 119 & 120 6. What is meant by Mechanical Equivalent of heat? How can it be determined experimentally?
(V. U.—1954, '59; C. U.—1959; Utk. U.—1963;
Mys. U.—1962; Gau. U.—1953; Dac. U.—1962;
Del. H. S.—1951; Nag. U.—1954; Del. U.—1952, '61;
R. P. B.—1946, '49; Pat. U.—1944, '52)

Art. p20 7. A meteorite weighing 200 kgm. falls on the moon with a velocity of 100 metres per sec. Assuming that all its energy is converted into heat, work out how many calories of heat will be produced ($J=4{\cdot}19\times10^7$)

Ans. $2{\cdot}4\times10^8$ cal.

8. A lead ball, dropped from an aeroplane at a temperature of 15°C just melts on striking the ground. Supposing the whole of its kinetic energy is converted into heat, find out the height of the aeroplane at the moment the ball was dropped. (Sp. ht. of lead = ·03; melting point of lead = 350°C; latent heat of lead = 35 cal./gm.) (Gau. U.—1961)

Ans. 19287 m.

Art. 120 9. One kilogram of coal generates $1{\cdot}7\times10^4$ calories of heat during combustion. Assuming that only $\frac{1}{10}$ of the energy so generated is utilised for mechanical purposes, find the mass of coal required per hour for running a 10 horse-power engine. (1 h. p. = 746×10^7 ergs per second; $J=4{\cdot}2\times10^7$ ergs/calorie.)

Ans. 18806 kgm./hour.

Arts. 119 & 120 10. What is meant by 'mechanical equivalent of heat'? In an experiment to determine if 800 gm. of lead shot were placed in a long card-board tube held vertically, the length of which was such that on reversing the tube the shot fell through 100 cm. The original temperature of the shot was 25°C and after 50 inversions of the tube the temperature was found to have risen to 28·84°C. Find J in ergs per calorie. (Sp. ht. of lead = ·031, $g.=980$.) (C. U.—1950)

Ans. $4{\cdot}12\times10^7$ ergs/cal.

Art. 120 11. Mercury falls from a height of 50 metres in vacuo upon a perfectly non-conducting surface. What will be the rise in temperature after the fall, if the specific heat of mercury be ·02 and $J=4{\cdot}2\times10^7$ ergs per calorie? (R. P. B.—1956)

Ans. 5·84°C

Art. p20 12. A cannon ball of mass 100 kgm. is projected with a velocity of 400 metres per sec. Find the amount of heat that would be produced, if the ball was suddenly stopped. ($J=4{\cdot}2\times10^7$ ergs per calorie.) (And. U.—1971)

Ans. $1{\cdot}905\times10^6$ calories.

Reference

Art. 120

13. A mass weighing 200 grammes falls from a height of 300 cm. If all the energy is converted into heat, find the amount of heat developed. (Mechanical equivalent of heat $=4{\cdot}2\times10^7$ C. G. S.) (Del. H. S.—1961 ; C. U.—1960 ;

Ans. 14 calories.

Art. 120

14. A block of metal slides down a rough inclined plane, from rest at a point 10 metres above the ground. Just before reaching the ground the velocity of the block is 10 metres per second. If half of the heat produced by friction remains in the block, by what amount has its temperature been raised while sliding down ? (Sp. heat of the metal = 0·1)

Ans. 0·057°C.

Art. 120

15. A copper calorimeter weighing 200 gm. contains 280 gm. of water. A mechanical stirrer is able to raise the temperature of water at the rate of 0·3°C per minute. Calculate the energy in watts used by stirrer. (Sp. ht. of copper = 0·1 and J = 4·2 joules per calorie.) (Mad. U.—1970)

Ans. 6·3 watts.

Art. 120

16. A bullet is suddenly stopped and all its energy is converted into heat causing a rise of temperature of the bullet by 100°C. If the specific heat of the bullet is 0·21, calculate the velocity with which the bullet was moving. $(J=4{\cdot}2\times10^7$ erg./cal.) (Raj. U.—1963)

Ans. 42×10^3 cm./sec.

Art. 120

17. A lead bullet strikes a target with a velocity of 200 metres per sec. Calculate its rise in temperature, assuming that all heat developed remains in the bullet. (Sp. ht of lead = ·03.) (P. U.—1962)

Ans. 158·7°C.

Art. 120

18. A lead bullet strikes against a steel armour plate with a velocity of 480 metres per sec. If the bullet falls dead after the impact, find the rise in temperature, it being presumed that the heat produced is equally shared between it and the target. (Sp. ht. of lead = ·03 ; $J=4{\cdot}2\times10^7$ C. G. S.)

Ans. 457·1°C.

Art. 120

19. A lead bullet travelling with a speed of 200 metres per second is brought to rest after hitting a target. Assuming that 80% of the kinetic energy is converted into heat in the bullet itself, find the rise in temperature of the bullet after impact. (Sp. ht. of lead = ·03 ; $J=4{\cdot}2\times10^7$ ergs/cal.) (Nag. U.—1962)

Ans. 126·9°C.

Arts 120 & 121

20. Describe a method for the determination of the mechanical equivalent of heat and mention clearly the unit in which it is measured. (U.P.B.—1963 ; P.U.—1966 ; Nag. U.—1970 ; Raj. U.—1955 · B. H. U.—1963 ; C. U.—1959, '61)

Art. 120

21. Describe how the mechanical equivalent of heat is determined by the frictional cone method.
(Raj. U.—1963 R.P.B.—1961 ; U.P.B.—1961 ; Mad. U.—1972)

Art. 120

22. A copper calorimeter of mass 122 gm. contain 1680 gm. of aniline oil which is stirred by rotating paddle requiring a couple of 10^3 dynes cm. to drive it. The temperature of

Reference

the liquid is raised 0°C after 450 revolutions. Calculate the value of J. (Sp. ht. of aniline $= 0·5$ and that of copper $= 0·095$.)

Ans. $4·15 \times 10^7$ erg/calorie.

Art. 120 23. Describe Joule's method for determining the mechanical equivalent of heat. (Nag. U.—1961, '73 ; C.U.—1954 ; Del. U.—1963 ; Bom.U.—1973 ; Gau.U.—1961,'74 ; Poona U.—1963 ; Del. H. S.—1954)

Arts. 119 & 120 24. What is meant by mechanical equivalent of heat? Describe Searle's method of determining the same, precautions to be taken being mentioned. (B. H. U.—1970)

Art. 120 25. How much work must be done for supplying heat necessary to convert 10 gm. of ice at -5°C into steam at 100°C? (Sp. ht. of ice 0·5 and mechanical equivalent of heat $= 4·2 \times 10^7$.) (R. P. B.—1962)

Ans. 30219×10^7 ergs.

Art. 122 26. Explain how the difference of specific heats of a gas enable you to evaluate the mechanical equivalent of heat. (R. P. B.—1969 ; U. P. B.—1972)

Art. 122 27. Prove that $C_p - C_v = \frac{R}{J}$.

(M.P.B.—1962 ; U,P,B.—1965 R. P. B.—1968)

Art. 122 28. Calculate the value of the mechanical equivalent of heat from the following data for air :—

specific heat at constant pressure $= 0·2375$ cal. per gm. ;
specific heat at constant volume $= 0·1690$ cal. per gm. ;
atmospheric pressure in dynes per sq. cm. $= 1·013 \times 10^6$
and volume of 1 gm. of air at N. T. P. $= \frac{1}{0·00129}$ c.c.

Ans. $4·19 \times 10^7$ ergs per calorie. (U.P. B.—1959, '65)

Art. 122 29. Specific heat of argon at constant pressure is 0·125 calorie per gm., and at constant volume 0·075 calorie per gm. Calculate the density of argon at N.T.P. ($J = 4·18 \times 10^7$) ergs/cal, normal pressure $= 1·0p \times 10^5$ dynes/sq. cm. (R. P. B.—1969)

Ans. $1·8 \times 10^{-3}$ gm./c.c.

CHAPTER X

HEAT ENGINES

123. Heat Engines—These are machines, by means of which heat energy is transformed into mechanical work. There are two principal types of engines, *viz.*. (*i*) External Combustion Engine, and (*ii*) Internal Combustion Engine. In the former type, water is converted into steam in a chamber *outside* the working portion of the engine, while in the latter the fuel used to generate heat is either a gas or a liquid and the combustion of the fuel takes place *inside* the working portion of the engine.

Conversion of Linear Motion into Rotatory Motion—If from a machine we can obtain a *continuous motion*, either linear or rotatory, it can be converted into any desirable type of motion. Almost in every class of engines, the primary object is to secure a rotatory motion of a massive wheel, called the fly-wheel. From this fly-weel every other motion is derived by direct or indirect coupling processes.

Consider a heavy wheel W (Fig. 87) rigidly attached to a shaft and capable of rotation. One end of a stout metal rod R, called the *crank rod*, is hinged at one spoke of the wheel and its other end is also hinged at the end of B to another rod capable of moving *to-and-fro* within a cylinder. The hinged combination of the crank rod R and the piston rod at B is known as a *stuffing box*. This second rod is provided with an air-tight piston P moving within the cylinder C. If now some force acting on the piston causes it to move *to-and-fro* within the cylinder, the link rod would rotate the fly-wheel continuously. All classes of engines are provided with such appliances.

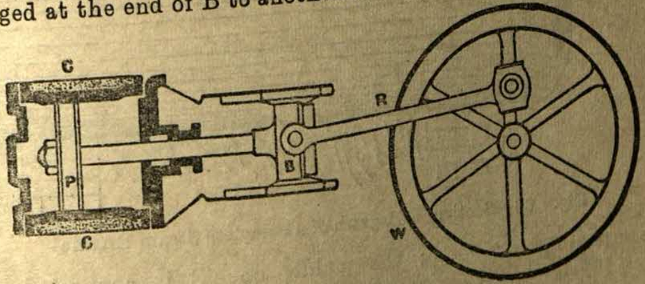


Fig. 87—Piston rod and Fly-wheel

124. Steam Engine—The principle of the steam engine was invented by James Watt in 1758. Although many important improvements have since been introduced by Trevithick, Stephenson and others, the fundamental features remain the same.

The essential *parts* of a steam engine (Fig. 88) are :-

(1) *Boiler* (L)—Here steam is generated by boiling water in a strong metal vessel over the furnace F in which coal is burnt. The steam is led away by a metal tube into a chamber S called the *steam chest*. With the tube there is a regulating key M, by means of which the supply of steam into the chest can be controlled.

(2) *Steam chest or Valve chest*—This is a stout box S, into which steam is injected from the boiler through the pipe. It has three openings or *ports*; the middle one E is connected to the *exhaust pipe*, while the other two A and B lead into the cylinder C.

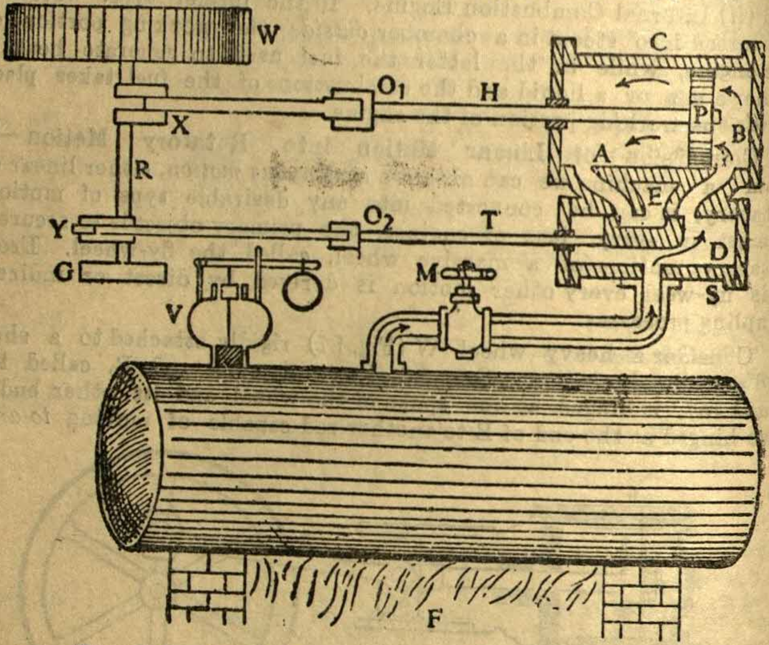


Fig. 88—Sectional diagram of a complete Steam Engine

(3) *Slide Valve* (or the *D-valve*) - This is D-shaped valve moving within the steam chest and regulating the supply of steam through the ports. The slide valve is connected to the rod T through a stuffing box O_2. The rod is finally connected to the *eccentric disc* X of the main shaft R.

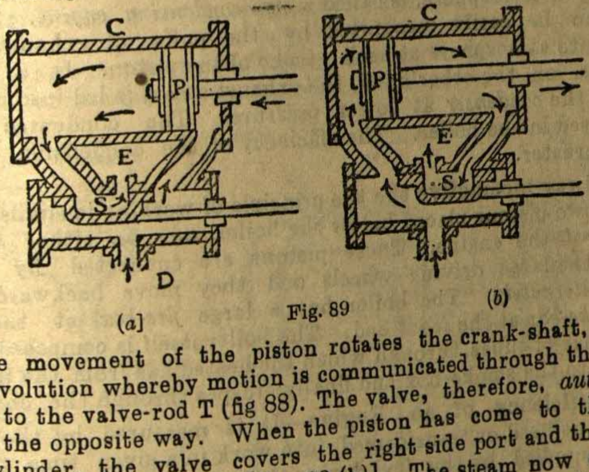
(4) *Cylinder* C—It is a strong cylindrical vessel communicating with the steam chest through the two tubes. A steam-tight piston P of cast steel just fits and works within the cylinder. The *piston* works through a *stuffing box* O_1 on the *driving rod* A which is hinged at X to the *main shaft* R.

(5) *Safety valve* V—To protect against a prohibitive *rise* of steam

pressure within boiler the safety valve is a mechanism for the safety of the boiler. If the steam pressure goes above the safe limit, the valve automatically opens and a part of the steam escapes, the pressure being thereby lowered.

(6) *Fly wheel* W—It is a large wheel connected to the main shaft.

Action—From the boiler superheated steam at a very high pressure (20 atmospheres or even more) is led through the pipe into the steam chest and thence into the cylinder C through any one of the ports say, the right side one [Fig. 89 (*a*)]. When the steam enters the cylinder, the slide valve covers the exhaust port and the left side port also, so that these two are put into communication. The pressure of the steam within the chest pushes the piston P within the cylinder along the left side.



(*a*) Fig. 89 (*b*)

The movement of the piston rotates the crank-shaft, through a half revolution whereby motion is communicated through the eccentric disc V to the valve-rod T (fig 88). The valve, therefore, *automatically* moves the opposite way. When the piston has come to the end of the cylinder, the valve covers the right side port and the exhaust, but opens the left side port [Fig. 89 (b)]. The steam now enters the cylinder through the other passage and pushes the piston in the opposite way, until it almost comes to the other end ; at the same time the exhaust steam goes out through the exhaust pipe. This operation is repeated succesively ; the slide valve opens and closes one of the ports alternately just at the proper moment, so that the piston moves *to-and-fro* within the cylinder. The *to-and-fro* motion of this piston is converted into a rotatory motion of the main shaft through the crank. Twice during each revolution (*i.e.*, at the top and the bottom of each stroke) the crank and the connecting rod lie along the same line, when the piston momentarily comes to rest and has no turning effect on the shaft : these are called the '*idle centres*' or 'dead points'. Again, at two points during each revolution the crank is perpendicular to the connecting rod, when the turning effect on the shaft becomes maximum. The shaft carries a large and heavy *fly-wheel* W which due to its large moment of inertia smooths down any variation of motion of the crank shaft.

From the boiler the supply of steam is regulated into the cylinder by a valve known as the *throttle valve*, which is worked by a piece of self-acting machinery known as the *governor*. If the engine works too fast, the balls of the governor move upwards and pull down the collar and in doing so operate the throttle valve so as to reduce the supply of steam. The motion of the shaft may to communicated to any machinery by means of a belt passing over a pulley attached to the main shaft. The type of engine described here is called a *double-acting engine*, for the steam acts alternatly on the two faces of the piston and thus produce a *to-and-fro* motion.

Condensing and Non-Condensing Engines—The type of engine, for which the waste steam escapes through the exhaust-pipe into the open atmosphere, is called a *non-condensing engine*. Such an engine can be easily recognised by the puffs of exhaust steam escaping into atmosphere at each stroke of the piston. In a *condensing engine*, on the other hand, the exhaust steam is led into a chamber called the *condenser* at lower pressure. The condensed steam in again used in the boiler. The efficiency of the condensing type of engine is greater.

In a railway **locomotive** the the principle of working is similsr; but there are two pistons placed below the boiler, one on either side of the body of the engine. These pistons are connected by crank shafts to the large driving wheels and they move backwards and forwards *alternately*. The boiler has a large *fire-box* at the rear and a *smoke-box* at the front end. The boiler itself is composed of a large number of tubes, through which the heated gas from the furnace goes out into the smoke-box and finally escapes through the chimney.

125. Efficiency of an Engine—It is measured by the ratio of the heat utilised or transformed into work in a given time to the total heat supplied by the fuel during the same time. This is more often called *thermal efficiency*. If Q be the heat produced by the fuel consumed per unit of time and W be the power developed (*i*, *e*. the amount of mechanical work done per second, the thermal efficiency γ is given by

$$\gamma = \frac{W}{JQ} \qquad \text{...(125.1)}$$

Thus γ gives the fraction of the total heat converted into work and is always less than unity, The efficiency of the best type of steam engine is not more than 17%. The output energy is measured in terms of **horse-power (H. P.)**. An engine is said to have one horse-power when it performs 550 ft. lbs. of work per second or 33,000 ft.-lbs. of work per minute. Heat that is taken in. may be measured by the quantity of steam used in a given interval and by the initial and final temperatures and pressures of the steam.

The power developed by steam pressure inside the cylinder of the engine is known as *indicated horse-power*, which depends upon the mean effective pressure of the steam on the piston during a stroke, the cross-sectional area of the piston, the length of the stroke and the number of strokes per minute. This power cannot be fully utilised by the fly-wheel, as by way of friction, weight of moving parts, etc., a certain part of this power is used up in the driving the engine itself. Thus the available power known as the *brake horse-power*, is always less than the indicated horse-power. The ratio of the latter to the former is called the *Mechanical Efficiency* Thus

$$\text{Mechanical Efficiency} = \left(100 \times \frac{\text{brake horse-power}}{\text{indicated horse power}}\right)\%.$$

126, Internal Combustion Engines—In this type. the combustion of the fuel is carried out *inside* the engine cylinder. These are, therefore termed *internal combustion engines*. According to the nature of the fuel used, an internal combustion engine may be (1) a petrol engine or (2) an oil engine or a gas engine.

Petrol engine—This is a type of *internal combustion engine*, in which combustible liquids (such as petrol, benzene, etc.) are used as fuel. It is commonly employed in motor cars and air-ships.

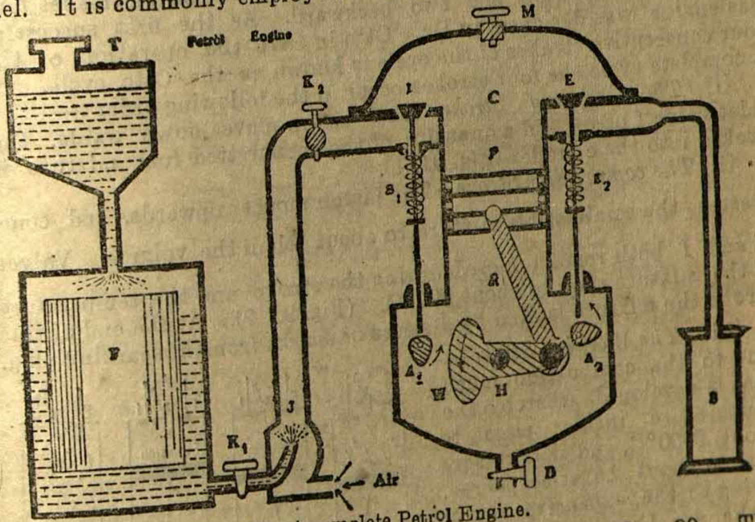


Fig. 90—A complete Petrol Engine.

A complete picture of a petrol engine is shown in Fig, 90. Th petrol is stored in the *petrol tank* T, whence it passes into a chambe containing float F. Due to pressure of the float and an automati compressing pump (not shown in the diagram) the petrol is forced int the *curburettor*. The quantity of fuel admitted into the curburetto may be adjusted with a key K_1. Within the curburettor the petrol i vaporised and is mixed with a proper proportion of air. A regulate supply of the mixture, which forms the fuel passes through an adjus able valve called the *throttle valve* K_2 into the *combustion chamb* placed just above the cylinder C. In this chamber the ignition caused by electric sparks from a *plug* M, the electric current bei supplied by a small dynamo called the generator (not shown in the diagram) which is driven by the engine itself.

Inside the cylinder C an air-tight piston P works, The cylinder is provided with two inlet and exhaust *valves* I and E with projecting rods and operted respectively by the rotating *cams* A_1 and A_2, which are driven by the main shaft. Each cam has a protruding end and is designed to rotate at half the speed of the main shaft H. In course of rotation of a cam when the end points upward, it pushes the valve upwards and thereby the corresponding valve opens. For other positions of the cams the valves are kept pressed on the holes by powerful springs S_1 and S_2. Hence the opening and the closing of the valves are controlled by the motion of the piston. In order to prevent the cylinder from becoming too hot during the operation, it is surrounded by water-jacket, through which there is a continuous circulation of water (not shown in the diagram).

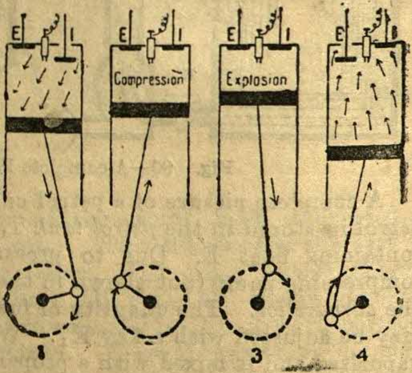
The engine is called a **four-stroke engine**, since the supply of the fuel is made only *once* during the time the piston makes *four* motions—two forward and two backward. As the first successful gas engine was designed by Dr. Otto in 1876. the operation of the four consecutive strokes of the cycle is known as the Otto cycle. In a complete cycle the four strokes occur in the following order :—

(i) *The Charging Stroke*—The piston moves down wards, the inlet valve I opens and a quantity of the curburetted fuel mixture is sucked into the cylinder (Fig. 91, 1).

(ii) *The compression Stroke*—The piston moves upwards, and compresses the combustible mixture to about $\frac{1}{5}$th of the volume. Valves E and I both remain closed during the stroke and the temperature of the mixture rises to about 600°C. (Fig. 91, 2). At the end of this stroke the mixture is fired by a series of sparks from the sparking plug.

(iii) *The Working Stroke*—Due to the combustion much heat is developed. whereby the temperature inside rises to about 2000°C, and the pressure rises to about 15 atmospheres. Owing to the expansive action of the gas the piston is driven outwards with a great force, both the valves still remaining closed (Fig 91. 3). This is the only stroke during which the engine does work.

As the explosive mixture requires a little time for full combustion, the maximum force of the explosion develops a little later than the sparking of the mixture.



Fsp. 91—Four Cycles of Operation

(iv) *The Exhaust Stroke*—The piston moves upwards. The exhaust valve E then opens and the spent products of combustion are forced out, the inlet valve I remaining closed (Fig. 91.4).

After the exhaust is complete, the initial condition is attained and a fresh cycle begins anew. As the engine receives energy only during the working stroke, the *fly-wheel must be heavy*, so that the inertia gained by it during this stroke will maintain the speed without any appreciable loss during the next three strokes. In Fig. 90. the fly-wheel is replaced by a heavy counter-weight W attached to the opposite end of the shaft.

In a motor car or aeroplane a petrol engine is provided with a number of cylinders, the number being a multiple of two (generally, four, six eight or even more). The pistons of all the cylinders work on the same shaft and thereby increase the working capacity for the horse-power of the engine. The arrangement of the different parts in a motor car engine is exhibited in Fig. 92, wherein A and B represent the two cylinders, M and N the piston rods, KSLQ the main shaft, W the fly-wheel and R the radiator of water circulation system. Since such a type of the engine requires to be started by external agency. H is the handle, by the rotation whereof the engine is started. There is also an electrical arrangement to give the first start of the engine ; this is called the *self-starting swtich*.

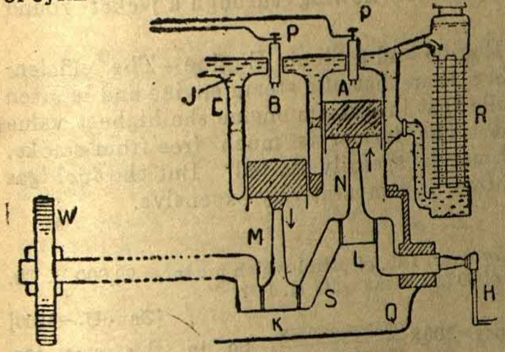


Fig. 92 – View of Motor Car Engine

127. Oil Engine—It is a type of *internal combustion engine* using as fuel an explosive mixture of air and some oil. The oil is admitted in the form of spray into the *vaporiser* which is a red-hot metal tube. Into the vaporiser air also is admitted. The oil evaporates, mixes with the air and explodes at suitable intervals. In this case four-stroke cycle also works. In a special type of machine, called the **Diesel Oil Engine** after the name of inventor, the cycle undergoes the following modification :—

During the *first stroke* a requisite quantity of air is sucked within the cylinder. During the *second stroke* the air is very strongly compressed, so that great heat is developed within the cylinder and the temperature rises to about 1500°C. At the begining of the *third stroke* oil spray is injected at a very high pressure into the cylinder containing the highly compressed and intensely heated air. The mixture catches fire and burns non-explosively, maintaining practically a constant pressure. During the *fourth* and the *last stroke*, as usual, the burnt gas is forced out of the cylinder.

The *efficiency* of the engine is very high. being about 70 per cent.

Adapted to a wide range of fuels from kerosene to tar spray, it is very economical. In submarines, in many merchant vessels and for various purposes on land oil engines are used.

Gas Engine—It works similarly as a petrol engine and employs as fuel a mixture of about 1 part (by volume) of coal gas and 8 parts of air. The same four-stroke cycle works and the valves are operated in the same way by cams and spring which regulate the inlet of the mixture and the exhaust. The firing of the gaseous mixture is effected within the cylinder by contact with the walls of a red-hot porcelain tube or by electric sparks occurring at properly timed intervals. The engine is also cooled by water flowing through a jacket round the cylinder.

Comparison of a Gas Engine and a Steam Engine—The efficiency of a gas engine is higher than that of a steam engine and is often as high as 70 per cent, while that in a steam engine the highest value is about 17 to 20 per cent. The former is much free from smoke, occupies smaller space and may be speedily started. But the fuel (gas or gasoline) used in the former is comparatively expensive.

Examples :

1. A petrol engine uses every hour 1 lb. of petrol which produces 22,000 B. Th. U. of heat and has an efficiency of 60%. What is its H. P. ? [Gau- U.—1963]

Ans. We know that $\text{efficiency} = 100 \times \dfrac{\text{output energy}}{\text{input energy}}$. So in this case the output energy $= \dfrac{60}{100} \times 22{,}000$ B. Th. U. per hour $= 13200$ B. Th. U. per hour $= 13200 \times 778$ ft. lbs. of work per hour.

Now 13200×778 ft.-lbs. per hour $= \dfrac{13200 \times 778}{60 \times 60}$ ft,-lbs. per sec.

Since 1 H. P. = 550 ft. lbs. per sec. the output H. P. of the engine

$$= \frac{13200 \times 778}{60 \times 60 \times 550} = 5{\cdot}18.$$

2. In drawing a train of weight 240 tons over a distance of 1 mile, the resisting force being equal to 35 lbs. per ton, an engine consumes 80 lb. of coal. The calorific value of a particular sample of coal is such that 1 lb. is capable of converting 15 lb. of water at 100°C into steam at the same temperature. Find the efficiency of the engine. (Latent Heat of steam = 536 ; J = 1400 ft.-lb. degree centigrade.)

Ans. The total resisting force = (35×240) lb.

∴ in moving the train through 1 mile the work done $= (35 \times 240 \times 1760 \times 3)$ ft.-lbs. Also heat Q developed by the consumption of coal $= (15 \times 536 \times 80)$ calories.

Hence the efficiency $\gamma = \dfrac{W}{JQ} = \dfrac{35 \times 240 \times 1760 \times 3}{1400 \times 15 \times 536 \times 80} = 0{\cdot}049$, *i, e*, 4·9%.

3. A motor car uses petrol whose calorific value is 11×10^4 B. Th. U. per gallon. Running at 24 miles an hour during which the average output is 10 H. P., the car covers on an average 20 miles to the gallon. Find the overall efficiency of the car.

Ans. Work done $= (10 \times 550)$ ft.-lbs. per sec.
Petrol consumed per hour $= (24 \div 20)$ gallons $= 1\frac{1}{5}$ gallons.

$$\therefore \text{ heat energy given per sec.} = \left(\frac{6}{5} \times \frac{1}{3600} \times 11 \times 10^4\right) \text{ B. Th. U.}$$

$$\therefore \text{ mechanical energy given per sec.} = \left(\frac{6}{5} \times \frac{1}{3600} \times 11 \times 10^4 \times 778\right) \text{ ft.-lbs.}$$

$$\text{Hence the required efficiency} = \frac{\text{work done}}{\text{energy given}}$$

$$= \frac{5500 \times 5 \times 3600}{11 \times 10^4 \times 6 \times 778} = 0{\cdot}193, = 19{\cdot}3\%.$$

EXERCISES ON CHAPTER X

Reference

1. Describe the principle and action of Steam Engine, giving a sectional diagram. (V. U.—1954 ; Del. U.—1962 ; U. P. B.—1960, '65 ; C. U.—1957 ; Del. H. S.—1970 ; E. P. U.—1962 , Pat. U.—1964) Art. 124

2. Describe with a sketch the principle underlying the construction of a Steam Engine and point out the transformations of energy which occur. (Utkal. U.—1957, '59, '72) Art. 124

3. Describe with a near diagram any form of a modern Petrol Engine. How does it act ? (U.P.B.—1964 ; C.U.—1960, '62, '68, ; Gau. U.—1960, '72 ; E. P. U.—1962) Art. 126

4. Describe the working of the internal combustion engine with diagram. (Dac. U.—1962 : U.P.B.—1967 ; B.H.U.—1960) Art. 126

p. A 33 H. P. engine consumes 22100 gm. of coal per hour. What proportion of the coal is wasted ? The calorific value of coal is such that 1 gm, of coal can convert 16 gm. of water at 100°C into steam at 100°C. (1 H. P.$=746\times10^7$ ergs per sec. and $J=4{\cdot}2\times10^7$ ergs per calorie ; latent heat of steam=536.) Art. 124

Ans. 0·9

6. A steam engine burns 100 gm. of coal per minute, the calorific value of the coal being 6000 calories per gm. The engine is used to turn a wheel, around which passes a rope that lifts a weight of 140 kgm. The wheel makes 30 revolutions per min, and its radius is 10 cm. Find the efficiency of the machine. Art. 125

Ans. 5·1%.

7. What is a Heat Engine ? Describe the construction and working of an external combustion engine and derive an expression for its horse-power. (Raj. U.—1963) Arts. 123 & 124

8. What is the essential difference between a steam engine and an oil engine ? (Raj. U.—1965 ; E. P. U.—1970 : C. U.—1958) Art. 127

9. The combustion of 1 pound of coal results in the evolution of 14,000 British Thermal Units. What is the efficiency of a steam engine pumping plant which consumes 20 pounds of coal per hour and raises each minute 20,000 pounds of water to a height of 12 ft. ? Art. 125

Ans. 6·6%.

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(in English) by N. N. Basu & J. Chatterjee
PART—I (General Physics, Heat & Sound)
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(Also for Central Modern, Pre-Eng. Pre-Professional, Pre-Medical and for new courses of studies for Classes XI, XII)

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HIGHER SECONDARY & INTERMEDIATE PRACTICAL PHYSICS (in English) by J. Chatterjee
(Intended for Higher Secondary, Intermediate & XI. XII Classes & Degree pass course.)

[Fifth edition of the Book has just been printed. It covers the Practical syllabi of all Indian Universities in as much as it embodies 180 experiments]

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by J. Chatterjee & H, Biswas (in English)
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কবি কঙ্কণ চণ্ডী—(বি-এ অনার্স ও এম-এর পাঠ্য ।
ডঃ ক্ষুদিরাম দাস সম্পাদিত কলিঃ বিশ্ববিদ্যালয়, রামতনু লাহিড়ী অধ্যাপক

ছোটদের মহাভারত—(তৃতীয় চতুর্থ ও পঞ্চম শ্রেণীর পাঁঠ্য) ফুলবালা রায় ।

SOUND

CHAPTER I

PRODUCTION OF SOUND

1. **Definition**—The term **Sound** is used to signify the sensation received by means of human ears. Also it refers to the external cause which produces such sensations. The branch of Physics, which deals with the production and propagation of sound, is commonly called **Acoustics**.

2. **Production of Sound**—It is a common experience that all material objects, when properly struck, are capable of producing sounds. The most commonly available sounding bodies possess shapes which may be broadly classified into any one of the following types—a string, a reed, a bar, a plate and a diaphragm. But, in all cases, whenever any type of them emits sound, it is in a state of rapid *vibration*. This is evident from the following experiments.

Vibration of Stringed Instrument—Stretch a string tightly between two posts on a table (Fig. 1). Pluck the string at the middle part. A note is heard and at the same string appears to be hazy and drawn out. If a small paper rider be placed on the string while emitting sound, the rider flies away. On touching the string the haziness of the string disappears and the sound ceases. This demonstrates that a string in a state of vibration produces sound. To this class belong all kinds of stringed instruments, *viz.*, violin, harp, sitar, etc.

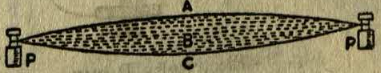

Fig. 1—Vibrating String

Vibration of a Bar—Clamp one end of a metal bar to a rigid support, such as a vice. Draw aside the other end of the bar and then release it. A dull sound is heard and at the same time a blurring of the free end is observed. This shows that the rod vibrates while emitting sound.

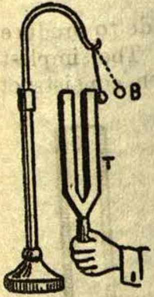

Fig. 2

The most usual source of uniform tone in the laboratory is a tuning fork. A tuning fork is a steel bar bent in the form of U with a stout metal stem at the bend (Fig. 2).

Take a tuning fork and strike it with a padded hammer. It gives out a uniform sound. Note that the projecting ends of the fork appear blurred. This is due to the rapid to-and-fro motion of its projecting ends. Bring a suspended pithball to touch its end. The pithball is found to jump aside as it touches the tuning fork. Now touch the ends of the fork with fingers; the blurring disappears and the sound ceases immediately. This proves that the fork while producing sound is in a state of rapid vibration. The vibration, however, gradually dies away and the sound grows fainter.

In order to produce a continuous sound, a tuning fork may be run by an electro-magnet. A short electro-magnet M is fixed symmetrically within the space between the prongs (Fig. 3). There are gaps between the pole-pieces and the prongs, which are sufficient for allowing free vibrations of the fork. One end of the coil carrying the electro-magnet M is connected to the stem through a battery of cells B, and the other end to an adjustable screw head K. The screw terminates in a platinum point which just touches a light metal spring S attached to a point.

In order to start the action of the fork, the screw is made to touch the spring. when electrical contact is established. The electro-magnet on being excited, attracts the two prongs which simultaneously swing inwards. The electrical contact is thus broken and the electro-magnet is demagnetised. Being released from the pull, the prongs swing back, when the circuit is again completed. In this way a continuous vibration of the tuning fork is maintained so long as the electrical circuit is complete.

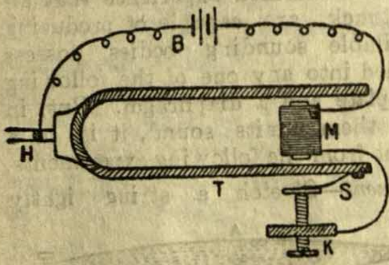

Fig. 3—Electrically Driven Tuning Fork

Vibration of Pipe Instruments—A pipe can also be made to produce a sound. To this class belong *flute*, *flageolet*, *piccolo*, etc. The simplest instrument of this type is an ordinary **organ pipe** which consists of a small tapering tube (Fig 5) connected to another larger hollow pipe of a circular or rectangular cross-section. The other end of the pipe may be open or closed. Within the larger tube there is a prism-shaped partition wall S almost covering all space except a narrow opening, which connects the inside and the outside air. Just near the opening a part of the outer wall of the tube projects and has a bevelled appearance. This is called the *lip* L of the organ pipe.

When air is forced into the tube at the narrow end, it escapes through the orifice and produces sound If a small flat dish containing a little sand be lowered into the pipe, while sound is being emitted, it is found that sand particles begin to dance at some fixed places of the pipe. This shows that air within the pipe vibrates at some definite regions producing sound.

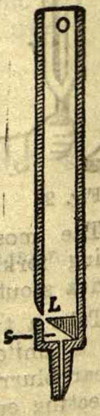

Fig. 4

Vibration of Reed Instruments—To the class of reed instruments belong *clarionet. aboe, harmonium*, etc. A simple reed is shown in Fig. 5. The air is forced into a rectangular air chamber through an opening, at one side of which there in attached a thin metal foil called the *reed*. In course of streaming into the pipe the air periodically presses against the reed which is thus made to vibrate rapidly. A magnified figure of reed, fixed to the air chamber, is shown in Fig. 6. If the reed is touched while sound is being emitted, the sound stops. This shows that while producing sound, the reed is in a state of vibration.

Vibration of a Bell Jar—In addition to these, there are other sounding bodies such as bells, plates and diaphragms To show the vibration of a jar while producing sound, take a glass jar and pour some water into it. Suspend one or two pithballs from a suitable stand so as to touch its rim (Fig. 7). Bow at an edge of the jar when a peculiar note is heard. Water within the jar is also thrown into violent agitation and the pithballs keep on jumping aside. This indicates a rapid vibration of the jar.

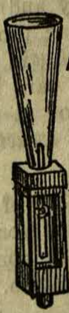

Fig. 5

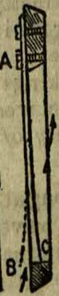

Fig. 6

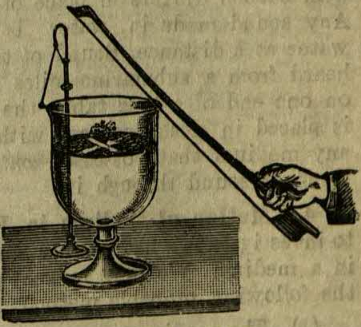

Fig. 7—Viabration of a 1

To produce soun from a gong or a bell, it requires to be struck with a hammer. The process of hammering gives the gong a violent blow. Due to this the gong is thrown into vibration and it emits a note. The vibrational sensation of a sounding bell may be felt by lightly touching it with fingers. Sounding of drums is an example of vibration of diaphragms. Hence we may conclude that while emitting sound, all bodies must be in a state of rapid vibration. Even massive bodies (*e,g*., a table or the walls of a room) may be set into vibration. The branch of Acoustics deals with the mechanical vibrations of various bodies producing sound.

3. Propagation of Sound: Medium necessary—In order that sound may propagate from one place to another, some sort of a material *medium* is always necessary. The following experiment helps to understand the fact.

The Bell Jar Experiment—A electric bell is suspended within a glass jar by two metal rods, which project outside. These are fitted with binding screws. On connecting the terminals of a battery of cells to the electrodes, the bell operates (Fig. 8). The jar can be placed on a round table which acts as the receiver of an air pump. When the bell works within the jar under ordinary conditions, a distinct ringing sound is heard. On gradually evacuating the receiver the sound grows fainter and fainter. When it is almost completely exhausted of air, the sound is almost inaudible.*

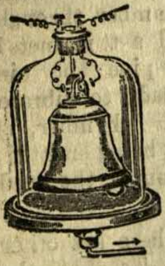

Fig. 8

If now air is slowly re-admitted into the receiver by partially opening the stop-cock which is fitted with the pipe, the sounds becomes gradually stronger, till when the jar is again full of air, the original sound is regained. The air surrounding the electric bell is, therefore the medium, through which sound is conveyed.

If the jar be filled with any gaseous medium or a liquid or a solid, the sound of the bell is heard. This proves that not only air but also any material medium (either a solid or a liquid) can transmit sound more or less. If the above experiment be repeated with the jar filled with carbon dioxide in place of air, similar result will be obtained. Any sound made in water is distinctly audible to a person under water at a distance. Sound of the rotating propeller of a ship can be heard from a submarine miles away. Again, if a watch is placed on one end of a long table, the tickings can be heard when the ear is placed in closed contact with the wood at the other end. In fact, any medium that possesses *continuity of structure* and *elasticity*, can transmit sound through it.

4. Motion of Medium in Propagation of Sound—We are now to investigate in what manner sound travels from one point to another in a medium. We may make a preliminary conjecture by considering the following probable methods of propagation.

(1) The medium moves bodily from one region to another, carrying sound with it, just as a railway compartment carries passengers from one place to another.

(2) Or, the medium as a whole is stationary, but sound is transferred from point to point within the medium due to some motion within it, somewhat like the motion of wave over the surface of water. Here the mass of water as a whole does not move; but the motion is conveyed by the up-and-down motion of the waves. The following experiment supports the latter view.

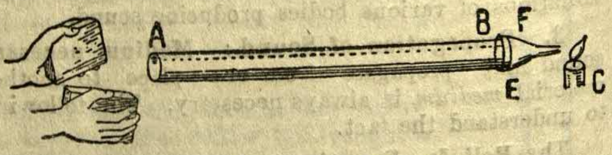

Fig 9—Propagation of Sound

AB is a hollow pipe (Fig. 9), about 6 feet long and having a diameter of 2 to 4 inches. One of its ends is provided with a conical mouth-piece with a gradually tapering end F. It is held horizontally and some smoke is introduced into it through the wider end A. A lighted candle C is placed in such a way that its flame is just against F. On smartly clapping two blocks of wood near the end A, a sound is heard and the flame is found to be ducked immediately, but the smoke is not forced out. This indicates that a momentary disturbance passes through air without, however, causing it to be pushed out. Such a motion of the disturbance constitutes the propagation of sound in air. More about such motion would be considered in the next chapter.

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* A faint sound is heard even when the inside of the receiver is nearly vacuous. The reason is that sound partly finds its way through the material medium of the bell, etc.

EXERCISES ON CHAPTER I

	Reference
1. Describe experiments to show that it is the mechanical vibration which produces sound. (Pat. U.—1963)	Art. 2
2. Draw a diagram to show the working of an electrically driven tuning fork. (Raj. U.—1959)	Art. 2
3. Explain why a medium is necessary for the propagation of sound and describe any experiment to prove the statement. (C. U.—1953)	Art. 3
4. How can you show that there is no translatory motion of air during the propagation of sound through it ?	Art. 4

CHAPTER II

WAVE-MOTION

5. Periodic Motion—A particle is said to execute a *periodic motion* when after each equal interval of time it occupies the same position having an equal velocity in the same direction. The earth has a periodic motion round the sun, the period of rotation of the earth being one year. A uniformly rotating point on a circular path is another illustration of a periodic motion. It has been seen in Art. 2 that all bodies emitting sound are in a state of rapid vibration. Sounding bodies execute periodic motion. We shall now investigate the simplest type of periodic motion, which is called the Simple Harmonic Motion.

6. Simple Harmonic Motion—A very common illustration of simple harmonic motion (abbreviated as S.H.M.) is the motion of a simple pendulum for small oscillations. The motion of a particle is said to be *simple harmonic*, when it oscillates periodically in a straight line under the action of a force which is proportional to the distance of that particle from a certain fixed point in its course, the direction of the force being always towards that point.

Let a particle at M, possessing some initial kinetic energy, be movin towards A′ along the line AA′. Suppose that the force acting upo the particle is directed *towards* a fixed point O and is proportional to the distance of the particle from this point. As the particl continues to move towards A′, this force gradually increases in proportion and retards the velocity of the particle. Ultimately the particle would reach an extreme position, say A′, where it would lose all kinetic energy and would be at rest for a moment. But here the force upon it towards O is greatest. Due to this it begins to move in the direction A′O. As it moves nearer and nearer to O, its velocity is gradually increased. When it ultimately comes to O, its velocity is greatest and owing to inertia it swings past this point along the direction OA. On crossing the point O along OA the direction of the force is now reversed, because the body is now on the other side of O. Its motion now gets gradually arrested until it is brought to rest at some point A. Due to the absence of any other force considerations of symmetry suggest that OA′ is clearly equal to OA. From the point A again, it will move in the direction AO and for a similar reason travel up to A′. Thus the particle will continue to oscillate backwards and forwards about its position of equilibrium. This type of motion is called *Simple Harmonic*.

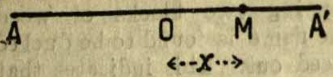


Fig. 10

Motion of a Simple Pendulum—The nearest approach to an S. H. M. is the motion of simple pendulum for small oscillations. Let O be the point from which a spherical bob of mass m is suspended by a light inextensible string (Fig 11). The *equivalent length* of the pendulum is the distance between O and the centre of the bob. When the pendulum is at rest at A, the tension of the string balances the weight of mass m, and so this is its position of stable equilibrium.

When the bob is so displaced that the string makes a *small* angle θ to the vertical, the tension of the string and the weight of the bob are not acting in one line. These two forces would produce a resultant motion of the bob. To find the resultant force we can proceed from the following simple consideration. In the displaced position the tension **T** of the string balances the component of the weight mg of the bob along BE, which is OB produced.

$$\therefore \quad T = mg \cos\theta.$$

The only effective force is, therefore, $mg \sin\theta$ acting along BD. Since θ is small, not being much greater than 4°.

$$\sin\theta = \theta = \frac{AB}{OA} = \frac{\text{displacement}}{\text{length of pendulum}}.$$

Hence the effective force $= \frac{mg}{l} \times$ displacement

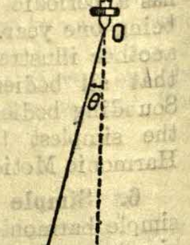
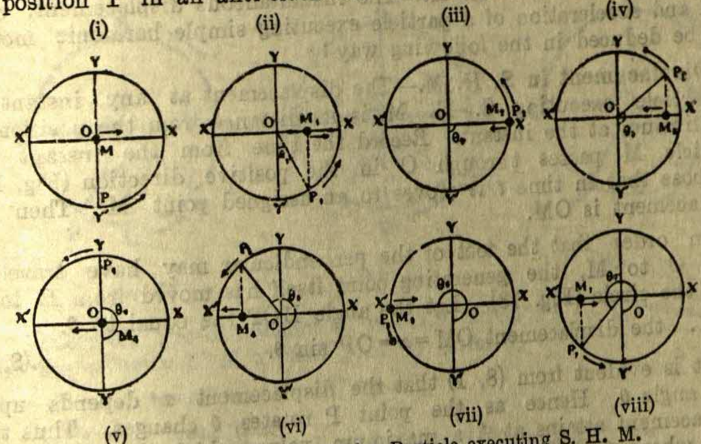


Fig. 11

The shows that the motion of the simple pendulum is simple harmonic.

7. Geometrical Representation of S. H. M.—The nature of Simple Harmonic Motion may be conveniently represented by the following geometrical method. Imagine a point P to be moving uniformly on a circle (Fig. 12) and let this rotating point start from the position Y′ in an anti-clockwise direction (Fig. 12, i).



(i) (ii) (iii) (iv) (v) (vi) (vii) (viii)

Fig. 12—Various Positions of a Particle executing S. H. M.

On any diameter XX′ drop the perpendicular PM from P (Fig. 12. i). As the point P revolves in the direction of the arrow-head towards X, the foot of the perpendicular M also moves along the diameter from O to X. This is illustrated in Fig. 12 (ii) by an intermediate position at P_1, the corresponding foot of the perpendicular being M_1. Thus when the point P moves from Y′ to X, M also moves from O to X (Fig. 12, iii). When P has crossed X and moves over the arc XY, the foot of the perpendicular moves from X towards O in the reverse direction as shown in Fig. 12 (iv). When P has come over to the position Y, its foot is again at O but tends to move in the opposite direction (Fig. 12, v). In this way, it is easy to follow that during the period the point P has completed one revolution round the circle starting from Y′, its foot M on XX′ has undergone *one oscillation* about its position of equilibrium O. Thus while P rotates continuously over the circle with a uniform speed, the point M oscillates to-and-fro along the diameter XX′.

This circle is known as the **circle of reference** and the rotating point is called the **generating point,** *its motion in the* anti-clockwise *direction being taken* positive. The centre O of the circle is taken as the origin of the rectangular Cartesian co-ordinates.

If T be the period of revolution of the point, the angular velocity ω of the radius OP is given by $\omega = 2\pi/T$ ... ... (7, 1)

The number of revolutions per second of the generating point over the reference circle is called its *frequency*; it is denoted by *n*. Plainly, then, $n = 1/T$.

$\therefore\ \omega = 2\pi n$ ... ... (7, 2)

It is clear that the number of complete oscillations of the particle M would be identical with the number of revolutions of the generating point P. Hence the frequency is also defined as the *number of complete oscillations* of the particle *per second.*

8. **Analysis of S. H. M.**—The instantaneous displacement, velocity and acceleration of a particle executing simple harmonic motion can be deduced in the following way :

Displacement in S. H. M.—The displacement at any instant of a particle executing S. H. M. is its distance from the position of equilibrium at the instant. Record the time from the instant the particle M passes through O in the positive direction (Fig. 13). Suppose that in time t it moves to an assigned point M. Then its displacement is OM.

In order that the foot of the perpendicular may have travelled from O to M, the generating point itself has moved from B′ to P over the circle (Fig. 13). Let the angle B′OP the equal to θ.

$\therefore$ the displacement $OM = x = OP \sin \theta$. ...(8, 1)

It is evident from (8, 1) that the displacement x depends upon the angle θ. Hence as the point P rotates, θ changes. Thus the displacement attains at A a maximum value which is positive in sign when $\theta = 90°$ and at A′ a minimum value which is negative in sign when $\theta = 270°$. The maximum displacement from the equilibrium position O of the particle is called the **amplitude of vibration.**

Thus $OA =$ the amplitude $= a$, say.

Since $\theta = \omega t = \frac{2\pi t}{T} = 2\pi n t$, it follows that

$$\left.\begin{aligned} x &= a \sin \theta = a \sin \omega t \\ &= a \sin \frac{2\pi t}{T} = a \sin 2\pi n t \end{aligned}\right\} \quad ...(8, 2)$$

Any one form of the above expressions represents the displacement of the particle at a time t.

If, however, the time is recorded from where the generating point passes through any other point P′ (Fig. 14) and if it has come to P in time t, then $\angle P'OP = \omega t$, so that

$$\angle B'OP = \angle P'OP - \angle P'OB = \omega t - \alpha.$$

Accordingly, $x = a \sin (\omega t - \alpha)$. ...(8, 3)

Velocity in S. H. M.—The velocity of a particle executing an S. H. M. changes from point to point in the path, attaining a maximum value while passing through the origin. At the extreme ends of the swing it comes *momentarily* to rest while changing its direction of motion. The velocity of the particle at any instant may be found with reference to the generating point P which is revolving round the circle at constant speed v, say. If the angular velocity be ω and the radius of the circle be v, then $v = a\omega$.

At *any* instant the generating point P has a tangential velocity v along the tangent PT of the circle (Fig. 14). The component of this velocity parallel to the direction A′A is the velocity of M along A′A. Draw PV parallel to A′A. Then $\angle VPT = \angle B'OP = \theta$.

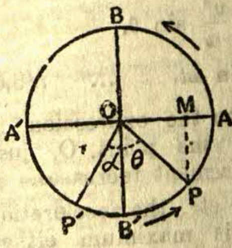


Fig. 13

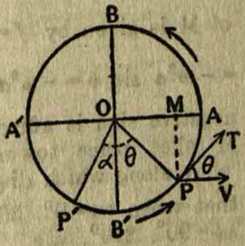


Fig. 14

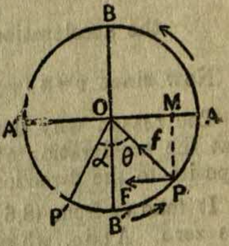


Fig. 15

Therefore the component of velocity along PV is V cos $\angle B'OP$.

So the velocity of M at that point is given by ... ...(8,4)

$$v = a\omega \cos\theta = a\omega \cos \omega t.$$

It is now clear that

(*i*) When $\theta = 0^\circ$, velocity $= a\omega$, being maximum along the positive direction ;

(*ii*) when $\theta = 90^\circ$, velocity $= 0$, being minimum ;

(*iii*) when $\theta = 180^\circ$, velocity $= -a\omega$, maximum along the negative direction ;

(*iv*) when $\theta = 270^\circ$, velocity $= 0$, being again minimum ;

and (*v*) when $\theta = 360^\circ$, velocity $= a\omega$, being once again maximum along the positive direction.

Thus *during one complete period of revolution the velocity is zero at* either *end of the swing and is maximum when passing either way through the origin.*

Those, who are acquainted with rudiments of Differential Calculus, will find it very easy to find instantaneous velocity v from the crresponding displacement x, in the following way. Displacement being x ; corrsponding velocity $= v = \dfrac{dx}{dt} = \dfrac{d}{dt}\,[a \sin \omega t] = aw \cos \omega t.$ ...(8,5)

Acceleration in S. H. M.—Since there is a change of velocity from point to point of the particle executing S H. M., it must have a certain acceleration at any point in its path of motion. When the generating point P moves uniformly over the circle of radius a with a velocity v, it is shown in Dynamics that the point has got an acceleration f directed always towards the centre. the magnitude of which is v^2/a. The component of this acceleration paralled to A′A is the acceleration of the particle M (Fig, 15).

Suppose the generating point occupies the position P at a time t. The acceleration of the point is directed towards O. The component of this acceleration parallel to A′A is along PF, which shows that the particle M has an acceleration directed towards O.

The component of the acceleration along PF is $\frac{v^2}{a}$ sin ∠B′OP.

∴ the acceleration of M is given by $f = -\frac{v^2}{a} \sin \theta$.

Now since $v = a\omega$; $f = -\omega^2 a \sin \theta = -\omega^2 a \sin \omega t$. ... ...(8,6)

The reason why the negative sign is affixed to the expression is that the acceleration of M takes place towards the point O, just opposite to the direction of A along which displacement increases.

It follows from (8,6) that when $\omega t = 0°$, 360°, etc., the acceleration f is zero. Again, when $\omega t = 90°$, 270°, etc., f is maximum either along the positive direction or along the negative direction.

The **force** acting on the particle M of mass m at any instant is given by

$$mf = -m\omega^2 a \sin \theta$$
$$= -m\omega^2 \times \text{displacement at that instant.} \quad \ldots \quad \ldots(8,7)$$

Hence force ∝ displacement.

As this is the essential condition of S. H. M., the particle executes Simple Harmonic Motion, and the relation (8.7) is characteristic of S. H. M.

In case of acceleration also, the expression can be obtained from Differential Calculus in the following manner. We know that the velocity expression is given by $v = aw \cos \omega t$.

Now acceleration $f = \frac{dv}{dt} = \frac{d}{dt}[aw \cos \omega t] = -a\omega^2 \sin \omega t$, which is the same equation of as that obtained in eqn. (8, 6).

Phase in S. H. M.—The phase of an oscillating particle is the factor which determines its position and velocity at any instant in its path. It is generally expressed—

either (*i*) by the angle traced by the generating point with reference to one of the co-ordinate axes, as in the first form of the equation (8,2).

or (*ii*) by the fraction of the period elapsed since the particle passed the equilibrium position as in the other three cases of the self-same equation.

The phase of the particle at the instant, from which time is recorded is called **epoch**. Thus in equation (8,3), when $t = 0$, the phase is $-\alpha$, which is the epoch. The algebraic sign of the epoch may be positive or negative depending upon the position of the particle from which time is counted.

Examples :

1. A particle is revolving in a verticle circle of radius 2 feet at the rate of 400 revolutions per minute. Find its period and angular velocity. Also obtain an

expression connecting the time and displacement of the foot of the perpendicular dropped from this particle on a horizontal diameter of the circle when the time is reckoned from the topmost position of the particle.

Ans. The frequency n of the particle $=400$ cycles per min. $=\frac{20}{3}$ per sec.

$\therefore$ the period $T=\frac{1}{n}=\frac{3}{20}$ sec. $=0{\cdot}15$ sec.

Also the angular velocity $\omega=2\pi n=\left(2\times3{\cdot}14\times\frac{20}{3}\right)$ radians/sec. $=41{\cdot}86$ radians/sec. $=2400^\circ$ per sec.

The amplitude a is always equal to the radius of the reference circle. Hence $a=2$ ft. For $t=0$, the particle is at the topmost position of the circle and a perpendicular dropped from it would have its foot at the centre of the circle. Hence at the instant $t=0$ the displacement of the foot $x=0$. This can only happen when of all the expressions for S. H. M., choice falls upon the equation $x=a \sin \omega t$.

By substitution we get $x=2 \sin \frac{40}{3}\pi t$.

2. The time displacement equation of a particle executing an S. H. M. is given by $x=3 \sin 10t$. Find its amplitude, frequency and maximum velocity while passing through the centre of the path.

Ans. Elementry Trigonometry suggests that the maximum value of the sine of an angle is $=1$. Hence the maximum value of the displacement x, (*i.e.*, the amplitude) is 3, in whatever unit of length it is expressed. From the equation it is noticed that.

$2\pi n=\omega=10$. Hence the frequency $n=\frac{10}{2\pi}=\frac{5}{3{\cdot}14}$ cycles/sec. $=1{\cdot}59$ cycles/sec.

Further, the maximum velocity attained is $=a\omega=(3\times10)$ or 30 units of length per second.

9. Graphical Representation of S. H. M.—Since the displacement of a particle executing an S. H. M. changes with time, we can

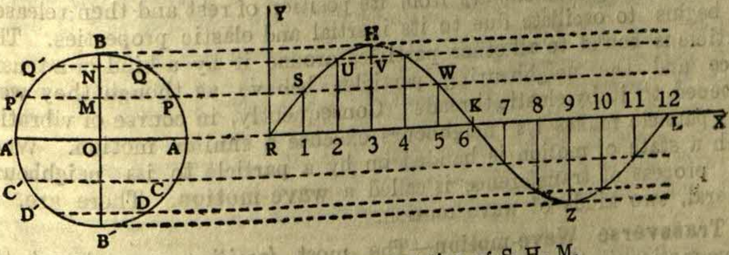


Fig. 16—Graphical representation of S. H. M.

represent graphically the relation between the time measured as a fraction of the period and the corresponding displacement. Let the particle oscillate along the line BOB′ having an amplitude OB or OB′ (Fig. 16). Take the period to be T. Draw a circle with BOB′ as diameter. Now assume that on this circle of reference a point is revolving uniformly with a time period T. Divide the circumference into any number of equal parts (say, m). For the sake of convenience and simplicity take m to be a multiple of 4.

Draw a diameter AOA′ perpendicular to BOB′. Produce AA′ to a point X and mark two points R and L on this line at a convenient distance. Divide the length RL into m equal parts. Let the line RL represents the time axis with a unit of T/m. From R draw the perpendicular RY, along which the displacement of the particle executing S. H. M is to be measured. Represent the positive and negative displacements by ordinates drawn above and below this line. In this particular case as shown in Fig. 16, m is taken as 12.

Referring to the S. H. M. along BOB′, we record the time when the particle is passing through O along the positive direction. On the reference circle the corresponding point is A when $t=0$. So for zero time, there is no displacement. After a time T/12 the particle on the reference circle is at P, where arc AP is $\frac{1}{12}$th the circumference of the circle. Through the point 1 representing T/12 on the axis RX draw a perpendicular in the positive direction and through P draw the line parallel to RX. Let these two lines meet at S. Then S is the first point on the time-displacement graph. The next position of the reference point after an interval T/12 is the point Q and the corresponding time on the line RX is at 2. Draw two lines through the points marked 2 and Q so as to meet at U. Then U is the second point on the graph. In this way we get points H, V, W, etc. During a complete period if such points are joined in succession, a complete sine-curve RHZL (0° to 360°) is obtained.

The nature of the *time-displacement graph* does not depend on the direction of vibration of the particle, since displacement in the positive direction is to be designated by a perpendicular standing above the time axis, and a displacement in the negative direction by a perpendicular below it.

10. Motion of Wave—When a particle of an elastic material medium is displaced *slightly* from its position of rest and then released, it begins to oscillate due to its inertial and elastic properties. This particle is bound to all other particles around it by a kind of a tensile force and the neighbouring particles behave as though they were fastened to it by elastic threads. Consequently, in course of vibration this particle makes its neighbours execute a similar motion. When such a state of motion is handed on by a particle to its neighbours, the process of transference is called a **wave-motion.** There are, in general, two kinds of wave motion.

Transverse Wave-motion—The most familiar example of the wave-motion is the motion of waves on the surface of water. Fig 17 represents a rectangular vessel partly filled with water and provided with a flat piece of wood S at one end. The plank may be periodically moved up and down so as to generate a series of waves as shown. The topmost position of a wave is called a **crest** and the lowest position is called a **trough**. The distance between two successive crests equals the distance between two successive troughs. The length (such as AC) occupied by crest and a trough is one **wave-length** and is denoted by λ. The inherent tendency of such a group of waves is to move away with its characteristic form unchanged. At some particular instant the position of a wave is denoted by the form APCDE (Fig. 18); but at a subsequent instant the wave moves forward to occupy the position $a\ p\ c\ d\ e$. In this way the wave *with its form unchanged* moves continuously over the surface of water *with a definite velocity.*

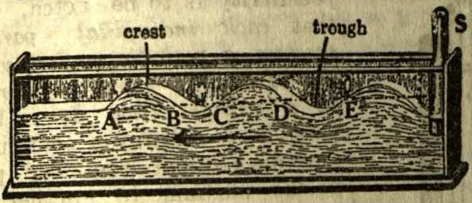


Fig. 17—Water Waves

Now imagine that a very light object (*e.g*, a piece of cork) is floating on water. When the waves pass by, this object continues to move up and down. On examining minutely it is found that during the time such a wave moves through a distance from one crest to the next, or from one through to the next, the small floating object would *once* move up and down. The time required for the particle to move once up and down is called the **period** of the vibration or of the wave passing through the particle. Hence we may also define the **wave length** as the distance traversed by the wave in one complete period of the wave. Thus the particle, which vibrates n times per second, would transmit n waves through it in one second, each of length λ. Hence the velocity V, with which a motion is carried through any point of the medium, would be $n\lambda$ per second. So that $V = n\lambda$.

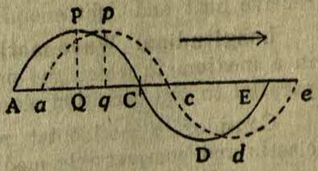


Fig. 18

Therefore, a wave-motion in which any particle in the path of the wave oscillates at right angles to the direction of motion of the waves, is called a *Transverse Wave-motion*. Propagation of transverse waves may be examined along a thin uniform rope hanging freely from a support. If the support be given a slow periodic motion in a direction at right angles to the length of the rope, the gradual progress of the transverse waves may be studied from point to point.

Another familiar model for the demonstration of transverse wave propagation is shown in Fig. 19. A number of straight and parallel rods, each provided with a small ball at the top and resting on an eccentric wheel, is fitted to a stand. Each rod passes through a hole of a crosspiece at the top of the stand. All the eccentric wheels have got a common axis of rotation. When the handle is continuously revolved, the motion of the balls at the top is analogous to that of the ripples. Note that during the period a particular ball makes a complete oscillation the wave curve moves through a distance equal to the wave-length.

In all cases of transverse wave propagation one common peculiarity is to be noted, *viz.*, that *each individual particle executes a periodic motion, while the wave curve moves uniformly in a straight line.* It is thus possible to convey a continuous periodic motion from one part of the medium to another by producing a series of waves at *any* part. For example, if we set up waves at one end of the rope, any point of it moves periodically, so long as waves are produced. But every motion is invariably associated with kinetic energy. Hence we may say that a wave form carries kinetic energy with it. It is due to the effect of some form of the transverse wave that we receive heat and light energies from a source.

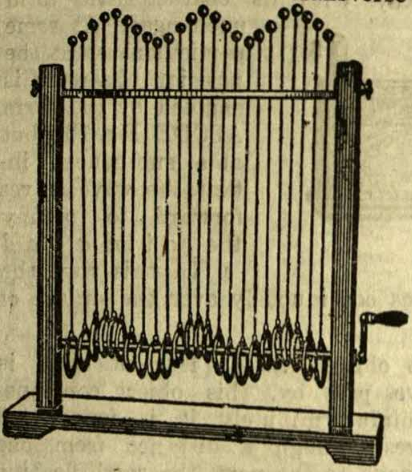


Fig. 19

Longitudinal Wave-motion—This is a type of the wave-motion in a medium in which the particles in the path of the wave oscillates parallel to the direction along which wave moves.

Consider a horizontal row of equally spaced particles in an elastic and compressible medium. The positions of the particles in an undisturbed condition are indicated by the dots marked 1, 2, 3, 9, (Fig 20). We assume that the particles are bound to each other by some elastic force. To the first particle a motion is given towards the second. As the first particle begins to move towards the second, the space between the first two particles diminishes and hence the pressure on the second particle increases, as indicated by the proximity of the first two particles in the second row. The increase of pressure causes the second particle to move towards the third row, and so on.

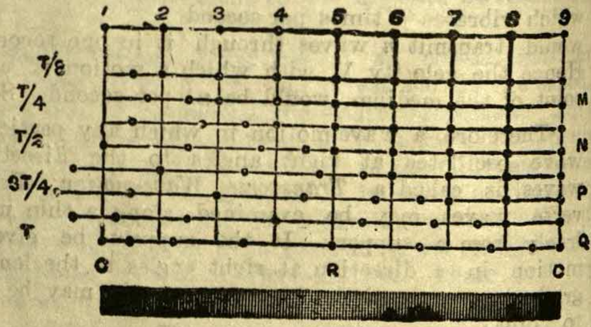


Fig. 20—Progress of Longitudinal Wave

Again, when the first particle has gone over to the extreme position as indicated in the third row, the force of restitution imparts to it a tendency to come back to the original position. So it begins to retrace the path, while due to the forward inertia the second particle still continues to move in front and passes on the third particle which in its turn begins to move forward (4th row). The space between the first two particles now increases. Due thereto a state of rarefaction is brought about between the two. In this way a state of rarefaction is transferred from particle to particle. The condensed and the rarefied parts follow each other with a definite velocity within the medium. The combined thickness of any two consecutive and rarefied systems of particles constitutes a wave-length of longitudinal wave. After the first particle has completed a period of vibration, the displaced positions of the successive particles are shown in the last row Q.

When the periodic motion is persistent within the medium, the sequence of motion as shown by the eight rows (Fig. 20) is repeated over and over again for all particles surrounding the source.

Propagation of sound waves through a gas is an example of longitudinal wave-motion. The wave-length λ is the combined thickness of a condensed layer and a rarefied one, the wave velocity V is the product of the frequency π and wave length ; *i.e.*, $V = \pi\lambda$.

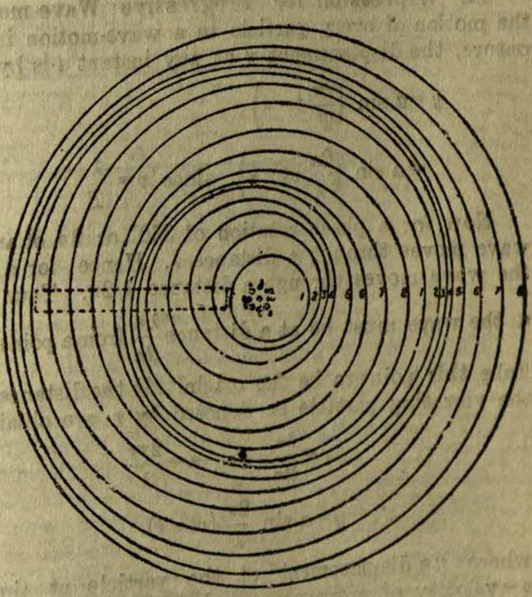


Fig. 21—Crova's Disc

Crova's Disc— The nature of the continuous state of motion of the particles of the medium during a longitudinal wave propagation is difficult to realise. To make some idea of the actual state of motion in a longitudinal wave propagation, the following simple experiment may be done.

Draw small circle and mark a few equidistant points (8 points in Fig. 21) on the circumference of this circle. With the first point as the centre draw a small circle and number it as 1. Then with the second point as the centre draw another circle 2 with a slightly longer radius. In this way draw a series of circles with the third, fourth,..., eighth points as centres and with equally increasing radii. Such a diagram was constructed by Crova and hence it is called after his name. Place a disc on a turn table so that it can revolve with the centre of the smallest circle as axis. Mount a card board provided with a long rectangular slit in front of the disc such that a small region of the circles drawn is seen through the slit. On revolving the turn table, the small circular arcs keep on moving about the slit so as to represent waves of condensation and rarefaction following each other in the propagation of the longitudinal waves.

Examples :

1. Sound travels in air with a velocity of 330 metres per second at 0°C. What is the wave-length of a note of frequency of 200 per second ? (B.H.U.—1950)

Ans. **Since $V = n\lambda$, the wave-length λ_1 for a frequency 200 per sec. is given by $200 \times \lambda_1 = 330$, whence $\lambda_1 = 1{\cdot}56 \times 1$ m. $= 165$ cm.**

2. A tuning fork vibrating in air sends waves of length 100·6 cm. The same tuning fork sends waver of length 382·4 cm. in hydrogen. If the velocity of sound waves in air be 332 metres per second, calculate the velocity of sound in hydrogen. (V. U.—1950)

Ans. **We know velocity = (frequency) × (wave-length). If the frequency of the tuning fork be n, then $n = 33200/100{\cdot}6 = 330$.**

$\therefore$ the velocity in hydrogen $= n \times 382{\cdot}4 = (330 \times 382{\cdot}4) = 126190$ cm./sec.

11. Expression for Progressive Wave-motion—Assuming that the motion of every particle in a wave-motion is simple harmonic in nature, the displacement y at any instant t is by (8,2) represented by

$$y = a \sin\left(\frac{2\pi}{T}t - \alpha\right)$$

$$= a \sin\left(\frac{2\pi}{\lambda}vt - \alpha\right), \text{ since } v = \frac{\lambda}{T} \qquad \ldots \ldots (11,1)$$

Now for a phase rotation of 360° or 2π of any single particle, the wave moves through a distance λ. Hence for a unit phase rotation the wave moves through a distance $\lambda/2\pi$. Thus for a phase rotation α, the wave must be at a distance $\frac{\lambda\alpha}{2\pi}$ from a point whose epoch is zero. Take this point to be the origin. If the distance from the source of the vibrating particle be denoted by r, we obtain

$$r = \frac{\lambda\alpha}{2\pi} \quad \text{or} \quad \alpha = \frac{2\pi r}{\lambda}.$$

$$\therefore \quad y = a \sin\frac{2\pi}{\lambda}(vt - r). \qquad \ldots (11, 2)$$

where y = displacement of the particle at time t, λ = wave-length, v = velocity of propagation and r = distance of the particle from the origin.

Wave-front—In a homogeneous medium a wave is propagated from a point source with an equal velocity in all directions. At any time the propagated wave lies upon the surface of a sphere, whose radius is equal to the product of the velocity and the time. On this sphere the state of motion (or the phase of vibration) is the same at all points. Such a surface having the same phase all over it is called an **wave-front**. When, however, the source is at very great distance, the wave-front has a very small curvature, *i.e.*, sensibly it is a plane surface.

12. Mechanism of Transmission of Sound through Air— We shall now examine the state of motion of air, when sound waves propagate through it. Vibrate a metallic prong *b* so as to emit sound. Its vibrations consist of a series of rapid *to-and-fro* motions. Each complete vibration comprises a movement from *b* to *c*, *c* to *a* and back to *b*. This movement needs an interval of the time which is called the **period.**

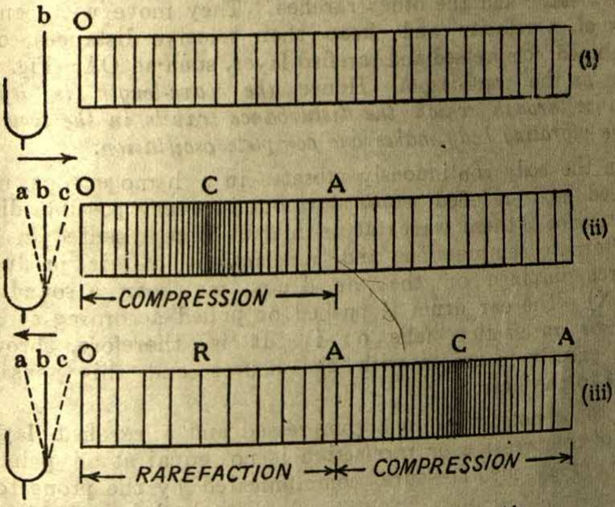


Fig. 22—Compression and Rarefaction in Air

Imagine the undisturbed air in front of the prong to be divided into thin vertical layers of equal thickness as shown in Fig. 22 (i). In course of vibration, the prong starts its course from *b* towards *c* and compresses the first layer of air in its front (Fig. 22ii). This layer has its volume diminished and consequently its pressure is increased. Due to elastic property, this layer tends to expand; but since the prong on its side behaves as a barrier, it compresses the second layer of air on the other side. Being so compressed, the second layer acts upon the third; and so on. *The state of compression travels from layer to layer with a definite velocity.* During the time the prong moves from *b to c* and back to *b*, it keeps on compressing all the layers in front of it. Suppose that during this interval the compression moves through a distance OA (Fig. 22 ii). Hence a thickness OA of air is under compression during half the period of vibration of the prong. With a definite velocity this compressed state moves forward.

Again, as the prong moves back from *b* in the direction *ba*, a partial vacuum is produced in the adjacent layer. Being thus relieved of pressure, the first layer expands on the side of the prong and its pressure is consequently diminished. For a similar reason the second layer moves towards the first and thereby expands itself. Thus every successive layer of air in its turn expands. Hence a *state of rarefaction is handed on from layer to layer, travelling with the same velocity as that of compression.*

Since the time occupied by the movement of the prong from *b* to *a* and back to *b* is another half period, the total thickness of the expanded layer is equal to that of the compressed one, so that AA' = AO (Fig. 22iii). It is evident that during a complete vibration of the prong, two such thin air layers are formed, one of which is in a compressed state, and the other rarefied. They move with an equal thickness of a velocity and keep their relative distances constant. The combined compressed and rarefied layer, such as OA' (Fig. 22, iii), is known as the *wave-length*. Hence *the wave-length is defined as the distance through which the disturbance travels in the time during which the vibrating body makes one complete oscillation.*

When the body continuously vibrates in a homogeneous medium, compressed and rarefied *waves* start out in every possible direction. The appearance of these waves at an instant it represented in Fig. 23. When alternate compressions and rarefactions of air produced by the source impinge on the ear drum, we hear a sound. (Vide Chapter X). The ear drum is pushed or pulled according as a compression or rarefaction falls on it It is, therefore, thrown into vibration exactly in unison with the source and this sensation is conveyed to the brain.

It is to be noted that in a compressed and a rarefied layer, the amount of compression or rarefaction is not equal at all points. The reason is that the pressure communicated by the prong to air at any instant depends upon the velocity of the prong at that instant. The velocity of the prong in its course of vibration varies from point to point, being maximum at the centre and minimum at the ends. Hence the pressure also of the compressed layers is maximum at the middle and normal at the ends.

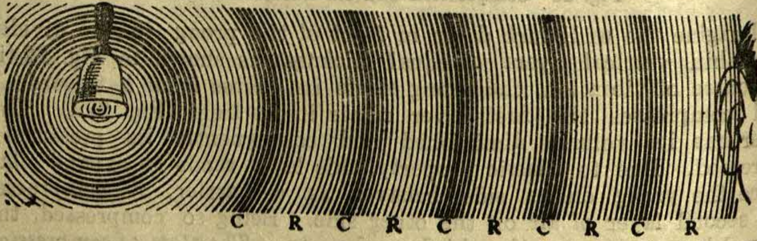


Fig. 23—Propagation of Sound Waves

13. A model for the Propagation of Sound through Air—The nature of the motion of the successive layers of air in a particular direction can be demonstrated by the following apparatus:

Spiral wire—It consists of a brass-wire of about 0·1 inch in diameter, made into a coil of diameter 3 inches nearly, the length of the coil is about 6 feet to 7 feet. By double suspension arrangement from two parallel bars in a manner, as shown in Fig. 24, this coil is hung up at several points by threads.

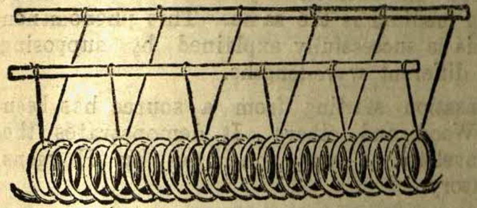


Fig. 24

On giving a smart push lengthwise at one end of the coil, a few turns of wire at that end are compressed. This compression moves along the spiral with a certain velocity towards the other end. This is very nearly what happens in air when a condensation travels through it, each layer being compared to each turn of the spiral. Again, on suddenly pulling the same end of the spiral, the turns become widely separated and this expanded state travels to the other end with an *equal velocity*. This is equivalent to a state of rarefaction travelling through air. Now if the end of the spiral be alternately pushed and pulled, alternate condensations and rarefactions pass through the spiral which travel with a constant velocity. Figure 25 shows the condition of the turns of the spiral *at any particular instant*, when a series of pulses is passing from the right to the left. In the figure C represents the position of condensed turns and R the rarefied ones.

It is interesting to note that during the propagation of a disturbance, the turns of the wire are not permanently displaced from their natural positions, but simply oscillate equally on both sides of their mean positions.

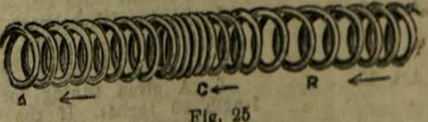


Fig. 25

14. Sound is a wave-motion—Sound may be considered to be a wave-motion for the following reasons:

(*i*) Sound requires a medium for its propagation. The medium does not move as a whole, but allows sound to pass through it.

(*ii*) Sound moves with a measurable velocity through a gas. The velocity of sound is greater in solids and liquids than in gases. This can only be explained by assuming that sound is conveyed by way of wave-motion.

(*iii*) Sound undergoes reflection and refraction according to certain laws, which can be explained on the assumption that it is a wave-motion.

(*iv*) Under certain conditions two sound waves might combine to produce silence, which is called *interference of sound*. This can only be proved by assuming that sound is a wave-motion.

(*v*) Sound waves can bend round an obstacle and tones of different pitches bend round an obstacle by different amounts, although their velocity in a medium is the same. This phenomenon is known as *diffraction*. This is successfully explained by supposing the different notes have got different wave-lengths.

(*vi*) A wave of condensation starting from a source has been photographed by R. W. Wood and others It demonstrates the existence of secondary wavelets, as was conceived by Huygens (1629—1693) in his wave theory.

(*vii*) That sound in a gas undergoes no polarisation proves further that it consists of longitudinal and *not* transverse vibration.

EXERCISES ON CHAPTER II

Reference	
Art. 6.	**1. Define S.H.M. and explain it with reference to any familiar example. (G.U.—1965 ; All. U.—1966)**
Art. 7.	**2. Define amplitude, frequency, wave velocity and wave-length. What is the relation between velocity and wave-length ? (Pat. U.—1963 ; R. P. B.—1971 ; C. U.—1956 ; Del. H. S.—1955, '64 ; *Cf.* Del U.—1966)**
Art. 10	**3. Compare the wave lengths in air of sounds given by two tuning forks of frequencies 128 and 384 respectively. (C. U.—1950)**
	Ans. **3 : 1.**
Art. 3	**4. Explain Simple Harmonic Motion and state its characteristics. Show that the motion of a simple pendulum is simple harmonic. What part does S.H.M. play in sound ? (U. P. B—1963 ; Nag. U.—1969)**
Art. 10	**5. Distinguish between longitudinal and transverse waves. (Del. H. S.—1951, '73 ; P. U.—1961, '67 ; And. U.—1950. '71 ; V. U.—1955 ; C. U.—195[illegible])**
Art. 10	**6. A given tuning fork produces sound waves of wave-length 30 inches. If the velocity of the wave is 1100 ft. per sec., what is the frequency of the fork ? (Guj. U.—1961)**
	Ans. **440 per sec.**
Arts. 10 & 12	**7. Explain what is meant by longitudinal wave-motion in a medium. Indicate what goes on in the medium which transmits a sound. (P.U.—1941)**
Art. 10	**8. Taking 1129 ft. per sec. as the speed of sound in air, find the number of vibrations which a fork of frequency 264 must make before the sound is heard at a distance of 154 ft. (C. U.—1962)**
	Ans. **$n = 36.01$.**
Art. 10	**9. A tuning fork of frequency 250 is sounded in air and the sound waves are found to travel with a velocity of 1120 feet per second. Find the wave-length of the note emitted. (Del. H. S.—1965)**
	Ans. **4·48 ft.**

Art. 10 **10. State what is meant by longitudinal and transverse wave-motions. Give an example of each type. Define wave-length and establish the relation between the wave-length and velocity of wave-motion in a free medium.** **(C. U.—1949)**

Art. 12 **11. Explain the terms wave-length and frequency.** **(Dac. U.—1961; P. U.—1960)**

Art. 12 **12. A sound generated in water has a wave-length of 580 cm. If the velocity of sound in water be 145,000 cm. per second, calculate the frequency of sound in water.** **(Dac. U.—1961)**

Ans. 250 per sec.

Art. 12 **13. Explain, as far as you can, the mode of propagation of sound through air.** **(Utk. U.—1968; Dac. U.—1958; Pat. U.—1959, '63)**

Art. 14 **14. What reasons are there for believing that sound is conveyed by wave-motion?** **(Utk. U.—1951; Dac. U.—1962, '70; C. U.—1953; *Cf.* R. P. B.—1971)**

CHAPTER III

VELOCITY OF SOUND

15. Velocity of Sound in Open Air—It is a matter of common experience that when there is thunder and lightning, we see the flash of the lightning first before we hear the report of the thunder. Similarly, when a locomotive engine blows a whistle at a considerable distance from a railway station, a casual bystander on the railway platform observes first the puff of gas at the nozzle of the engine and then hears the whistle. Thus where there is something to see and hear from a distant source, the eye is quicker to receive than the ear.

All these cases are due to the fact that light moves with a far greater velocity than sound. Thus if lightning occurs in a cloud one mile away from an observer, the flash would reach him in a period of 0·0000054 second, of which we can form no idea. On the other hand, the report of the thunder from the same distance requires approximately 5 seconds to reach him Hence the interval between seeing and hearing in this case, which is really the difference of the two timings, can be recorded practically as the time taken by sound to travel through the distance.

Experimental determination—The velocity of sound in *open air* has been determined by different observers at different times. The following method was originally undertaken by Arago (1786—1853) in 1829. Two observers are stationed on hills at a known distance, several miles apart. One of them is provided with a gun, and the other with a stop-watch which can read a fifth or tenth of second. The first man fires his gun, while the second man starts the watch *just* when he sees the flash of light from the nozzle of the gun and keeps a continuous record of time till he hears the report. If the interval recorded be equal to t seconds and if the distance between the two stations be x feet, the velocity of sound is given by $V = x/t$ ft. per sec. A large number of observations is taken under similar atmospheric conditions and the mean value of the velocity is found.

But in such an experiment there arise two principal *sources of error*. First, the velocity of sound, being greater in the direction of the wind and less against it, is partly affected by motion of the wind. This is known as *the error due to the wind velocity*. Secondly, any man is always apt to delay a certain fraction of a second to record the time after he has actually seen an event. The error is called the *personal equation of the observer*. It is different for different persons; but it is always a small fraction of a second. The personal equation generally varies, on an average, from one-fifth to one-fifteenth of a second.

The first error is eliminated by taking what is known as the *reciprocal observation*. Both the observers are provided with a gun and a watch. One of them fires the gun and the other records the time t_1 between the sight of the flash and the hearing of the report. The other man then fires in his turn and the interval t_2 is now recorded by the former in a similar way. Such similar sets of readings are repeatedly taken and the mean value is calculated.

Suppose the wind is blowing in the direction from the first station to the second with a velocity of v ft. per sec. Let the velocity of sound in still air be V ft. per sec.

Then $V + v = x/t_1$ and $V - v = x/t_2$.

On elimination of v we find

$$V = \frac{x}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right) \qquad \text{...(15, 1)}$$

The *personal error* or equation, is avoided by making electrical arrangements for recording the exact moments of the gun-fire at one station and the reception of the sound at the other. An electrical clock automatically records the instant of firing. The receiving station is provided with an automatic *microphone* (Chapter IX). As soon as the sound reaches the microphone, its arrival is recorded on a moving photographic film. Thus the actual interval can be determined and the velocity of sound at a particular condition of the atmosphere is accurately found.

16. Some Classical Experiments—The first attempt at an accurate determination of the velocity of sound in open air was made by the Commission of the Paris Academy in 1738. They chose 3 or 4 stations, the total distance covering about 12 kilometers (about 7·5 miles). Alternately, from each of the two end stations guns were fired at regular intervals. The middle station recorded the time. The distances between the stations being known, the mean velocity of sound could be found. The experiments were conducted under different conditions of temperature, pressure and humidity. Their observations led to the following conclusions:

(i) *The velocity of sound does not depend upon a change of atmospheric pressure.*

(ii) *It increases with the temperature and humidity.*

(iii) *It increases the direction of the wind and decreases against it.*

(iv) *The velocity of propagation of sound in still dry air at 0°C is approximately 332 metres per second.*

A series of careful determinations by Moll, Van Beck and other Dutch physicists in 1823 gave the mean value for the velocity of sound at 0°C to be 332·26 metres per second.

The next important investigation on the velocity of sound along a slope was made by Bravais and Martins. The upper station was Faulhorn and the lower one was the Lake of Briez, the difference of the altitudes being 2079 metres and the actual distance being 9560 metres. They made use of the firing method and found that the velocity, after being corrected by 0°C, was 333·37 metres per second. Experiments were also carried out at very low temperatures during Arctic expeditions by Parry and Greely and almost the same result was arrived at.

Because of uncertainties of the velocity of sound in open air due to wind, humidity and temperature fluctuations, Regnault in 1864 carried out a series of experiments near Versailles on the velocity of sound in air enclosed in long pipes. His method of recording was all electrical. Reciprocal observations were made every time. He found that for *sounds of great loudness* (*e.g.*, a heavy explosion) the velocity was slightly greater than the ordinary value. He observed also that under similar conditions of temperatures and humidity, the velocity of sound in a pipe was slightly smaller than that in free air. Determination of the velocity of sound in open air were made during the Great World War (1914-18) in connection with the location of the sources of sound (such as, gun firing or a charge exploding) by a special device known as the the *Sound-Ranging* method (Vide Chapter IX). An actual laboratory method of determining the velocity of sound in air under controlled condition of temperature, pressure and humidity was devised by Hebb in 1904. (Vide Art. 31)

17. Theoretical Investigation—From a theoretical standpoint Newton came to the conclusion that the velocity of periodic compression or rarefaction in an elastic medium is given by

$$V = \sqrt{\frac{E}{\rho}}, \quad \ldots \quad \ldots \quad \ldots \quad (17{,}1)$$

where E is the modulus of elasticity, and ρ the mean density of the medium. By definition, the modulus of elasticity $= \frac{\text{stress}}{\text{strain}}$.

In all fluid media E represents *volume* elasticity or bulk modulus and is represented by k, so that (17,1) reduces to

$$V = \sqrt{\frac{k}{\rho}}. \quad \dots \quad \dots \quad \dots \quad (17,2)$$

where $k = \dfrac{\text{increase of pressure per unit area}}{\text{consequent change of volume per unit volume}}$.

When a pulse of compression travels through air or any other gaseous medium, the later is momentarily compressed. Let V be the volume of a layer of the gas of unit cross-section under normal atmospheric pressure P (Fig. 26). Let the pressure be increased by p owing to the arrival of a compression and let the consequent decrease of volume be v.

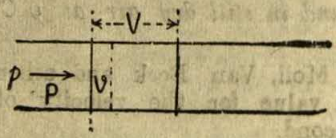


Fig 26

If the changes of pressure and volume take place within the medium under *isothermal* condition (*i.e.*, if the temperature does not change) as was assumed by Newton, Boyle's law gives

or $\quad PV = (P + p)(V - v) \quad$ or $\quad PV = PV - Pv + pV - pv,$

$Pv = pV$, assuming pv to be negligibly small.

Thus $\quad P = p\Big/\dfrac{v}{V} = \dfrac{\text{stress}}{\text{strain}} = k.$

Use of eqn (17, 2) readily leads to

$$V = \sqrt{\frac{P}{\rho}} \quad \dots \quad \dots \quad \dots \quad (17,3)$$

Applying the equation to the case of air at normal temperature and pressure (N.T.P.) at 76 cm. of mercury and 0°C, we get,

$$P = 76 \times 13{\cdot}6 \times 981 \text{ dynes/sq. cm. and } \rho = 0{\cdot}001293 \text{ gm./c.c.}$$

$$\therefore\ V_0 = \sqrt{\frac{76 \times 13{\cdot}6 \times 981}{0{\cdot}001293}} \text{ cm./sec.} = 280 \text{ metres/sec.(approximately).}$$

The value thus obtained is much lower than what is experimentally found.

Laplace's Correction—Laplace pointed out the source of discrepancy in Newton's assumption. He argued that during the transmission of sound the change of volume of any layer of air is so quick that all the heat given out or absorbed, during a compression or a rarefaction, is wholly confined within the layer itself and so the temperature does not remain constant. This process is known as an *adiabatic transformation.* In such a case the relation between the pressure P and volume V of a gas is given by

$$PV^{\gamma} = \text{constant}$$

where $\gamma = \dfrac{\text{specific heat of the gas at constant pressure}}{\text{specific heat of the gas at constant volume}}$.

For a diatomic gas (such as oxygen, nitrogen or air) γ is 1·41 and for a monatomic gas (such as argon) γ is 1·66.

Hence when a particular layer of air undergoes an adiabatic contraction v due to an increase of pressure p, we get

$$PV^{\gamma}=\left(P+p\right)\left(V-v\right)^{\gamma}=V^{\gamma}\left(P+p\right)\left(1-\frac{v}{V}\right)^{\gamma}$$

By aid of the Binomial theorem we get

$$PV^{\gamma}=V^{\gamma}\left(P+v\right)\left[1-\gamma\frac{v}{V}+\frac{\gamma(\gamma-1)}{2}\left(\frac{v}{V}\right)^{2}+\cdots\right]$$

or

$$P=\left(P+p\right)\left(1-\gamma\frac{v}{V}\right)$$

neglecting higher powers of v/V which are very small quantities.

Thus
$$P=P-P\gamma\frac{v}{V}-\gamma.\frac{pv}{V}+p=P-P\gamma\frac{v}{V}+p,$$

or
$$\gamma P=p\Big/\frac{v}{V}=k. \qquad \ldots \qquad \ldots \qquad \ldots \qquad (17,4)$$

Returning to the original expression for the velocity, we find that

$$V_o=\sqrt{\frac{k}{\rho_o}}=\sqrt{\frac{\gamma P}{\rho_o}}=\sqrt{1·41}\times 280 \text{ metres/sec.}$$

$$=332 \text{ metres/sec. (approximately).}$$

Thus the value obtained by Laplace from adiabatic considerations agrees fairly well with experimental results.

18. Influence of Various Factors on the Velocity of Sound in Air—The following are the effects of a change of atmospheric pressure, change of temperature and humidity on the velocity of sound in air.

Effect of Pressure—Whenever there is a change of pressure on a given mass of gas, there is a corresponding change of volume and density. Let the velocities of sound under pressures P_1 and P_2 be v_1 and v_2 respectively. The ratio γ is supposed to be independent of pressure. Any change of atmospheric pressure is so sufficiently slow as not to affect the temperature and is, therefore, governed by Boyle's Law. If V_1 be an element of volume of atmospheric air at pressure P_1 and if V_2 be the volume of the same element at pressure P_2 under an isothermal condition, Boyles Law asserts that $P_1V_1=P_2V_2$.

$\therefore \frac{P_2}{P_1}=\frac{V_1}{V_2}=\frac{\rho_2}{\rho_1}$, and since $\rho_1V_1=\rho_2V_2$,

whence $\frac{P_1}{\rho_1}=\frac{P_2}{\rho_2}$. ...(18,1)

By taking square root it shows that $v_1=v_2$

Thus *the velocity is independent of a change of pressure.*

Effect of Temperature—With a change of temperature there is a change of density of the atmospheric air and consequently the velocities at differet temperatures will be different. Let V_0 and V_t be the velocities at 0°C and t°C respectively. Then

$$V_0 = \sqrt{\gamma}\sqrt{\frac{P}{\rho_0}} \text{ and } V_t = \sqrt{\gamma}\sqrt{\frac{P}{\rho_t}},$$

where ρ_0 and ρ_t are the densities of the gas at the corresponding temperatures.

But by Charle's Law $\rho_0 = \rho_t\ (1+\alpha t)$, where α = the ceefficient of cubical expansion of gas = 0·00366.

Consequently,

$$\frac{V_t}{V_0} = \frac{\rho_0}{\rho_t} = \sqrt{1+\alpha t},$$

whence $V_t = V_0\sqrt{(1+\alpha t)} = V_0(1+\frac{1}{2}\alpha t + \ldots..)$ by Binomial Theorem ...(18,2)

or, $V_t = V_0\ (1 + 0{\cdot}00183 \times t)$.

Taking $V_0 = 1100$ ft. per second at 0°C, it is found from (18,2) that for each degree centigrade rise of temperature, the velocity of sound in air increases by 2 feet per second.

***Effect of Humidity**—Since for the same temperature and pressure the density of water vapour is less than of air, the presence of moisture in some volume of air would lower the density of the mixture. The velocity of sound is inversely proportional to the square root of the density and so it would be *greater in moist air than in dry air.*

Let the velocity of sound in dry air at temperature t°C and normal pressure be V_d. Let the velocity of sound is moist air at temperature t°C and pressure P be V_m.

$$\text{Then } V_d = \sqrt{\frac{\gamma \times 760}{\rho_d}} \text{ and } V_m = \sqrt{\frac{\gamma P}{\rho_m}},$$

where ρ_d and ρ_m are the respective densities of dry air and moist air.

Let the saturation pressure of aqueous vapour at t°C be = e cm. of mercury.

Also the total pressure of the moist air = P cm. of mercury.

$\therefore$ the pressure of dry air at the same temp. = $P - e$

So ρ_m = mass of 1 c.c. of dry air at a pressure $(P-e)$
+ mass of 1 c.c. of water vapour at the pressure e.

From Boyle's Law the volume of 1 c.c. of dry air at a pressure $(P-e)$ becomes $\frac{P-e}{760}$ c.c. at normal pressure.

Hence the mass of 1 c.c. of dry air at normal pressure $= \frac{p-e}{760} \times \rho_d$

Again, the volume of 1 c.c. of aq. vapour at normal pressure $= \frac{e}{760}$

$\therefore$ mass of 1 c.c. of aq. vapour at normal press. $= \frac{e}{760} \times \rho_d \times 0{\cdot}625$,

since the density of water vapour (relative to dry air) is 0·625.

$$\text{Thus } \rho_m = \frac{P-e}{670}\rho_d + \frac{e}{760} \times \rho_d \times 0{\cdot}625$$

$$= \frac{\rho_d}{760}\left(P - e + 0{\cdot}625 \times e\right) = \frac{\rho_d}{760}\left(P - 0{\cdot}375 \times e\right).$$

$$\text{Hence } \frac{V_d}{V_m} = \sqrt{\frac{760\rho_m}{P\rho_d}} = \sqrt{\frac{P-(0{\cdot}375 \times e)\rho_d}{P \times \rho_d}},$$

$$\text{so that } V_d = V_m \times \sqrt{1 - 0{\cdot}375 \times \frac{e}{P}} \quad \ldots \quad \ldots(18{,}3)$$

Examples :

1. An echo from a cliff is heard 5 seconds after the sound is made. If the temperature of the air is 15°C, how far away is the cliff? The velocity of sound at 0°C is 1090 feet per second.

Ans. The velocity of sound at $0°C = V_o = 1090$ ft./sec. Hence the velocity of sound at 15°C, (say V_t) is given by the equation.

$V_t = V_o\ (1 + {\cdot}00183 \times t) = 1090\ (1 + {\cdot}00183 \times 15)$ ft./sec. = 1119·1 ft./sec.

Let the distance of the cliff from the observer be d ft. Then for an echo to come back to the observer, the distance travelled by sound is $2d$ in a period of 5 secs.

Thus $2d = 1119{\cdot}1 \times 5$, whence the desired distance $d = 2599{\cdot}5$ ft.

2. An observer sets his watch by the sound of a signal gun fired at a distant tower. Yet he finds that his watch is slow by 5 seconds. Find the distance of the tower from the observer. The temperature of air during the observation is 20°C and the velocity of sound in air at 0°C is 332 metres per sec. (Utk. U.—1953)

Ans. Velocity of sound at 0°C = 332 m./sec. Hence the velocity of sound at 20°C is = $(1 - {\cdot}00183 \times 20)$ m./sec. = 344·15 m./sec.

If the distance of the tower be d metres, the time for the sound to travel from the tower to the observer is $= \frac{d}{344{\cdot}15}$ sec. The time is given to be 5 sec., and thereby gives rise to the equation;

$\frac{d}{344{\cdot}15} = 5$, whence the required distance $d = 1720{\cdot}75$ metres.

3. The thunder accompanying a lightning is heard 6 seconds later than the flash. Assuming the temperature of air to be 27°C, calculate the distance at which the lightning must have occurred. (Velocity of sound in air at 0°C = 332·0 metres per second).

Ans. In relation to the velocity of sound, the velocity of light may be taken to be so great the flash may be considered to have crossed the distance in practically no time. Consequently the duration of the passage of sound is 6 seconds.

The velocity of sound at 0°C = 331·3 metres per second. Hence the velocity of sound (say, V_1) at 27°C is given by

$V_1 = \{331{\cdot}3(1 + {\cdot}00183 \times 27)\}$ m./sec. = 347·67 metres/sec.

∴ lightning occurred at a distance
$=(347{\cdot}67\times6)$ m. $=2086{\cdot}02$ m.

4. In an experiment for the determination of the velocity of sound in a well the value was found to be 345·3 metres at 20°C under a pressure of 74·8 cm. of mercury. What is the true value of the velocity at N. T. P.?

Ans. The saturated pressure of aqueous vapour at 20°C as obtained from any vapour-pressure table = 17·4 mm. Hence, by 18·5 at the same temperature,

$$V_a=345{\cdot}3\times\sqrt{\left(1-\frac{{\cdot}375\times1{\cdot}74}{75{\cdot}8}\right)}=343{\cdot}8 \text{ m.}$$

If the required velocity be V_0 metres per second, we get $V_0(1+{\cdot}00183\times20)=343{\cdot}8$, whence $V_0=331{\cdot}6$.

***19. Velocity of Sound in Water**—The experimental determination of the velocity of sound in water was made by Colladon and Sturm in 1825. The experiments were carried out at night. Two boats were moored at a definite distance in the Lake of Geneva. One boat was provided with a bell which lay inside water. The bell could be sounded with a hammer through a lever, the motion of which fired some gunpowder. The other boat had a larger funnel-shaped ear trumpet, the lower end of which was closed by a flexible membrane and was immersed in water, while the upper end was applied to the observer's ear. The time taken by the sound to travel a known distance in water was thus obtained. It was found that the mean velocity of sound in water at 8·1°C was 1437 metres per second. These experiments are of historic interest nowadays.

Theoretical discussion—The velocity of sound in any liquid is given by the expression :

$V=\sqrt{\dfrac{k}{\rho}}$, when k is the volume elasticity or bulk modulus of that liquid. In the case of water at 15°C, $k=2{\cdot}23\times10^{10}$ dynes/sq. cm. and $\rho=1$ gm./cc.

Hence the velocity of sound in water at 15°C is

$$=\sqrt{\frac{2{\cdot}23\times10^{10}}{1}}\text{ cm./sec,}=1500\text{ m./sec.}$$

***20. Velocity of Sound in a Solid**—The velocity of sound in a long cast iron bar was determined by Biot (1774—1862). By joining several pipes end-to-end, he obtained a total length of 915 metres. A bell fixed at one end was rung At the other end two sounds were heard, one travelling through the material of the pipe, the other through the air. The interval between the two sounds was accurately recorded. The velocity of sound in air being known, that in the solid could be calculated. The velocity was found to be approximately 5300 metres per second. When a solid is available in the form of *string*, the velocity of disturbance can be experimentally determined in the laboratory (Vide Art. 42). The velocity of compression through a *uniform* rod can be determined in the laboratory by Kundt's Tube Method (Art. 54).

Theoretical investigation—A compressional disturbance conveying sound travels through a solid with a velocity given by

$$V=\sqrt{\frac{y}{\rho}},$$

where y = Young's modulus for that material.

For steel, $y=2\cdot14\times10^{12}$ dynes/sq. cm. and $\rho=7\cdot6$ gm./c c,.

$$\text{the velocity of sound through steel}=\sqrt{\frac{2\cdot14\times10^{12}}{7\cdot0}}\ \text{cm./sec.}$$

$$=5300\ \text{m./sec.}$$

*21. Velocity of Sound in different Gases

—The velocities of sound in two gaseous media can be conveniently compared by Kundt's Tube Method.

Under the same condition of pressure and temperature let the velocities of sound in air and in another diatomic gas be V_a and V_b,

$$\text{Then}\quad V_a=\sqrt{\frac{\gamma P}{\rho_a}}\ \text{and}\ V_b=\sqrt{\frac{\gamma P}{\rho_b}}.$$

$$\therefore\quad \frac{V_a}{V_b}=\sqrt{\frac{\rho_b}{\rho_a}} \qquad \text{...(21, 4)}$$

whence V_a can be determined, if the other three factors be known. It is evident from (21, 1) that the velocities of sound in different gases are inversely proportional to the square roots of their densities.

Examples :

1. Calculate the velocity of sound in hydrogen gas, assuming its velocity in air to be 332 metres per sec. and having also given that 1 litre of hydrogen weighs 0·089 gm and a litre of air 1·298 gm.

Ans. If ρa and ρh denote the densities of air and hydrogen respectively, usual notations give

$$\frac{Vh}{Va}=\sqrt{\frac{\rho a}{\rho a}}=\sqrt{\frac{1\cdot298}{0\cdot089}}=\sqrt{14\cdot3}=3\cdot8,$$

whence $V_h=Va\times3\cdot8=332\times3\cdot8$ m./sec. $=1261\cdot6$ m./sec.

2. An explosive percussion signal on a rail is set off by a locomotive passing over it. A listener, 1 kilometer away, with one ear on the rail, hears two reports. Explain the phenomenon and calculate the time interval between the two sounds. Given y for steel $=2\times10^{12}$ dynes/cm.^{2}; ρ of steel $=7\cdot8$ gm./cm.^{3}, ρ of air $=\cdot0013$ gm./cm.^{3}, γ for air $=1\cdot4$; $P=10^6$ dynes/cm.^{2}. [Gau. U.—1953]

Ans. With the ear on the rail the listener gets the sound of the signal passing through the rail and with the other ear in air he gets another sound passing through air. Now the velocity of sound in the two media being different the two reports are heard at different instants.

Now the velocity of sound through steel rail $=V_1=\sqrt{\dfrac{y}{\rho}}=\sqrt{\dfrac{2\times10^{12}}{7\cdot8}}$ cm./sec.

$$=506400\ \text{cm./sec.}$$

$\therefore$ the time t_1 for the sound to travel 1 km. (10^5 cm.) in steel is given by

$$t_1=\frac{10^5}{506400}\ \text{sec.}=0\cdot2\ \text{sec.}$$

Again, the velocity of sound in air $= V_1 = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1{\cdot}4 \times 10^5}{{\cdot}0014}} = 32816$ cm/sec.

$\therefore$ the time t_2 for the sound to travel 1 km. in air is found from:

$$t_2 = \frac{10^5}{32816} \text{ secs.} = 3 \text{ secs.}$$

Hence the required time interval $= (3 - 0{\cdot}2)$ secs. $= 2{\cdot}8$ secs.

Velocity of Sound in Common Media

Media	Metres/Sec.	Media	Metres/Sec.
GASES N. T. P. :—		SOLID :—	
Air (dry)	331 7	copper	3600
,, at t°C	331 7 + 0·54 × t	iron	5000
carbon dioxide	259	steel	5100
hydrogen	1262	glass	5000
oxygen	316	wood	3090-4900
LIQUID :—		nickel	4900
water at 20°C	157	India Rubber	30-60
,, t°C	190 + 33 × t		

22. Rectilinear Propagation of Sound—It has already been stated that a vibrating source in a homogeneous gaseous medium generates spherical waves of compression and rarefaction which expand in quick successions with a definite velocity. Imagine the source to be placed at a point S (Fig. 27) An element of a wave surface travelling in the direction SA reaches a point r_2 after a short interval of time. The same disturbance reaches a point A_2 an instant later, where the points S, r_2 and A_3 are collinear. For a similar consideration, another element of a wave surface, moving from S in the direction SA_2, will pass through all the points lying in that straight line. It is in this sense that the propagation of a sound wave is said to be *rectilinear*. The same argument holds in the case of the propagation of light waves. Hence, as in light, the track of an element on the wave-front is called a *ray*. So any straight line Sr_2A_3, along which an element of a sound wave is propagated, may be termed a *sound ray*.

We know that when some opaque object is placed in the path of a beam of light, a shadow of the same shape is cast upon a screen placed behind, which fact verifies the rectilinear propagation of light. But when an obstacle (*e.g.*, a piece of wood) is placed at r_2 in the path of sound (Fig. 27), an ear placed at A_2 would hear the sound from S without much obstruction. Thus to some extent sound rays bend round a barrier.

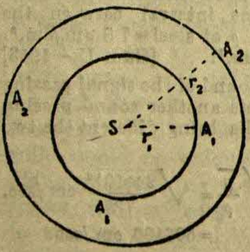


Fig. 27

The reason for this difference in propagation lies in the fact that the amount of bending of rays depends upon the wave-length. The longer the wave-length, the greater the bending round any obstacle. The wave-length of audible sound varies between 1·5 cm. and 11 metres. For this range of wave-length the bending is appreciable. An acute sound, which is of shorter wave-length, obeys the laws of rectilinear propagation more closely than a grave one. A watch is often taken as a source of sound in different experiments on reflections and refractions, since its note is of short wave-length.

23. Reflection of Sound - When a sound wave travelling in a medium reaches the surface of separation of another medium, a part of the incident wave is reflected back into the original medium and the remainder is partly absorbed and partly transmitted into the second. Partial reflection may also take place at the surface of separation of the two portions of the same medium having unequal densities The reflection of sound takes place according to two definite laws.

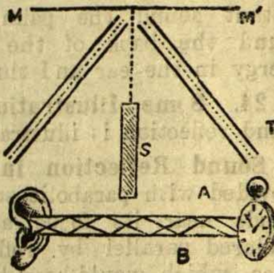


Fig. 28

Reflection at a Plane Surface—In front of a vertical wooden board fix up a straight hollow tube at a definite angle with the plane of the board (Fig. 28). Determine the point where the axis of the tube meets the board. Take another similar tube fitted at one end with a hollow rubber tubing. Insert the end of the rubber tubing in the ear so as to receive the sound. Now rotate the receiving tube in all positions till the sound heard appears maximum. Fix the tube in this position with suitable clamps.

It will be noticed

(i) that the axes of the two tubes together with the normal at the point of the board, where the axes meet, approximately lie in one plane and (ii) that the axes make equal angles with the normal.

In large halls the voice of a speaker becomes indistinct owing to reverberations from the walls and the ceiling which are commonly known as the *howling*. This can be minimised by hanging up curtains, the effect of which is to absorb partly the incident sound energy. The same purpose is also served to some extent, when the hall is packed up with audience.

Reflection at a Concave Surface—When the reflecting surface is concave, the effect of reflection is more pronounced.

Take two large concave mirrors of polished wood or metal and place them coaxially facing each other at a fair distance on a table (Fig. 29). Measure the radius of curvature of one mirror with a spherometer and place a ticking watch at a distance of half the radius from the pole of the mirror.

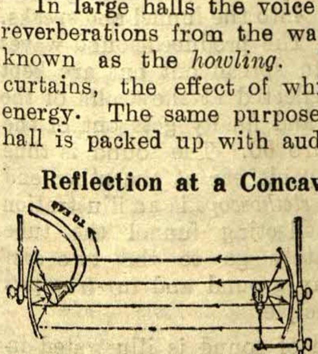


Fig. 29

Evidently, the source of sound is at the principal focus of the mirror. Move a receiver of sound along the axis and place it at a point where you get the best possible sound. The point is found to coincide approximately with the principal focus of the second mirror.

On being reflected from the first mirror, rays of sound are rendered parallel. The group of parallel rays is incident upon the second mirror and is finally made to converge to the principal focus of the second mirror. It is a matter of common experience that to hear a distant sound the palm of the hand is often put in a curved way round the back of the ear. This helps to concentrate the sound energy in the ear and thus make it more audible.

24. Some Illustrations of Sound Reflection—The principle of sound reflection is illustrated in the following cases :

Sound Reflection in an Auditorium—Some very big halls are provided with parabolic sound reflectors behind the dias. The speaker stands near its focus and the rays of sound due to his voice are rendered parallel by reflection at the parabolic surface. Thus his voice, which would have been otherwise inaudible at a distance, passes over to the audience to the extreme end of the hall and is made audible there. The old Calcutta University Senate House had got a reflector of this kind. Unfortunately the building has been dismantled. For concentration of sound, many churches and show houses have the back walls suitably curved.

Speaking tubes—In some large building and steam ships *speaking tubes* are often installed. In its essential form such a tube consists of a metal pipe of small diameter connected at one end with a funnel-shaped piece called the *mouth-piece*. The speaker puts his mouth into the funnel and speaks. The produced sound waves entering the mouth-piece are not allowed to spread out but are kept confined within the tube. *Successively* reflected at the walls of the tube, the sound travels down the tube and finally gets out of the other end in a manner shown in figure 30. The sound is thus concentrated and so the hearer on placing his ear at the other end can distinctly hear the speech. Physician's *stethoscope* is an illustration of this principle. A little above the collecting funnel the tube bifurcates into two branches which separately go to the two ear. Speaking horns or *megaphones* used on play ground and *ear-trumpets* used by deaf people work on the same principle.

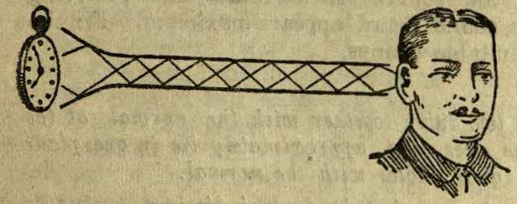


Fig. 30—Successive reflections in a tube

Echoes—A familiar example of reflection of sound is illustrated in the formation of **echoes**. An echo is repetition of the speaker's own words, caused by reflection at a distant surface, *e.g.*, a row of buildings, trees or any other extended surface.

In order that two very sharp sounds may be distinguished from one another, the interval of reception between the two should be, on an average, about $\frac{1}{10}$ of a second, because the sensation of sound persists in our brain for about that period. This interval is known as the period of *persistence for the sensation of sound*. When a sound is made before a reflector, an echo of it would be distinguishable, provided that the interval between the cessation of sound and the reception of the reflected sound is $\frac{1}{10}$th of a second. During this period the sound travels a distance of 112 feet approximately, so the minimum distance of the reflector for the reception of an echo for any sharp sound is about 560 feet. But for an echo of words the interval is different. It is an observed fact that a person can on an average utter 5 syllables per second. Thus to have an echo in which a mono-syllablic word is to be distinctly heard, the time interval should be $\frac{1}{5}$th second and during this period sound traverses a distance of about 224 feet. So the distance of the reflecting wall should be 122 feet from the observer for the reception of *mono-syllablic echo.* If the distance of the reflector be twice, etc., of 112 ft., disyllablic or trysylleblic echoes would be distincly heard.

When there are two opposite reflecting surfaces at a proper distance (*e.g.*, two parallel walls), repeated reflections take place and successive echoes are heard. When there are several surfaces at suitable distances, a number of reflections takes place and several echoes are produced. When successive echoes occur within $\frac{1}{10}$th of a second, they cannot be distinguished separately and a long drawn sound is heard. The rolling of thunder is due to successive reflections of the original report from surfaces of clouds at different heights or unequally heated layers of air.

Examples :

1. A man stationed between two parallel cliffs fires a gun. He hears the first echo after two seconds and the next after five seconds. What is his position between the cliffs and when will he hear the third echo ? (Velocity of sound in air=1120 ft./sec.)

Ans. Let the velocity of sound in air be V cm. per sec., the distance of the man from one cliff d and that from the other l cm.

As the first echo is heard after 2 seconds, $2d/V=2$, or $d=V$. When the direct sound is reflected from the second cliff, the second echo is produced. As the second echo is heard after 5 seconds, $2l/V=5$, whence $l=\frac{5}{2}V$.

Hence $d : l=2 : 5$. Thus the position of the observer divides the distance between the cliffs in the ratio of 2 : 5.

From this position either of the sound waves on reflection from the opposite walls reaches the man and the third echo is heard. As the time necessary for this is 7 secs., the third echo will be heard 7 secs. after the firing of the gun.

2. A sharp sound is made in the neighbourhood of an extended wall and the echo is heard 1·6 seconds after. On proceeding towards the wall through a distance of 33 metres the echo is heard after 1·4 seconds. Find the distance of the wall and the velocity of sound.

Ans. Let the distance of the wall from the first position be d metres and the velocity of sound be V metres per second.

Then $2d/V=1.6$ or $1.6\times V=2d$. ... (1)
Also $2(d-33)/V=1.4$ or $0.7\times V=d-33$. ... (2)

On solving (1) and (2) the required velocity is 330 metres/sec. and the distance is 204 metres.

***25. Nature of the Reflected Wave**—It has been already stated that a sound wave may be reflected at the surface of separation of two media or of two layers of the same medium at different densities. In reflection, therefore, a sound pulse may meet a more or less dense reflecting surface according as it travels from a rarer medium to a denser one or *vice versa.*

Reflection at a Rigid Surface—When a pulse of condensation travelling in a medium strikes against a rigid wall, the latter does not move on easily and so due to reaction the pulse comes back with its form unchanged, *i.e.*, as a *pulse of condensation.* Hence *the form of a wave remains the same at reflection at a rigid wall.* This is equally true for a transverse wave,—the crest coming back as a crest and the trough as a trough. If one end of spiral is a fixed to a piece of wood placed on a suitable stand and if a pulse of compression is produced by suddenly pushing the spiral at the other end, it is noticed that the reflected pulse also is a compression. For a similar consideration a pulse of rarefaction due to reflection at a rigid wall comes back as a pulse of rarefaction.

Reflection of an Yielding Surface—When, on the other hand, a pulse of compression meets the surface of a rarer medium, the surface of separation is disturbed by the motion. The surface is pushed a little and produces a partial rarefaction This rarefaction travels in the opposite direction, so that the original pulse of compression is reflected as a *pulse of rarefaction.* Thus *due to such a reflection a change of type of the pulse is brought a out.* In the experiment on the motion of compressional wave in a copper spiral, if both ends of the spiral are free, a pulse of condensation travelling to the other end comes back as a pulse of rarefaction. Similarly a rarefaction is found to return as compression. This is called a phase change of π in reflection.

26. Refraction of Sound—When a sound-wave travelling in one medium meets the surface of separation of another, a portion of it is transmitted into the second medium, approximately obeying the usual laws of refraction. The refraction of sound may be demonstrated by taking a lens-shaped bag of fine rubber, containing some gas denser than air (e.g., carbon dioxide).

Hold a convex gas lens by a suitable stand and determine its axis. Somewhere on the axis of the lens place a watch on one side of the lens.

Determine the point on the other side, where the maximum sound is heard in a receiver. Find the object distance u and the image distance v from the lens. Take a number of readings for u and v. It is found that in each case

$$\frac{1}{v}+\frac{1}{u}=k=\frac{1}{f},$$ where f stands for the focal length of the lens.

27. Some Phenomena due to Sound Refraction—The following phenomena concerning the refraction of sound are explained :—

Bending of Rays in a Temperature Gradient—During cold winter nights the air in contact with the surface of the earth is coldest and so it has got the maximum density. The upper regions are less and less dense. When sound is produced near the surface, rays start out in all directions. Moving upwards in an oblique direction, some of the rays meet layers of air of less and less densities. They are, therefore, gradually bent away from the normal, till at a stage total internal reflection takes place. The rays then bend downdards and finally return to the surface of the earth. This is akin to the phenomenon of optical mirage. The reverse effect takes place during a hot summer noon. As the sound waves pass from rarer to denser layers, the rays get gradually bent towards the normal ; they, therefore, continually move upwards and never come back to the earth. This explains the fact of the common saying that *sound travels a longer distance in night than in day.*

Bending of Rays due to Wind—If there were no wind, a sound wave-front Pa_0 [Fig. 31 (A)], during propagation would occupy the successive dotted positions 1, 2, 3, etc., after equal intervals of time. When wind blows in the direction of the sound the velocity of wind in upper layers is greater than that nearer the ground, and the upper layers move with greater speed than the lower ones. Consequently, each vertical column of air during a strong wind moves through an appreciably greater distance at the top than at the bottom, so that it is gradually bent downwards as shown by 1′, 2′, 3′. Since the direction of each ray is normal to the column, the rays will gradually bend downwards causing concentration of sound near the surface of the earth.

If, on the other hand, the wind is blowing against the sound, its velocity is to be subtracted, so that the velocity of sound gradually decreases upwards. Hence the columns would be gradually bent in the reverse direction [Fig. 31, (B)] and the rays go up into the sky. Hence sound travels *a longer distance over the surface of the earth in the direction of wind than against it.*

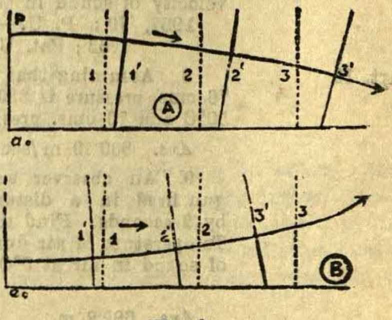


Fig. 31

EXERCISES ON CHAPTER III

Reference

1. A stone is dropped into a well 240 feet deep and the sound's impact is heard 4·1 seconds later. Find the velocity of sound ($g=32$ ft./sec.^{2}). Art. 15

Ans. 1044 ft./sec.

2. Calculate the velcoity of sound in air on a day when the barometer reads 760 millimetres. Given the density of air Art. 18

Reference

= ·0001293 gm./c.c. and $\gamma = 1·41$. (Nag. U.—1963 ; R. P. B.—1971)

Ans. 332·5 m./sec.

Art. 15 3. Explain fully how the velocity of sound in open air has been determined. (U. P. B.—1963 , B. H. U.—1971)

Art. 18 4. If the velocity of sound in air at 0°C and 76 cms. of mercury pressure is 330 metres per second, calculate the velocity at 29°C and 74 cms. pressure. (Coefficient of expansion of air = 0·0086651). (Nag. U.—1971 ;

Ans. 346·3 m.|per sec.

Art. 18 5. If the velocity of sound in dry air at 0°C be 332 metres/sec., calculate the temperature at which the velocity would be 345 metres/sec. (Coefficient of expansion of air = 1/273) [Utk. U.—1969]

Ans. 21·3°8C.

Art. 24 6. Standing between two parallel cliffs, a man fires a rifle. He hears a first echo after $1\frac{1}{2}$ secs., then a second $2\frac{1}{2}$ sec. after the shot, and then a third echo. Explain how these three echoes are produced. Calculate how many seconds elapsed between the shot and the third echo, and calculate the distance between the two cliffs.

Ans. Taking $v = 1120$ ft. per sec., $t = 4$ secs., and $d = 2240$ ft.

Art. 18 7. Discuss the effects of temperature and pressure on the velocity of sound in gases. (V. U.—1962 ; Utk. U.—1962, '70 ; P. U.—1959, '72 ; U. P. B.—1961, '73 ; C. U.—1963 ; Pat. U.—1971 ; Nag. U.—1971 ; Raj. U.—1963)

Art. 18 8. Assuming that the velocity of sound in air at 0°C and 76 cms. pressure is 330 metres per second, find its velocity at 50°C and 70 cms. pressure.

Ans. 360·19 m./sec.

9. An observer sets his watch by the sound of signal gun fired in a distant tower. He find that his watch is slow by 2 seconds. Find the distance of the tower from the observer. Temperature of air during observation is 13°C and the velocity of sound in air at 0°C is 332 m./sec. (Gau. U.—1961 ; Pat. U.—1959)

Ans. 682·2 m.

Arts. 17 & 18 10. Give Newton's formula for the velocity of sound. What correction has been suggested by Laplace, and why ? (Bom. U.—1961, '72 ; Del. U.—1968 ; P. U.—1967, '69, V. U.—1954 ; C. U.—1957 ; E. P. U.—1952 ; Nag. U.—1933 ; U. P. B.—1951, '65 ; *Cf.* Guj. U.—1962, '64)

Art. 17 11. Explain how speed of sound has been determined in water. (U. P. B.—1962 ; E. P. U.—1963)

Arts. 19 & 20 12. How would you show that sound travels faster in air than in carbon dioxide and slower in air than in iron ? (Utk. U.—1962)

Art. 17 13. The mass of 1 litre of air at N. T. P. is 1·293 gm. Calculate the velocity of sound in air at N. T. P., it being given that the ratio of the specific heats of air is 1·41.

Ans. 332 m./sec. (approx.)

Reference

14. How do the velocities of sound in different gases depend on densities? Calculate the velocity of sound in hydrogen at 130°C given that at 0°C and 76 cms. of mercury 11·2 litres of hydrogen weigh 1 gm. (Velocity at N. T. P. in air = 332 m./sec.). Art. 17

Ans. 1585 m./sec.

15. How would you show that sound waves get reflected and that the waves obey the law that the angle of incidence is equal to the angle of reflection? Explain how the formation of an echo and a physician's stethoscope are due to the reflection of sound waves. (Del. H. S.—1964) Arts. 23 &25

16. Explain the production of echo. An echo repeated six syllables. The velocity of sound is 1120 ft. per sec. What was the distance of the reflection surface? (C.U.—1960) Art. 24

Ans. 672 ft.

17. A whistle is blown by a ship while approaching a cliff and an echo is heard after 8 seconds. Five minutes after the first whistle, the whistle is blown again and the echo is heard after 3 seconds. Find the distance of the cliff from the latter position of the ship and find the velocity of the ship in a direction normal to the cliff. (Velocity of sound in air = 1120 ft./sec.) Art. 24

Ans. $169\frac{3}{17}$ ft. from the position of second whistling; $9\frac{7}{16}$ per sec.

18. Explain: A brick wall reflects wave of sound but not waves of light, whereas a small plane mirror will reflect waves of light but not of sound. (Gau. U.—1952) Art. 23

CHAPTER IV

SUPERPOSITION OF SOUND

28. Superposition of Waves—When two systems of waves pass simultaneously through some point in a medium at any instant each system would tend to produce its own displacement at that point. As a result, the point would have a displacement which is the resultant of the two component waves. This is known as the *principle of superposition* of waves.

In order to find the nature of vibration of a particle due to superposition the following graphical method is utilised. Let the two dotted curves (Fig. 33) denote the time-displacement graph of the two waves at *any* point in their common path. The abscissa denote the time, and ordinate the corresponding displacement. At any time corresponding to P the displacement due to one component wave is found to be PL and that due to the other is PL′. The two have a resultant displacements PC. If, on the other hand, the

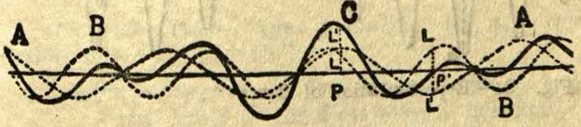


Fig. 33—Superposition of waves

displacements at any time are oppositely directed, their difference gives the resultant displacement. In this manner final displacements are obtained from point to point on the time axis and a thick curve is plotted out, which represents the resultant displacement of a particle with time. For the type of the superposition as shown in Fig. 33 its amplitude of vibration is found to be minimum at an instant corresponding to B and maximum near C.

29. Beats—When two sources of nearly equal frequencies and of moderate intensities are sounded together, the resultant displacement at any point due to the superposition periodically fluctuates, thus causing waxing and waning in the loudness of the resultant sound. This phenomenon of periodic waxing and waning of the sound is known as **beats.**

Let the two curves A_1A_2 and B_1B_2 (Fig. 34) represent the displacement graphs for the two component waves at any point. At a particular instant as at D of the first curve and at E of the second, they are in the opposite phase of vibration. The resultant amplitude at any point is the difference of the two component amplitudes as given by the point F of the curve [Fig. 34, (iii)]. It will be shown subsequently that the loudness of a sound depends upon the amplitude of vibration of the source and consequently on the ordinate of the displacement curve at a point. So that at the instant denoted by F the sound heard is minimum. As time advances, the component curves gradually come in steps, since their wave-lengths

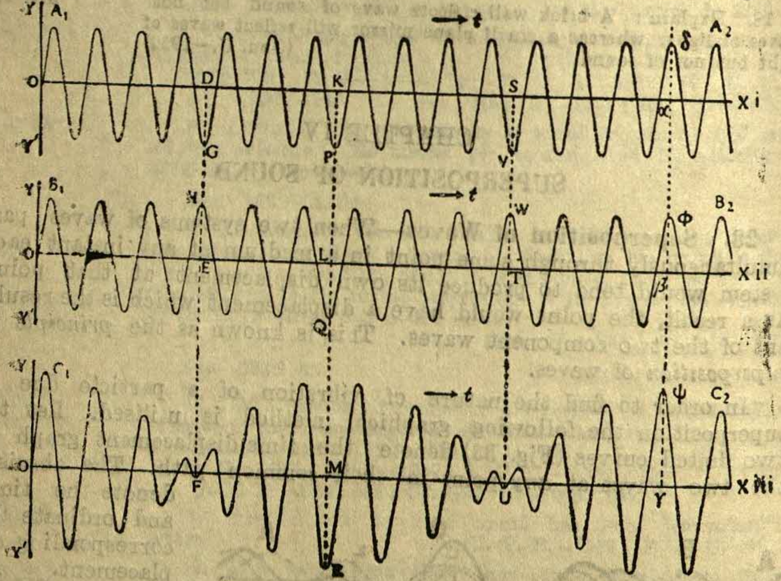


Fig. 34—Representation of Beats

are not equal. At K or L (fig i & ii) the phase difference between the component vibrations is zero; and the resultant amplitude at M is a maximum, being the sum MR of the components At this instant, the sound heard is maximum. Again, it decreases and becomes a minimum near U, when the phase difference is zero, *i,e.*, when one gains over the other by a complete vibration. It is, therefore, evident from the graph that there will be a periodic fluctuation in the loudness of the sound heard. During the interval between F and U, one of the components has gained over the other by one complete vibration and therefore one beat occurs. The number of beats heard in one second will therefore, be equal to the number of times the interval FU or Mγ occurs in one second.

To take an illustration, let two tuning forks, having frequencies 280 and 284, be sounded together and placed on a table. Each vibrating fork has got a displacement curve. The two forks start to vibrate together in the same phase as at M (Fig. 34). After a quarter of a second, the slower one completes 70 vibrations and the faster one 71. This state is represented at γ During this interval one beat is heard. The number of beats per second is therefore, 4 which is the *difference* between the vibration frequencies of the two forks. Similarly, it is found that in every case *the number of beats produced by two sources is equal to the difference of their vibration frequencies.* Under proper conditions beats can be generated by various sources. Thus two violin strings, which are nearly turned with each other, or two consecutive reeds of harmonium, when sounded together, can produce beats. *The only necessary condition is that the two sources should produce notes similar in nature and of* nearly *equal frequencies.*

Beat tone—When the number of beats per second is more than 10, they cannot be separately distinguished owing to the persistence of the sensation of sound. The beats then produce a continuous note, called the *beat tone.*

***Mathematical Analysis of Beats**—Let n_1 and n_2 be the frequencies of two sources. On the simplest assumption that they start with the same phase, the individual displacements at a point at an instant due to the two wave systems accrding to Art. 11 are given by

$$y_1 = a \sin 2\pi n_1 t \text{ and } y_2 = b \sin 2\pi n_2 t.$$

If n_2 is greater than n_1 by n, then $n_2 - n_1 = n$,

or, $y_2 = b \sin 2\pi (n_1 + n)t$.

Therefore from the principle of superposition, the resultant displacement y is given by

$$y = y_1 + y_2 = a \sin 2\pi n_1 t + b \sin 2\pi (n_1 + n)t$$
$$= \sin 2\pi n_1 t\,(b \cos 2\pi n t + a) + b \cos 2\pi n_1 t \sin 2\pi n t \quad \ldots(29{,}1)$$

Equation (29,1) represents another wave equation. Rewrite it in the form—

$$y = F \sin (2\pi n_1 t + \rho) \quad \ldots(29{,}2)$$

where F is its amplitude and ρ is the epoch of the resultant wave.

To find the values of F and ρ, we get on expansion

$$y = F \sin 2\pi n_1 t \cos \rho + F \cos 2\pi n_1 t \sin \rho \quad .. (29,3)$$

Now comparing (29,1) and (29,3) we get on equating the coefficients of sin $2\pi n_1 t_1$ and cos $2\pi nt$.

Accordingly, $F \sin \rho = b \sin 2\pi nt$

and $F \cos \rho = b \cos 2\pi nt + a.$

On squaring and adding, we get

$$F^2 = b^2 \sin^2 2\pi nt + b^2 \cos^2 2\pi nt + 2ab \cos 2\pi nt + a^2.$$

or $$F^2 = a^2 + b^2 + 2ab \cos 2\pi nt. \quad ...(29,4)$$

Also $$\tan \rho = \frac{b \sin 2\pi nt}{b \cos 2\pi nt + a} \quad .. (29,5)$$

Equation (29,4) shows that as t changes, the amplitude F of the resultant displacement assumes maximum and minimum values.

Thus when $t = 0$, $\cos 2\pi nt = 1$ and $F = a + b$, which is a maximum

,, $t = 1/2n$, $\cos 2\pi nt = -1$ and $F = a - b$, ,, minimum

,, $t = 1/n$, $\cos 2\pi nt = 1$ and $F = a + b$, ,, maximum

Hence between any two maximum amplitudes, there is one minimum present, *i.e.*, between two maximum sounds a minimum sound occurs at an interval of $1/n$ second. In similar way, it can be proved that between any two consecutive minimum sounds, a maximum occurs in an interval $1/n$ sec. So the number of beats per second $= n = n_1 - n =$ the difference of the two frequencies.

From equations (29,2) and (29,5) it is seen that the phase of the resultant wave is $2\pi n_1 t + \rho$, where the value of ρ also depends on time. Hence the frequency of the resultant wave is not actually n_1, but lies at some intermediate value between n_2 and n_1. So the fluctuating sound heard would have a pitch lying between the pitches of the component sounds. To take a simple case, if in the above illustration the amplitudes a and b assumed equal, the frequency of the resultant component wave can be shown to be the mean of n_2 and n_1.

Determination of Frequency by Counting Beats—Take two tuning forks of nearly equal frequencies n_1 and n_2, one of them (say, n_1) being known. Strike the forks and place them together on the table. Beats would be produced. Count the number of beats heard. If the number of beats be n, the unknown frequency (n_2) is either $n_1 + n$ or $n_1 - n$.

Now fix up a little wax (or a sliding weight) at the head of any prong of the tuning fork of frequency n_1. *By adding* the *load its frequency is lowered.* Now strike both the tuning forks and place them together on the table. Beats would again be heard. Find the number of beats produced. If the number be decreased, the unknown frequency n_2 is $n_1 - n$. But if it increases n_2 is $n_1 + n$.

***30. Interference of Sound**—When two sources of sound have got the same frequency and of a similar wave form, the displacement graphs are identical in nature. When two such similar systems of waves travelling almost along the same direction superpose on each other, the phase difference between the two systems of waves *at any point* depends upon the *distances of the sources* from that point, but it does not depened on time.

Let A and B be two exactly identical sources of sound emitting similar sound waves given by the equation:

$$b \sin \omega t = b \sin \frac{2\pi}{\lambda} vt. \qquad \text{.. (30,1)}$$

Let P be the point where the two wave systems superpose and let the distances of P from A and B be r_1 and r_2 (Fig. 35). The amplitude of the wave systems reaching P may be supposed sensibly equal and let the amplitude reaching P be a. Now if x_1 and x_2 be the displacements contributed by the waves at P at an instant t, then by virtue of equation (11,2) we may write

$$x_1 = a \sin\frac{2\pi}{\lambda}(vt - r_1) \text{ and } x_2 = a \sin\frac{2\pi}{\lambda}(vt - r_2).$$

Then the resultant displacement X contributed by both the waves is given by

$$X = x_1 + x_2 = a \sin\frac{2\pi}{\lambda}(vt - r_1) + a \sin\frac{2\pi}{\lambda}(vt - r_2)$$

$$\text{or} \quad X = \cos\frac{\pi}{\lambda}(r_1 - r_2) \sin\frac{2\pi}{\lambda}\left(vt - \frac{r_1 + r_2}{2}\right) \qquad \text{...(30,2)}$$

This shows that due to the superposition of two similar waves a displacement X is obtained at an instant t, and that this resultant displacement is another wave equation whose amplitude is

$$2a \cos\frac{\pi}{\lambda}(r_1 - r_2) \qquad \text{...(30,3)}$$

and whose phase is equal to

$\frac{2\pi}{\lambda}\left(vt - \frac{r_1 + r_2}{2}\right)$ at the instant t.

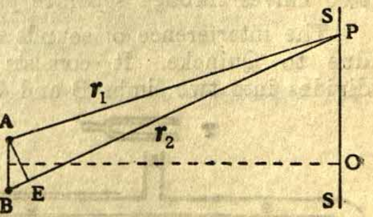


Fig. 35

The intensity or loudness of sound is proportional to the square of the amplitude of the resultant wave. If I be the intensity of sound,

$$\text{Then, } I \propto 4a^2 \cos^2\frac{\pi}{\lambda}(r_1 - r_2) \propto 4a^2 \cos^2\phi, \text{ say.} \qquad \text{...(30,4)}$$

We take note of the following two cases:

Case I—Let $\phi = 0, \pi, 2\pi, 3\pi, 4\pi \ldots$ *i.e.*, $\phi = n\pi$, when $n = 0, 1, 2, \ldots$, etc.

Then $\cos^2\phi = 1$ and $I \propto 4a^2$. Maximum loudless is then heard. Now for $\phi = n\pi$.

$$\frac{\pi}{\lambda}(r_1 - r_2) = n\pi, \text{ whence } r_1 - r_2 = n\lambda \qquad \text{...(30,5)}$$

Equation (30,5) shows that when $r_1 - r_2 = n\lambda$, *i.e.*, when the path difference between the sources of sound at any point P is a multiple of the wave-length of the sound, the sound heard is a maximum. This is called *coherent constructive superposition.*

Case II—Let $\phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$; *i.e.*, $\phi = (2n+1)\frac{\pi}{2}$, where $n = 0, 1, 2, \ldots$, etc.

Then $\cos\phi = 0$ and $I = 0$. No sound is heard in such cases.

Thus for $\phi = (2n+1)\frac{\pi}{2}$,

$$\frac{\pi}{\lambda}(r_1 - r_2) = (2n+1)\frac{\pi}{2}, \text{ whence } r_1 - r_2 = (2n+1)\frac{\lambda}{2}. \qquad \ldots(30,6)$$

Equation (30,6) gives the condition of no sound by superposition. This is called *coherent destructive interference.*

Experimental demonstration—The *conditions* for the interference of two sounds are :—(*i*) the sources must have the same frequency and amplitude, (*ii*) the nature of waves representing the sounds must be similar in appearance and they must have constant phase relation and (*iii*) the displacements produced by them must take place along the same line. In fact, no two separate sources are obtainable which will produce sound satisfying all the above conditions. So the usual practice is to divide the waves from a *single* source at one region and reunite them at another region after the two halves through separate paths of different lengths.

The interference of sounds can be demonstrated by an apparatus due to Quincke. It consists of a mouth-piece. A (Fig. 36) which divides into two limbs B and C combining again into one tube EF. D is a sliding tube by means of which the length of the path ABDE can be altered. In front of A, a vibrating tuning fork is held, or a Galton whistle is sounded. The ear is placed either directly at F or against the funnel which is connected by a piece of rubber tubing to F. When the sliding tube is at D and the paths ABE and ACDE are of equal lengths, equation (30, 5) is satisfied in which n becomes zero, r_1 *being equal* to r_2. The two wave trains now arrive at the same phase and produce a maximum effect at F. By drawing out the tube the path ACDE is then gradually increased for a minimum sound ; a position D′ is obtained. Evidently this is the condition attained in equation (30, 6) where $n = 0$ and so $r_1 - r_2 = \lambda/2$.

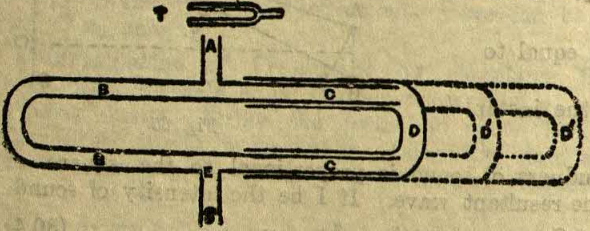


Fig. 36—Interference of Sound

If the path ABE is r then the path ACDE has been increased by $\lambda/2$. Evidently the difference in paths between ACDE and ACD′E is half a wave-length. By drawing out the tube to D″, a maximum effect is again obtained, so that the path difference between ACD′E and ACD″E is again one wave length. Thus the wave-length of the particular sound is obtained by this experiment.

Examples :

1. A tuning fork, originally in unison with another tuning fork of frequency 256, produces 4 beats per second, when a little wax is attached to it. What is its frequency now ?

Ans. When 4 beats are produced with a tuning fork of freqnency 256, the frequency of the tuning fork under observation may be 256 ± 4, (i.e., 252 or 260) per second.

Now to attach any little mass, such as wax, to the prong of the fork is equivalent to some extra loading, which diminishes the frequency of the fork. Hence the frequency is 252 per sec.

2. A fork of unknown frequency gives 4 beats per second, when sounded with another of frequency 256. The fork is loaded with a piece of wax and it again gives 4 beats per second with the same fork. What is the frequency of the unknown fork ?

Ans. Since the number of beats per sec. is 4, the unknown frequency of the fork is $256 \pm 4 = 260$ or 252 per sec. By loading the fork with wax, its frequency diminishes. If now the number of beats per sec. be 4, the possible case is that its frequency diminishes to 252. Thus the original frequency of the fork must have been 260 per sec.

3. Calculate the velocity of sound in a gas in which two waves of lengths 1 and 1·01 metres produce 10 beats in 3 seconds.

[U. P. B.—1954 ; R. P. B.—1959]

Ans. Let the velocity of sound in the gas be V metres per second. Then the frequency n_1 of the wave-length 1 metre is given by

$$V = n_1 \times 1, \text{ whence } n_1 = V.$$

For a similar reason the frequency n corresponding the wave-length 1·01 metres is given by

$$V = n_2 \times 1{\cdot}01, \text{ whence } n_2 = \frac{V}{1{\cdot}01}.$$

Since by hypothesis the difference of frequencies is $\frac{10}{3}$, we get

$$V - \frac{V}{1{\cdot}01} = \frac{10}{3} \text{ or } \frac{{\cdot}01}{1{\cdot}01} \times V = \frac{10}{3},$$ whence $V = 336{\cdot}66$ metres per sec. This gives the velocity of sound.

***31. Electrical Method of determining Velocity of Sound—** This is a laboratory method of determining accurately the velocity of sound in open air, in which the uncertainties of wind, temperature and humidity effects can be satisfactorily controlled. The room, in which the experiment is performed, may be 30 ft. to 40 ft. in length, having a good number of windows and matting on the floor and having screens from the ceilings so as to minimise sound reflections from the walls and to check reverberations. The apparatus consists of two parabolic reflectors R_1 and R_2 made of wood or plaster of Paris and having a facial diameter ranging between 4 feet and 6 feet.

Facing each other, these reflectors are mounted on two vertical stands moving in a long and straight groove so as always to be in one line (Fig. 37).

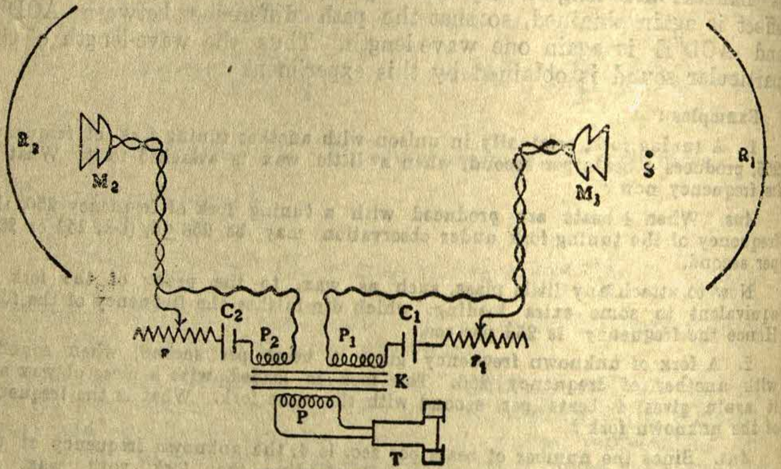


Fig. 37—Hebb's Method

A source of sound S, preferably of high frequency (for example, a tuning fork of frequency of about 1000 cycles per second) is placed at the focus of the reflector R_1. In more recent days a loud-speaker aproducing a continuous note is used as the source. In front of the loud-speaker there is a microphone M_1 which immediately receives the sound and sends a current fluctuation through a coil P_2 in an electrical circuit containing a battery and a rheostat r_1. The coil P_1 forms the primary circuit of an induction coil, its corresponding secondary coil being P. A periodic magnetic flux is thus generated in the secondary coil which actuates the telephone T in which the sound of the source is distinctly heard. This telephone receiver is generally kept in a *separate* room, so that direct sound from the source is not heard. The rheostat r_1 can be adjusted to a desired loudness of the tone heard in the telephone.

The source of sound, being at the focus of R_1, sends a parallel beam of reflected sound to R which again refocuses the sound to its focal point, where there is a second microphone M_2. This microphone collects the sound pulses and sends a similar current fluctuation to the associated electrical circuit containing a battery rheostat r_2 and another primary coil P_1 wound on the same cone. These electrical fluctuations in the coil P_2 also actuate the telephone receiver T.

Now if the fluctuating currents in the two primary P_1 and P_2 are in the same phase, the induced magnetic flux changes in the core would be additive and the sound heard in the telephone would be loud. If, however, the currents be in opposite phases, the magnetic flux changes would be the less and the sound would become feeble. By adjusting the rheostats r_1 and r_2 this feeble sound can be very nearly reduced to silence. We can assume for simplicity that the sound-wave of condensation or pressure produces a current surge in the coil in the positive direction while a sound-wave of rarefaction or suction produces a current surge in the negative direction.

The actual experimental procedure consists in setting the S, R and M_1 in combination fixed at a particular position and in moving the M_2R_2 combination gradually away, till the minimum sound is heard in the telephone, and finally adjusting the rheostats to have practically no sound. At this stage, the current fluctuations in the coils P_1 and P_2 are in opposite, phases, and by virtue of our assumption a pulse of condensation meets M_1, while a pulse of rarefaction meets M_2 at the same instant of *vice versa*. Or, in other words, the phases of vibration at M_1 and M_2 are opposite each other. This can only happen, if the total distance from S to the mirror R_1, R_1 to R_2 and R_2 to M_2 is an *odd* multiple of half the wave-length of the sound produced. If this distance is l_1 then

$$l_1 = \left(2s + 3\right)\frac{\lambda}{2}, \qquad \ldots(31, 1)$$

where s is *any* natural number and λ is the wave-length of sound. It is not possible to measure the wave-length from this single reading, since the total distance l_1 traversed by sound by two reflections cannot be accurately measured and s is *not* a definite number. To eliminate uncertainties of l and s, the M_2 R_2, combination is *slowly* moved further to some other position, where the sound in the telephone is again minimum. If now the total distance of the wave propagation is l_2, then

$$l_2 = \left\{ 2s+1 \right\}\frac{\lambda}{2} \qquad \ldots(31, 2)$$

Subtracting (1) from (2), we get

$$l_2 - l_1 = \lambda \qquad \cdots(31,3)$$

where $l_2 - l_1 =$ the distance which M_2 has been moved through. This distance can be accurately measured. In practice, a number of readings is taken for successive minimum sound positions and the mean value of such distances is taken. This gives the wave-length of the sound produced by S.

If the source of sound has a frequency of n cycles per second, the velocity v of sound propagation is given by

$$v = n\lambda \qquad \cdots(31, 4)$$

In practice the source may be a standard tuning fork. Or, if it be a loud-speaker, its frequency should be standardised by some accurate process. This method of measuring the velocity of sound was first employed by Hebb in 1904. In fact, this is the most accurate method of measuring the velocity of sound or determining the wave-length of sound in air.

32. Stationary Waves—When two systems of identical waves travelling in opposite directions in a medium are superposed on each other, they give rise to a phenomenon known as *stationary waves.* The wave-length and the amplitude of the two systems being the same, they are in a condition to interfere at certain regions. Some parts of the medium on the line of propagation are *thrown permanently into maximum vibration.* These points are called *antinodes.* Some other points midway between the antinodes are *permanently at rest* and are called *nodes.*

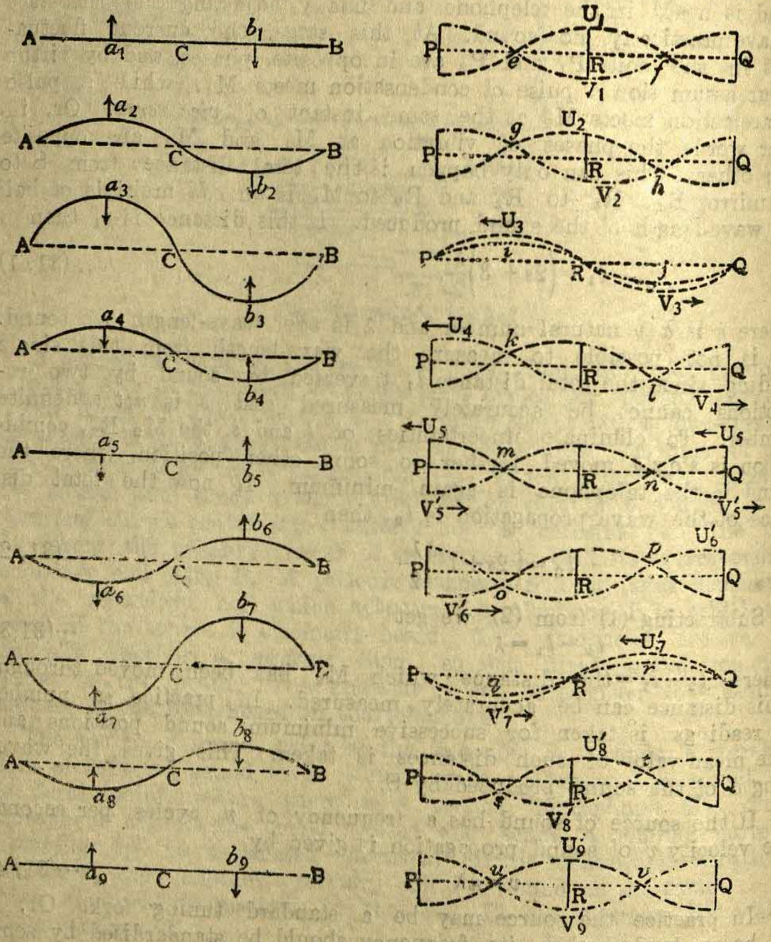


Fig. 38—Stationary Undulations

In order to study the mode of stationary vibration at any instant the resultant displacement graphs of both the incident and the reflected systems over that particular region of the medium is to be obtained. In Fig. 38 the figures on the left from (i) to (ix) represent the successive positions of a string thrown into stationary vibration. These positions are attained at successive intervals of T/8, when T is the period of vibration. On the right side, the displacement curves are drawn. Due to the superposition of these displacements curves the final shape of the string for any instant is obtained. To get the successive positions of the string from the superposition of the displacement curves, we have to assume that these displacement curves move over PQ in opposite directions with an equal velocity.

Since the nature of the profile of a transverse wave is similar in appearance to the displacement graph. a transverse stationary system of waves also changes form with time exactly as in the above diagrams. As for example, if one end of a string is attached to the prong of a tuning fork and the other end is stretched by a weight passing over a pulley, then by striking the tuning fork *stationary transverse vibrations* are set up in the string (Fig. 39). All the successive stages of movement in a complete period of the wave are presented before the eyes and the combined effect is observed. In consequence, the string appears to vibrate in several segments.

A study of the figures show that the wave-length of each of the component waves is double the length of each segment, *i.e.*, double the distance from node to node or from antinode to antinode. Hence by measuring the distances of three consecutive nodes the length of the wave can be determined.

In the case of *stationary longitudinal vibration*, the antinodal regions will have the greatest amount of oscillatory motion along the line of propagation and no motion at the nodal regions. But it is to be noted that *nodes are places of greatest variation of pressure, and the antiondes the* least. By taking a Galton whistle as the source and a sensitive flame as the detector, the presence of nodal and antinodal regions in air can be demonstrated by putting the sensitive flame at different points along the line of propagation. At the nodes the flame will flicker most.

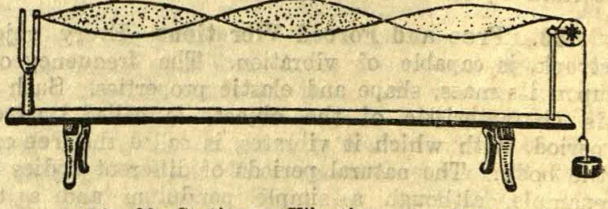


Fig. 39—Stationary Vibrations of a String

Distinction between Progressive and Stationary Waves—The following are the distinguishing features between the progressive and stationary waves :—

In a progressive wave—(*i*) Each particle of the medium executes a periodic motion about its mean position. (*ii*) At *any* instant there is a continuous change of phase from particle to particle.

(*iii*) Every part of the medium undergoes similar changes of pressure and density, as each complete wave passes across it and is restored to the original condition after each period. (*iv*) In a transverse wave a complete wave-length comprises a crest and a trough, while in a longitudinal wave a complete wave-length contains a condensation and a rarefaction. (*v*) The wave moves with a velocity depending upon the properties of the medium and some other factors. In other words, there is a transference of a certain phase as also of some energy from one part of the medium to the other with a definite velocity.

In a stationary wave—(*i*) All particles of the medium except those at some *equidistant* points, execute a periodic motion of varying amplitude, the period being equal to that of the component waves. The points, where the amplitude is zero, are called *nodes*. At some other equidistant points midway between the nodes, the amplitude is maximum; these are called *antinodes*. (*ii*) At any instant all particles between two consecutive nodes are in the same phase of motion, the phase difference between the particles, that lie on opposite sides of node, being 180°. (*iii*) The parts of a medium undergo the greatest change of pressure and density at the nodes, while at the antinodes the pressure and the density are *nearly* uniform. Twice in each complete period the particles lying along the direction of wave propagation comes to rest. (*iv*) The wave-length is equal to the distance between *three* consecutive nodes or antinodes, since the particles, separated by this distance, are in the *same* phase of vibration. (*v*) There is no permanent transference of the particles from one part of the medium to another. Condensation and rarefaction do not move from point to point; but simply appear and disappear at certain equidistant fixed points.

33. Free and Forced Vibrations—Every object, when properly struck, is capable of vibration. The frequency of vibration depends upon its mass, shape and elastic properties. Such a vibration, which is characteristic of the object, is called its **free vibration**. The period, with which it vibrates, is called the **free** or **natural period** of the body. The natural periods of different bodies are different. For example, although a simple pendulum and a tuning fork start to vibrate together, they would have different periods of vibration.

When a force changes in magnitude and direction with time but assumes the same magnitude and direction after each definite interval of time, it is called a *periodic force*. The interval, during which the force assumes the same magnitude and direction, is called the *period of the impressed force*. When such a periodic force acts upon a body, whose natural period is different from that of the impressed force, the body at the preliminary stage tends to vibrate with its own period, while the periodic force also tries to establish its natural period on the body. After a few swings of irregular amplitude the body yields to the impressed force and vibrates with the period of this force with some definite amplitude and phase. Such phenomenon is known as the **forced vibration.**

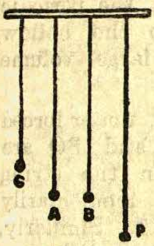


Fig. 40

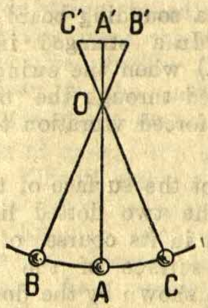


Fig. 41 Longer pendulum

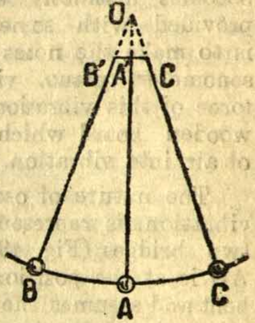


Fig. 42—Shorter pendulum

The following illustration furnishes a simple mechanical model to represent *approximately* the case of forced vibration.* From a flexible support (such as India rubber cord) stretched between two points suspend three pendulums C, A and P of unequal lengths (Fig. 40). Set the middle bob A to swing. The periodic force of vibration, characteristic of A, is transmitted to C and P through the cord. It is found that the two bobs C and P make some irregular oscillations for the first few minutes and finally swing with the same frequency as that of A, but with different amplitude and phase. The oscillation of the driver pendulum A would be found to die away periodically and this decay of oscillations should be compensated for by properly timed push at the extremities of oscillations occasionally. Since the period of the *driver* A is different from those of the *driven* C and P, it is approximately a case of *forced vibration*. The manner, in which the pendulums C and P adjust their lengths so as to attain the same period as that of the pendulum A, is shown in figures 41 and 42 When the pendulum P oscillates, the cord moves through a small length $B'C'$ (Fig. 4) with the same period. The pendulum C, whose natural period has a lesser value, is made to oscillate with the same period as that of A and thus becomes equivalent to a pendulum of length OA, which is equal to the length of the pendulum A (Fig. 42).

When a vibrating tuning fork is held in air, a very feeble sound is heard. But as soon as it is placed upright with its handle in contact with a table, the sound gets magnified to a great extent. The reason for the magnification is that the vibration of the tuning fork is communicated to the table through the handle. The table is thus thrown into a state of force vibration. The large surface, vibrating

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*** In fact, in the experiment stated above, the periodic manifests itself through the agency of a material body (driving pendulum) which is put into a material communication with the driven. In such a case there is a periodic interchange of energy between the driver and the driven. It is truly called *coupled vibration* instead of a forced vibration.**

with the frequency on the tuning fork, sets an extended mass of air into vibration and so increases the volume of sound, which thus becomes distinctly audible. Almost every musical instrument is provided with some sort of a sounding board, the purpose of which is to make the notes louder. In a stringed instrument (such as a sonometer, piano, violin etc.) when the string vibrates, the periodic force of this vibration is carried through the bridges to the hollow wooden board which due to forced vibration throws a large volume of air into vibration.

The nature of oscillation of the surface of the board, under forced vibration, is represented by the two dotted lines. AP and BQ are two bridges (Fig. 43). When in its course of vibration the string AB is at the position 1, the surface of the board is momentarily bent and assumes the shape as shown by the dotted line 1. Similarly, when the string is at 2, the board is at 2, the phase difference of vibration being nearly 180°. As the string keeps on vibrating between the position 1 and 2, it maintains a forced vibration of the board. It is to be noted that in all cases of forced vibration the kinetic energy of the sounding source is continuously transferred to the forced vibrator through the region of contact. Consequently, the sounding source will lose energy as it transfers energy to the surrounding medium and to the vibrator. Hence unless energy is continuously supplied to the source, its energy will die out more rapidly when coupled with a vibrator.

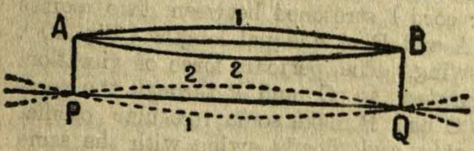


Fig. 43

34. Resonance—When the period of the impressed forces is equal to the natural period of the vibrator, it readily takes up the vibration and begins to oscillate with a considerable amplitude. This is called **resonance.**

Suspend from the same cord (Fig. 40) a pendulum B, equal in length to A. Whenever A begins to oscillate, B starts to swing immediately with an increasing amplitude. Since the two pendulums are equal in length, their natural periods of vibrations are equal and so a resonance occurs. **Strictly** speaking, it is a case of resonant coupled vibration, where there is a mutual transfer of energy between the driver and the driven ; as a result thereof the driver comes to rest momentarily after a regular interval. The energy is then fed back from the driven to the driver. Another simple illustration of synchronised forces in resonance is afforded in common garden swings. Two children stand face to face on a platform. One imparts to the platform a slight push in a certain direction. When the platform is at the extreme end of the swing, the other child now takes his chance of pushing it backwards. In this way at each extremity of oscillation the platform receives a separate impulse and thus its amplitude is gradually increased. The two children here apply alternately periodic impulses having a frequency equal to that of the swing.

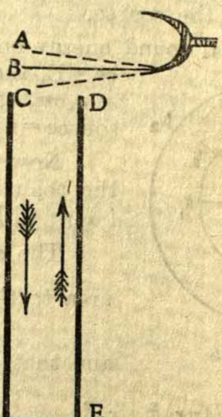
The principle of resonance is applied in some of the Indian stringed instruments, *e.g. sitar, esraj*, etc. In addition to the main strings there are several side strings tuned to various musical scales. When the main string is played upon, the side strings are successively thrown into resonance. The notes thereby increase in volume. The forced vibration of the body of the instrument produces a note which increases the sweetness of the tone also.

Two tuning forks of exactly the same frequency are taken and placed side by side. One of them being struck gives out its characteristic note. If the fork is stopped by being clapsed, the other fork will be found to give out a similar note though of much less intensity. In the case of resonance of two bodies, there is a mutual transference of energy. As a result of this, if the supply of energy be given once to the vibrator, after some time then the source will have no energy of vibration. Then the vibrator will act as the source transferring energy to the other which now acts as the vibrator. The natural process of transference of energy continues, till both of them give up all their energy to the surrounding medium.

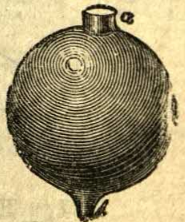
Electrical resonance play a very important part in the detection of wireless signals. Each broadcasting station transmits wireless waves of a certain specified frequency. When a radio receiver is to receive a message from a certain station, it is to be tuned to resonance with the frequency of that station. By turning the tuning knob of the receiver when the condition of resonance is attained, an appreciable current is obtained in the receiver. This current is further amplified by a system of radio valves before being passed through the loudspeaker, which gives an exact reproduction of programme.

Resonance of Air Column—Take a tall empty glass jar (Fig. 44). Hold a vibrating tuning fork just above the upper end on the jar and pour water into it until at a particular level, say at E, maximum sound is heard. If another fork of different frequency is taken, the maximum sound is produced at another water level.

This is due to the resonance of air within the jar. A definite mass and volume of air has each a particular natural period of vibration. When the period of the tuning fork is equal to that of column of air, resonance occurs. Organ pipes and different types of wind instruments work on this principle. (For further details *vide* Art. 48).



Fig, 44



Fig, 45

***Helmholtz's Resonators**—Helmholtz constructed a series of hollow metallic vessels of different sizes, each having two narrow openings *a* and *b* at the two ends (Fig. 45). The natural period of such a resonator can be determined by blowing across the neck, when a particular note is heard. The exact frequency, at which a vessel resonates, is indicated on the vessel. If a source of sound having an equal frequency is placed in front of the wider opening of a resonator, the sound is magnified to a great extent. A complex sound consists of more than one frequency. When a complex sound is made in front of them, there, will be resonance, provided that any constituent of the complex tone has the same frequency as that of the particular resonator. The resonance can be detected by placing the ear at the lower opening when a loud sound is heard.

35. Intensity of Sound—The *intensity* of sound Is a measure of its loudness or volume, as perceived by the ear. It has also an objective existence and is proportional to the the amount of energy contained in unit volume of the medium, through which sound waves are propagated. It is also measured by the amount of energy passing per unit area placed at right angles to the direction of propagation of sound.

The following are the *factors*, upon which the intensity or loudness of a sound depends :—

(*i*) *Amplitude of vibration of the source*—The larger is the amplitude of oscillation of the source, the louder is the tone produced. In fact, intensity is proportional to the square of the amplitude.

(*ii*) *Distance of the source*—The flow of energy per unit area across a surface, placed normal to the direction of propagation, is a measure of the intensity at *any* point on that surface. We can thus compare by the following procedure, the intensities at two points at known distances from the source.

Let the total sound energy emanating per second from the source be $= E$, the distance of one point from the source $= r_1 cm.$ and distance the other point from the source $= r_2\ cm.$

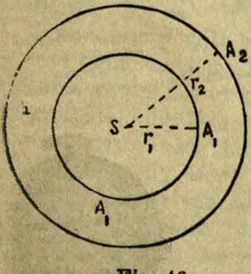


Fig. 46

Now draw two spheres with the source as the common centre and with r_1 and r_2 as radii (Fig. 46).

The amount of energy flowing across a unit area to the sphere of radius r_1 is $= \dfrac{E}{4\pi r_1^2}$

and that of the sphere of radius r_2 is $= \dfrac{E}{4\pi r_2^2}$

$$\therefore\ \frac{I_1}{I_2} = \frac{E/4\pi r_1^2}{E/4\pi r_2^2} = \frac{r_2^2}{r_1^2}.$$

Thus the *intensity at a point* **is** inversely *proportional to the square of its distance from the source.*

(*iii*) *Size of sonorous body*—A large vibrating area sends out a greater amount of energy. Accordingly, the larger is the size of the sonorous body, the louder is the sound produced by it. For this reason church bells, which have a bigger vibration area than an oridinary gong, can be heard from a much larger distance.

(*iv*) *Presence of other bodies*—A resonant body near the source sympathetically vibrates and so it intensifies sound. Also large reflectors concentrate sound energy and act as sound magnifiers.

(*v*) *Density of medium*—Since the kinetic energy of the medium increases with the mass per unit volume of the medium, it follows that *a more intense sound will be heard in a medium of higher density.* Thus cold air conveys a louder sound than hot air. Carbon dioxide is a better medium for sound than air. Motion of the medium often helps to concentrate sound energy and thereby increase the intensity.

EXERCISES ON CHAPTER IV

Reference

1. What are beats? How are they produced? **Art. 29**

(C. U.—1959, '63; Dac. U.—1960, '73; U. P. B.—1964, '71; P. U.—1964, '69; Del. U.—1963, Nag. U.—1961; Del. H. S.—1969; Gau. U.—1952)

2. Explain the phenomenon of 'beats' in a sound. How will you prove that the number of beats produced by two tuning forks is equal to the difference of frequencies? (Raj. U.—1952, '65; V. U.—1955; Guj. U.—1962; Poo. U.—1962; Bom. U.—1964) **Art. 29**

3. Two forks when sounded together give 4 beats per sound. One is in unison with a length of 93 cms. of a monochord string under a constant tension and the other with 97 cms. of the same string. Find the frequencies of the forks. **Art. 29**

(C. U.—1952; Del. U.—1960)

Ans. **284 and 388.**

4. Two organ pipes give 6 beats per second when sounded together in air at a temperature of 10°C. How many beats will be given when the temperature rises to 24°C? (Velocity of sound in air at 0°C = 1083 ft. per sec.) **Art. 29**

Ans. 6·15.

5. Two tuning forks A and B when sounded together produce 5 beats per second. If A is slightly loaded, the number is reduced. Find the frequency of A, if the frequency of B be 256 per sec. **Art. 29**

(Dac. U.—1963)

Ans. **261.**

6. Explain with the help of a diagram the production of beats. **Art. 29**
(Pat. U.—1962; P. U.—1969; C. U.—1949; Gau. U.—1961; Bom. U.—1970; Guj. U.—1951; Del. U.—1964)

7. You are given a standard tuning fork of known frequency and another tuning of slightly different frequency. How would you proceed to determine the frequency of the second tuning fork? (Gau. U.—1972)

Reference

Art. 29 8. A set of 24 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats with the preceding one and the last fork is found to be the octave of the first, calculate the frequencies of the first and last forks. (Bom. U.—1964)

Ans. 92 ; 184.

Art. 29 9. A siren having 16 holes in the disc makes 600 revolutions per minute. A sounding body when brought near the siren gives 4 beats per second. When the speed of the siren is increased to 630 revolutions per minute, the number of beats heard is again 4 per second. Calculate the frequency of the sounding body. (Guj.—1962)

Ans. 164 per sec.

Arts. 29 & 30 10. Distinguish between interference and beats and describe an experiment to show the interference of sound waves.

Art. 30 11. Under what conditions can two sound waves produce the phenomenon of interference? Describe the Quinke's method of producing interference and explain how it can can be used for determining the frequency of a source of sound. (R.P.B.—1970)

Art. 32 12. What are stationary waves and how are they produced?

(C. U.—1961 ; Dac. U.—1965 ; *Cf.* Del. H. S.—1961.

Art. 32 13. How is the stationary undulation produced by sound waves in air and what are the principal characteristics of such undulation? What is character of the motion of air particles when this condition prevails?

Art. 32 14. What are nodes and antinodes? How will you demonstrate their existence? What will be the effect on the distances between successive antinodes in a column of a gas by *increasing its temperature and pressure*?

(Del. U.—1969 ; P. U.—1959 ; C. U.—1960)

Art. 32 15. Write a note on stationary undulations.

(Guj. U.—1962, '75 ; Bom. U.—1960 ; Dac. U.—1962 ;
Poo. U.—1953, '64 ; B. H. U.—1953)

Art. 32 16. Distinguish between a progressive and a stationary wave. Illustrate your answer by diagrams.

(Pat. U.—1964 ; R. P. B.—1971 ; Gau. U.—1963 ;
C. U.—1953, '55 ; U. P. B.—1960.

Art. 33 17. Explain the difference between free viabration and forced viabration. (Raj. U.—1955 ; Poo. U.—1972)

Art. 33 18. Write a note on forced vibration.

(Pat. U.—1952)

Art. 33 19. A tuning fork is vibrating in free air, distinguish between the natures of vibration of the fork and of the medium. If the handle of the vibrating fork is held against a wooden board, the amount of the sound produced is considerably increased. Explain carefully why this happens. Is the time, during which the fork goes on vibrating, affected? If so, why so? (C. U.—1953)

Art. 33 20. Explain why when the handle of vibrating tuning fork is pressed against a thin wooden board, the intensity of sound is greatly increased. (C. U.—1955, '56 ; *Cf.* Pat. U.—1953, '61)

Arts. 33 & 34 21. Explain clearly the difference between forced vibration and resonance. Give mechanical and acoustical illustration.

(Bom. U.—1962, '65 ; P. U.—1971)

	Reference
22. A vibrating tuning fork is placed at the mouth of an open jar and water is poured into the jar gradually. Explain what will happen. (C. U.—1958, '65 ; P. U.—1971)	Art. 34
23. What is meant by resonance? Give mechanical and acoustical illustration.	Art. 34
24. Explain the principle of resonance. (Del. H. S.—1958, '60, '72 ; C. U.—1959 ; Pat. U.—1930 ; Guj. U.—1953 ; Utk. U.—1953 ; R. P. B.—1962)	Art. 34
25. What is meant by resonance? Calculate approximately the length of the resonance box closed at one end on which a tuning fork is to be mounted. The pitch is 256 and the velocity of sound is 1120 ft. per sec. Would the same resonance box answer for a fork of another pitch? If so of what pitch? *Ans.* 1·09 ft. Yes, it would be 768.	Art. 34
26. What are the factors determining the loudness of a musical note? (E. P. U.—1962, '64)	

CHAPTER V

MUSICAL SOUNDS

36 Musical Sounds and Noises—We ordinarily distinguish between a musical sound and a noise by a sense of comparative choice. **A musical sound** consists of regular and periodic vibration, whilst **noise** is a discontinuous sound produced by a sort of confused, sharp and irregular movements in a medium. In spite of all these differences no clear line of demarcation can be drawn between the two. For example, a person standing near a market place might experience noise coming from it ; but when he is at a distance, the noisy sounds, as they move on are blended together to form a regular hum. The gurgling of a fountain, the murmuring of a brooke and the pattering of rain drops often produce pleasing sounds.

Characteristics of a Musical Sound—Musical sounds are distinguished from one another in respect of the following features :

(*i*) **Intensity**—It is a measure of the loudness of a note. It has been discussed in details in Art. 35.

(*ii*) **Pitch**—It is the property, by which an acute, sharp or *shrill* sound is distinguished from a dull, flat or *grave* one. It will be shown that the pitch of a note depends upon the frequency of vibration of the source.

(*iii*) **Quality**—It is the property by which two notes of the same pitch (and possibly of the same intensity) given out by two different sources may be distinguished from each other. This property (discussed in details in Art. 40 under the heading 'Quality of a Musical note') is also termed *character*, *colour* or *timbre*.

37. Pitch—The relation between the pitch and the frequency can be demonstrated by an apparatus known as **Savart's Toothed Wheel.** It consists of four discs (Fig. 47) mounted co-axially upon a spindle which can be rotated at any known speed. All the discs are of the same diameter ; but the number of the teeth on the successive discs are in some fixed ratio 4 : 5 : 6 : 8. A cardboard C is clamped in front of the disc, so that it can be made to press lightly against any row of teeth

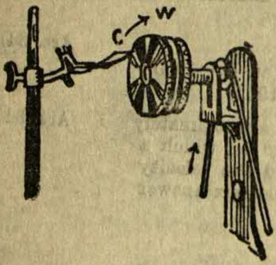


Fig. 47

When the spindle is gradually rotated a pattering sound is heard initially On gradually raising the speed of revolution a musical note is emitted, acuteness of which slowly rises. If *m* be the number of revolutions of the disc having *n* number of teeth, the frequency, with which the cardboard vibrates at any time, is $m \times n$. This shows that *with the increase of frequency of the source, the pitch of a note rises*

The pitch depends upon the frequency of a vibrating source in much the same way as the colour depends upon the frequency of light radiations. It is known that the electro-magnetic radiations of all frequencies as a whole, are not visible, but that only a very short range from the red to violet spectral colours produces the sensation of sight. Likewise our range of audibility is limited. Although different for different persons, the highest frequency, to which our ear is sensitive, varies between 30,000 and 40,000 cycles per second. This can be experimentally varified by an apparatus known as the **Galton's Whistle** or an audio-frequency oscillator with a loudspeaker. The lowest audible frequency lies between 16 to 20 cycles per second. Beyond these limits no mechanical vibration, can produce any effect in our ears. Frequencies above the highest limits of audibility are classed under *supersonics* or *ultra-sonics* The frequencies below the audible limits are called *infra-sonics*.

It may be noted in this connection that the pitch of note also depends upon the relative motion of the source, medium and the observer. This is explained in **Doppler's Principle** (See Chapter XI).

38. Determination of Pitch—There are mainly two ways of determining the pitch of a musical note :

(i) **Direct or Visual method by** (*a*) *Duhamel's Vibroscope*, or (*b*) *Stroboscopic Wheel*, or (*c*) *Falling Plate Method*.

(ii) **Indirect or Auricular method** by (*a*) *Savarts Toothed wheel* or (*b*) *Siren*, or (*c*) *Sonometer*, or (*d*) *Resonance Column*. The methods, mentioned above, will now be dealt with one by one.

Duhamel's Vibroscope—When the frequency of vibration of a body is to be directly recorded, this apparatus is very suitable. It consists of a cylindrical drum D (Fig. 48) which is mounted on a screw cut axle and is provided with a handle on one side. A piece of smoked paper is wrapped round the drum. The tuning fork T, the frequency of which is to be determined, is provided at one of its prongs with a style which lightly touches the smoked paper. The fork is then struck. On rotating the drum uniformly, a wavy line is traced out by the style on the smoked paper The nature of the wavy line traced on the smoked paper is shown in Fig 48. If two points be marked on the wavy line at known interval, the number of complete vibrations of the fork during that interval can be directly counted.

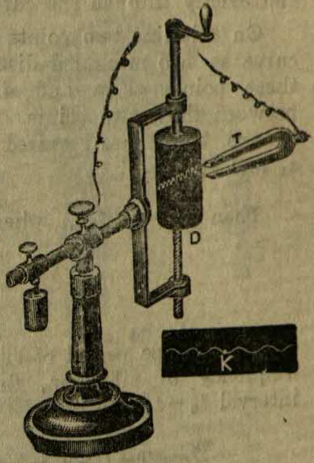


Fig. 48—Duhamel's Vibroscope

Due to friction of the style with the paper, the amplitudes of successive vibrations gradually die away. In actual practice the tuning fork is run electro-magnetically (Art. 2) and the recording of time is made with an electrical pendulum, which produces sparks at the end of the style at known regular intervals. These sparks give spots on the wavy line. The number of complete waves between any two consecutive sparks gives the number of vibrations during that period.

Stroboscopic Wheel—For a detailed description of such an apparatus, *vide* Chapter IX.

Falling Plate Method—In its simplest construction the apparatus consists of a rectangular glass plate capable of falling freely under the action of gravity in a space between two parallel grooves, so that the plate always remains vertical. The tuning fork, whose frequency is to be determined, is provided with a style at one prong as shown in Fig. 48 and the tuning fork is held rigidly in such a way that its prong just touches the surface of the plate in a normal position. The tuning fork may be excited by a padded hammer. Else it may be run continuously with an electro-magnet and a contact switch

The glass plate is taken out and smoked lightly with lamp black. Then it is suspended within the groove by tying it with thread from a suitable stand in such a way that the prong touches the lower part of the plate. The tuning fork is now made to vibrate, so that the prong etches a horizontal line on the lamp back surface. The plate is then released by cutting off the thread when it falls freely under the action of gravity. A trace of the style is found on the plate of the nature of

Fig. 49 (*Vide* GENERAL PHYSICS, Part I), the starting point of the curve being the top of the figure. A line is drawn vertically and evenly through the curves.

On this line two points are taken each at the junction of the curve at two measured distances from the top, say s_1 and s_2. Mark these points as a and d. Count the number of complete waves between these two points Let this number be m. Suppose t_1 and t_2 are the times required by the plate to fall through the distances s_1 and s_2.

Then $s_1 = \frac{1}{2} g t_1^2$, whence $t_1 = \sqrt{\frac{2s_1}{g}}$. Similarly $t_2 = \sqrt{\frac{2s_2}{g}}$

$$\therefore \quad t_2 - t_1 = \sqrt{\frac{2s_2}{g}} - \sqrt{\frac{2s_1}{g}}.$$

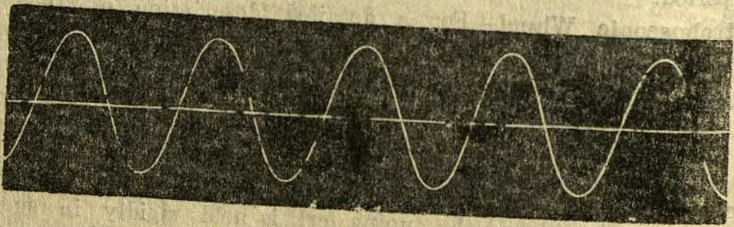
If T be the period of vibration of the tuning fork and n be its frequency of vibration, then since m vibrations take place in the interval $t_2 - t_1$, we find

$$\frac{m}{n} = m\text{T} = t_2 - t_1 = \sqrt{\frac{2q_2}{g}} - \sqrt{\frac{2s_1}{g}} = \sqrt{\frac{2}{g}} \quad \sqrt{(s_2 - s_1)},$$

$$\text{whence} \quad n = \sqrt{\frac{g}{2}} \cdot \frac{m}{\sqrt{s_2} - \sqrt{s_1}}.$$

Thus by knowing m, g, s_1 and s_2, the frequency n can be measured.

Savart's Toothed Wheel—This apparatus already been described in the preceding article To use the apparatus place the source of



Fig; 49—Wave trace of a Tuning Fork

sound on a table. Adjust the speed of the wheel, such that the note emitted by the cardboard is similar to that of the source. The frequency of the cardboard is then equal to the frequency of the source. If m be the number of revolutions per second of the disc having n teeth, the pitch is evidently mn.

Seebeck's Siren--It consists of a metal disc containing four rows of equidistant holes drilled along the circumference of concentric circles (Fig. 50). The disc can be rotated at any desired speed on a spindle provided with a speed counter. A narrow tube ending in a nozzle stands against the disc. The tube is connected to a foot bellows, from which air can be forced against the disc. The nozzle is set just opposite to one row of holes and the disc is rotated. The air is thus admitted or stopped, according as a hole or a stop passes by the nozzle. Thus a series of puffs of air is released at regular intervals, which emits a note of a particular pitch.

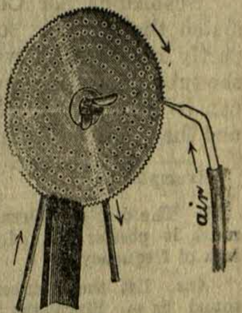


Fig. 50 – Siren

To determine the pitch of a note, regulate the speed of the siren disc, till the note emitted by the source appears to be identical with that given out by the siren. Then the pitch of the note heard is equal to that of the siren. If the number of holes for a complete round be *m* and the frequency of revolution be *n*, the pitch of the note is *mn*

Cagnaird de la Tour's Siren—This is a more improved type of siren which can produce sounds of high pitch. The essential parts of the apparatus are the following (Fig. 51):—

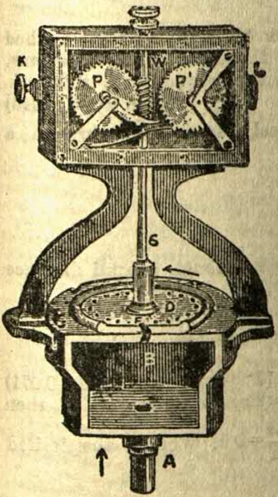


Fig. 51

(*i*) a cylindrical hollow box B, known as the *wind chest*, the lid of which contains equidistant concentric holes;

(*ii*) a disc D with an exactly equal number of holes, supported just above the lid over a vertical spindle, about which it can rotate with as little friction as a possible. (The two systems of holes make an angle with each other, so that air blown into the wind chest through a leading pipe rushes out and strike against the sides of the hole. This rotates the upper plate D);

(*iii*) a leading pipe A from a foot-bellows to the wind chest;

(*iv*) a speed counter P′P attached to the vertical shaft on the disc.

Every time the two rows of holes are opposite each other during the rotation of the disc, a puff of air escapes through each hole of the upper disc If the upper disc contains *m* holes and its frequency of revolution is *n* as indicated by the speed counter, the number of puffs issuing from any whole in the lid in one second is equal to $m \times n$. The *m* holes in the lid merely serve to magnify the sound *m* times, since at any particular instant *m* puffs come out simultaneously from the wind chest. The modern type of this siren is run electrically and is largely used as signals.

Sonometer—The determination of frequency of a source of sound by this method consists is tuning the note of a sonometer wire of the known mass by properly adjusting its tension and length with the note emitted by the source. (*Vide* Art. 45).

Resonance Air Column—With reference to Art. 36 it has been stated that when a source of sound is held at the mouth of a tube partly immersed in water, a resonance occurs at a certain height of the tube, depending upon the frequency of the source. The frequency can be determined, when the length of the exposed part of the tube and the velocity of sound are known (*Vide* Art. 49).

Examples :

1. The disc of a siren is making 10 revolutions per second. How many holes must it possess in order that it may produce 4 beats per second with a tuning fork of frequency 484 ? [C. U.—1943]

Ans. Let the number of holes in the siren responsible for the production of sound be m. Since the frequency of revolution is 10, the pitch of the note produced $=10m$.

Four beats are heard with a tuning fork of frequency 484. Consequently, the pitch of the siren may be 484 ± 4 *i.e.*, 488 or 480. Now either of these numbers would be equal to $10m$, where m is an integer. It, therefore, follows that if we put $10m=480$, the value of m is integral and equals 48. Not so is the case if we consider 488.

2. In an experiment to find the frequency of a fork by dropping plate method the plate was dropped through 2·5 cms. While falling through another 11·9 cms. the fork was found to make 24 vibration. Calculate the frequency of this fork, given that $g=980$ cms./sec.^{2}. [Del. H. S. 1946]

Ans. Let the time required by the plate to fall freely from rest through a distance of 2·5 cms., be t_1 seconds.

Since $s=\frac{1}{2}gt_1^2$ we have $\frac{1}{2}gt_1^2=2{\cdot}5$, or $490\,t_1^2=2{\cdot}5$.

$$\text{or, } t_1^2=\frac{1}{196}, \text{ or } t_1=0{\cdot}071 \text{ sec.}$$

Again let the time or during which the plate falls through a total distance of $2{\cdot}5+11{\cdot}9$ or 14·4 cms. be t_2. Then for a similar reason,

$$\frac{980}{2}\,t_2^2=14{\cdot}4, \text{ whence } t_2=\sqrt{\frac{14{\cdot}4}{149}}\text{ sec.}=0{\cdot}171 \text{ sec.}$$

Therefore the time to fall through a distance of 11·9 cms. is $=(0{\cdot}171-0{\cdot}071)$ sec. $=0{\cdot}1$ sec. If T be the period of vibration of the fork and n its frequency, then $T=\frac{1}{n}$. Since by the problem, $24T=0{\cdot}1$. we deduce $\frac{24}{n}=0{\cdot}1$, whence the desired frequency is 240 per sound.

39. Quality of a Musical Note—The quality depends on the form of the time-displacement curve. Figure 52 represents the displacement graphs due two musical sounds. The wave-length for both the curves is the length of the line MM′ and therefore they have the same frequency or pitch, since the velocities for both the waves are equal. The amplitude for both the curves are equal, though they do not occur at the same instant. So their intensity is the same. Yet

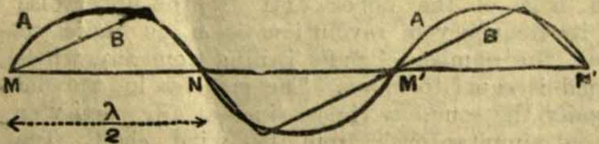


Fig. 52—Wave curves for Two Notes

because the shape is not the same for the two curves, the two notes heard may be distinguished from each other. The sine curve MAN is that due to a tuning fork and the sawtooth curve MBN is due to violin string (Fig. 52). It is due to the difference in the nature of these two displacement curves that two notes have got different qualities.

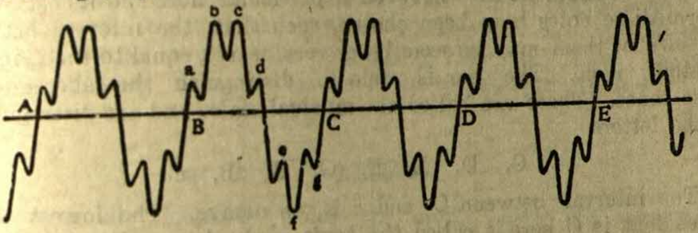


Fig. 53—Wave curve for a Complex Note

Figure 53 represents a wave form of another note, which is very much different from a simple harmonic curve. The line of reference is the central straight line and the wave curve is periodic in nature. The wave-length is the distance between the points B, C, D, etc. Such a complex wave form is due to the presence of harmonic frequencies in it. Thus the quality of a musical sound finally depends on the number of various harmonics present in it and their relative phases and intensities.

40. *Consonance and Dissonance—It is an observed fact that when *two simple tones of different frequencies* are sounded together their combination sometimes creates a pleasing sensation to the ear. This is known as **consonance** and the two sounds are said to produce a *concord*. Again, the combination of two tones, even when each is musical, is sometimes very much disagreeable. This is called *dissonance* and the tones produce a *discord*.

Helmholtz suggested that this discordance or dissonance is due to the production of beats between the component tones, which produce a sort of jarking sensation to the tympanic membrane of the ear. A similar effect is felt in the eye when a light is flickering before it.

41. Musical Interval—The amount of agreeableness produced, when two notes are sounded together, has been experimented over a wide range of frequencies. It has been found in all cases that two notes form a pleasing combination, when the ratio of their frequencies is expressible by two simple integers, neither of which is very large, The ratio is called the **interval** between the two given notes. For example, if n_1 be the frequency of one source and n_2 that of the other, the *interval* between them is n_1/n_2. Again, if n_1, n_2 and n_3 be the frequencies corresponding to three notes, the interval between the first and the second is n_1/n_2 and that between the second and the third is n_2/n_3. Also the interval between the first and the third is n_1/n_3. But as

$$n_1/n_3 = n_1/n_3 \times n_2/n_3,$$

the total interval (n_1/n_3) between two notes is obtained by multiplying the component intervals due to a note having an intermediate frequency n_2.

***42. Musical Scale**—Between a particular note and its octave *six* intermediate notes have been chosen, such that the interval between any two of them may be exactly, or very nearly, equal to the principal constant ratio. The ear is able to distinguish the succession of these tones. There are called the **musical** scales and are distinguished by the letters

C, D, E, F, G, A, B, c.

The interval between C and c is an octave. The lowest of the series that is C here is called the **tonic** or the **key note** of the scale. The musicians often name these scales as follows :

	Do	re	mi	fa	sol	la	si	do...(*Western system*)
or	Sa	re	ga	ma	pa	dha	ni	sa...(*Indian system*)

Do or Sa being the key-note. The intervals between the key-note and the successive scales are

$$1,\ \tfrac{9}{8},\ \tfrac{5}{4},\ \tfrac{4}{3},\ \tfrac{3}{2},\ \tfrac{5}{3},\ \tfrac{15}{8} \text{ and } 2.$$

The relative frequencies are

24, 27, 30, 32, 36, 40, 45 and 48.

The intervals between each pair of consecutive notes are

$$\tfrac{9}{8},\ \tfrac{10}{9},\ \tfrac{16}{15},\ \tfrac{9}{8},\ \tfrac{10}{9},\ \tfrac{9}{8} \text{ and } \tfrac{16}{15}.$$

It is seen that the intervals are a major tone ($\frac{9}{8}$), minor tone ($\frac{10}{9}$) or a semi-tone ($\frac{16}{15}$). There are two major tones D : C and G : F in the scale. For this reason it is called **Major Diatonic Scale.**

Tempered Scale—To suit the voice of a singer in music, any of the scale may be selected as the key-note. For this reason the diatonic system is not suitable as would be put in evidence from the following illustration :

Let the frequency of the tone of a scale be 256. According to the chosen intervals the frequencies of the scale corresponding to this tone are

C	D	E	F	G	A	B	c	d	e	f...
256	288	320	341	384	427	480	512	576	640	682

If the frequency 288 be chosen as the tone, the succeeding scales according to the same intervals are

D	E	F	G	A	B	c	d	e	f
288	324	360	384	432	480	540	576	648	720

It is seen from the above that the scales corresponding to C the key-note do not fit exactly into the scales corresponding to D as the key-note and so some new scales are necessary. If, again, E is selected as the key-note, there should be some more scales are to complete its octave. In this way innumerable scales are to be constructed so as to suit any one as the key-note.

To meet this difficulty some of the intermediate scales have been slightly altered in frequencies in such a way that the notes corresponding to one as the key-note may serve for any other key-note. The slight alternation in frequency is called **musical temperament,** The *successive intervals* are the *same* in the tempered scale, In an octave consisting of 8 scales, 5 additional scales have been introduced, making a total of 13 scales having 12 intervals. Since the interval of an octave is 2, the successive intervals being x, we find

$$x^{12} = 2,$$

$$\text{or} \quad x = 2^{\frac{1}{12}} = 1{\cdot}065946.$$

In equally tempered scales the notes with the corresponding intervals are

C	Cs	D	Ds	E	F	Fs	G	Gs	A	As	B	c
1	$2^{\frac{1}{12}}$	$2^{\frac{2}{12}}$	$2^{\frac{3}{12}}$	$2^{\frac{4}{12}}$	$2^{\frac{5}{12}}$	$2^{\frac{6}{12}}$	$2^{\frac{7}{12}}$	$2^{\frac{8}{12}}$	$2^{\frac{9}{12}}$	$2^{\frac{10}{12}}$	$2^{\frac{11}{12}}$	2

The sign S attached to a note denotes the corresponding **sharp** or *sharpened* note. In a piano or harmonium the white keys (*e.g.*, C, D, E, etc.) represent primary notes, while the black keys (*e.g.*, Cs, Ds, etc.) represent the sharp notes. Note that sharp note of a preceding key is equivalent to the **flat** or *flattened* note of the succeeding key; *e.g.*, Cs = DF, Ds = EF, etc., the sign F being indicative of a flat note.

The following table shows the change brought about by equal temperament.

	C	D	E	F	G	A	B	c
Natural scale .	1·000	1·125	1·250	1·333	1·500	1·667	1·875	2·000
Tempered scale...	1·000	1·122	1·160	1·325	1·498	1·682	1·888	2·000
Frequencies ...	256	287·2	322·5	339·2	383·3	430·6	438·8	512

44. National and Pitch for Musical Scales—Helmholtz's notation of the scales is usually adopted musicians and is represented over any octave by the letters :

Cs Ds Fs Gs As
C D E F G A Bc
DF EF GF AF BF
রে গা মা ধা নি
কোমল কোমল কড়ি কোমল কোমল
সা রে গা মা পা ধা নি সা

In music, many such octaves are necessary one after the other. They are all designated by the same letters but with different subscripts. The gravest octave begins which C_1, D_1, etc., C_1 having a frequency of 32. The note is the same as that of the fundamental of a 16-foot organ pipe. Hence the octave C_1 to B_1 is commonly spoken of as 16 *foot octave.* The initial and final letters with their corresponding frequencies of all the octaves of a pianoforte are

	1st	2nd	3rd	4th
Octaves	C_1...to ..B_1	C . to.. B	c...to...b	c'...to b'
Frequencies	32 .. to.. 60	64...to...120	128 . to...240	256...to...480

	5th	6th	7th
Octaves	c''...to...b''	c'''...to...b'''	c''''...to...b''''
Frequencies	512...to.. 960	1024...to 1920	2048 .. to...3840

As shown above, the musical range rarely exceeds 7 octaves since the fundamental frequency approximately lower than 32 or higher than 3840 loses its musical character. For this reason a complete musical scale, as that in a piano, contains 7 octaves. It must not be thought that in music frequencies higher than about 4000 are not required. To have the quality of a sound a few harmonics along with the fundamental tone are always required. The harmonics of higher pitch of musical notes are sometimes found to have frequencies as high as 10,000 or even higher. It is for this reason that to reproduce a music faithfully and sonorously, amplifier speakers should have a frequency response from 20 up to 10,000.

45. Some Technical Terms—The following terms are commonly used in music ;

Triad—Whenever three notes are sounded together, such that their frequency ratio is 4 : 5 : 6, they form a concordant combination. This is known as a *musical triad.* Thus C, E, G or F, A, C are triads.

Chord—If a triad be sounded with an additional note, such that this stands on octave to the lowest note of the triad, the combination is called a *chord.* The common chord is formed of the first, third and the fifth of the notes of the scale, the interval. between the first and the third being $\frac{5}{4}$, and that between the first and the fifth $\frac{3}{2}$. The intervals are known as the *major chord and minor chord.*

Harmony—Any combination of a number of notes, which being simultaneously sounded produces a sensation of a concordance, is known as a *harmony.* Western music, which mainly consists of chords and triads, is based on the principle of production of harmony.

Melody—When a series of notes is successively produced in such a way that the resulting combination produces a musical sensation, it is called a *melody.* Indian music, which mainly consists *of tans* and *gitkiris,* is based upon the production of melody.

Solo—When a single instrument is played upon, it is called a solo ; *e.g.*, a piano-solo or a violin-solo.

Orchestra—When a number of musical instrument is played upon to produce an atmosphere of harmony or melody or both it is called an orchestra.

***46. Human voice**—The organ, that enables a person to speak or sing, is called the **larynx** or the *voice box,* a transverse section of which is shown in Figure 54. It is situated in the **trachea** or the *wind-pipe* of the throat. It consists of two flat and thin membranous bands C, C, called the **vocal chords,** which are stretched across the trachea. The membranes are attached to the muscles, which regulate their tensions as well as the *air space* between them. This space is known as the **vocal slit.**

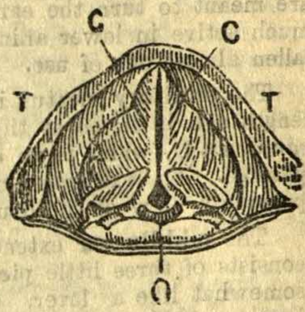


Fig. 54

When air from the lungs is forced out through the slit, these membranes are set into vibrations like reeds producing a sound. The nature of sound given out depends upon the air cavities of nose, mouth and throat, which act as resonators. The natural pitch of the voice can be altered by altering their capacity and the size of their openings. The movements of the tongue and jaws give rise to articulate sounds. The peculiar vibrations of the vocal chords can be observed with an apparatus known as a *laryngoscope.* In whispers the vocal chords remain open and are not set into vibration. The sound is produced by the expulsion of air through the opening of the throat and mouth. The resonating effect of the air cavities can be detected, when air is forced through the lungs with the mouth open. It is found that a faint dull sound is emitted in every case, the nature of which depends upon the shape of the mouth.

The pitch of the human voice depends on the tension and thickness of the vocal chords. The vocal chords of men are much larger and thicker than those of women and children. For this reason a male voice is much deeper than that of a female.

47. Human ear—The *human ear* may be subdivided into three parts, *viz.*, (1) *external ear*, (2) *middle ear* and (3) *internal ear or labyrinth.*

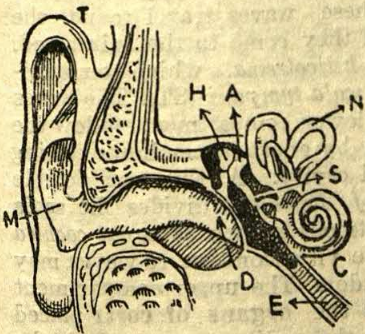


Fig. 55—Human Ear

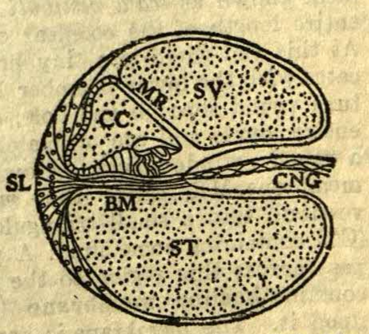


Fig. 56—Magnified section of Cochlea

The **external ear** consists of the **pinna** or the projecting extension T (Fig. 55). The form of the outer ear is slightly concave, so as to be able to collect sound waves and concentrate them into the ear passage. There are three small muscles attached to pinna and these are meant to turn the ear in various directions. These muscles are much active in lower animals ; but with human beings they have fallen almost out of use.

The **auditory** meatus is the *ear passage* M. It is about an inch in length and is closed at the inner end by **membrana tympani** or the *ear drum* D, which is an elastic membrane stretched across the ear passage. An external examination through the ear passage can provide a view of the drum.

The **middle ear** extends from the other side of the ear drum and consists of three little pieces of bone joined to one another and acting somewhat like a lever. One of these, H called the **malleus** or *hammer*, has one end attached to the tympanic membrane, while the other end is connected to the **incus** or *anvil* A. This again, is coupled with one end on the **stapes** or the *stirrup* S ; the other end is fixed on an oval membrane at the entrance of the internal ear. The function of these three inter-connected pieces of bones is to carry any vibration of the ear drum through the middle ear to the oval membrane on which rests the stirrup. In order to have an easy vibration of the ear drum, the pressure on either side of the drum is normally equal. This equality is preserved by the air filling the middle ear at atmospheric pressure and by maintenance of free communication of air by the tube E from the throat which is called Eustachian tube.

The **internal ear** consists of the central oval cavity, called the **vestibule**, just behind the membrane on which the stirrup rests. From this region there extends a bony tube C, called the **cochlea**, which is a spiral of two and a half turns and divided into two compartments throughout its length, but has no direct communication with each other except at their ends. Figure 56 represents a magnified transverse section of the cochlear tube. A watery fluid fills both the compartments. If any vibration is set up on the oval membrane, waves are produced within this watery fluid in the upper compartment known as *scala vestibuli* (SV). These waves travel round the entire length of the cochlear canal until they come to the spiral end. At this end there is a tiny hole, called *helicotrema.* which communicates with the lower chamber known as *scala tympani* (ST). The waves turn round through the hole, enter the lower compartment, follow the entire course and are ultimately damped by another membrane closing a round window just below the oval cavity. A thin and flexible membrane MR, called the *membrane of Reissner*, divides the scala vestibuli into a small triangular compartment known as *canal cochlea* (CC) containing some fluid. A part of the vibration due to waves may pass across this region to the round window. The upper compartment contains **basilar membrane** (BM) and the **organs of corti** placed upon it. The membrane is connected to the auditory nerves of the brain through the *hair cells* in corti.

The sonorous vibrations in the air are collected by the pinna and are led to the auditory meatus. The waves, as they travel down the ear passage, are more and more condensed and finally they impinge upon the ear drum, causing it to vibrate likewise. When the ear drum vibrates, it sets the malleus into similar vibration and thus a similar motion is handed on to the membrane of the fenestrá ovalis through the anvil and the stirrup. The vibration of fenestra ovalis is communicated to the fluid and throws into resonance that part of the basilar membrane which has a natural frequency equal to that of the incoming waves. The vibrations of the membrane set those hair-cells of corti into resonance which have the same natural frequency. Thence they are ultimately carried to the brain where the sensation sound is perceived.

Although not directly connected with the sensation of hearing the internal ear is provided with *three* semi-circular bony pipes N containing some fluid. These three pipes have got their bends in three perpendicular planes and are inter-connected with one another. They are known as *semi-circular canals* and are suppossd to maintain the stability of a man while standing up or walking. Any disturbance of the fluid in any one of these pipes would cause temporary instability.

EXERCISES ON CHAPTER V

Reference

1. Explain the terms loudness, pitch and quality of sound. (Anna. U.—1961; P. U.—1969; Dac. U.—1962; Utk. U.—1968) Art. 36

2. What is the essential feature of a musical note which distinguishes it from noise? (P. U.—1953, '69; C. U.—1951) Art. 36

3. Distinguish clearly between the loudness and pitch of a musical note. On what physical conditions of the body do they respectively depend? (Dac. U.—1940, '52, '63; C. U.—1956; Pat. U.—1958; P. U.—1955, '63, '66) Art. 36

4. Describe some form of siren and explain how you would use it to determine the frequency of a note. (Utk. U.—1961; Gau. U.—1951, '65; C. U.—1950, '63, '66; Pat., U.—1957, '58, '70) Art. 38

5. A siren having a ring of 200 holes is making 132 revolutions per minute. It is found to emit a note which is an octave lower than that of a given tuning fork. Find the frequency of the latter. (Gau. U.—1965) Art. 38

Ans. 800 per sec.

6. Describe some form of siren and explain its action. The disc of a siren is making 10 revolutions per second. How many holes must it possess in order that it may be in unison with a tuning fork of frequency 480? (Dac. U.—1962) Art. 38

Ans. 48.

7. Describe a direct method of determining the frequency of a tuning fork. (Del. H. S.—1959; Del. U.—1960, '62) Art. 38

8. The cylinder of a Duhamel's Vibroscope is of diameter 14 cms. When it is rotated once per second, the number of wavy curves present in a length of 44 cms. on the drum is found to be 100. Find the frequency of the tuning fork tracing such a curve. ($\pi = 22/7$). Art. 38

Reference

Ans. Since the radius of the cylinder is 7 cms. and as it undergoes one revolution per sec., the distance moved through by *any* point on the cylinder in one second $=(2\times7\times\frac{22}{7})$ cm. $=44$ cms.
As the number of waves in 44 cms. on the drum is 100,
,, ,, ,, ,, 44 cms. ,, $=\frac{100}{44}\times44=100$.
Hence the frequency of the fork $=100$ cycles per second.

Art. 47 — 9. Describe a human ear with a sectional diagram. (*Cf.* Guj. U.—1964 ; C. U.—1962)

Art. 47 — 10. Give a brief description of the human ear with a neat diagram and mention the functions of the different parts. (C. U.—1958)

Art. 41 — 11. What are musical intervals. Explain the various ways of dividing an octave into musical intervals. (B. H. U.—1960)

Art. 42 — 12. What is meant by diatonic scale? (Bom. U.—1962 ; Guj. U.—1953, '65 ; E. P. U.—1960)

Art. 43 — 13. Write a note on Tempered Scale. (Bom. U.—1970 ; Guj. U.—1961, '65)

Arts. 41 & 45 — 14. Define musical interval, harmony, melody and chord. Show that the interval *sa* and *ga*, is obtained by multiplying the intervals *sa* and *re*, *re* and *ga*, but not by adding them. (P. U.—1958)

Art. 41 — 15. Write the relative frequencies of notes comprising a diatonic scale and find the interval between each pair of consecutive notes. (P. U.—1953, '65)

Art. 41 — 16. Give the relative frequencies of notes in a diatonic scale showing where half notes come in. (P. U.—1969)

Arts. 40 & 41 — 17. What do you understand by the terms fundamental notes, musical interval and octave? Illustrate these by reference to diatonic scale and show that the interval between two notes is equal to the product of the notes lying between them. (P. U.—1968)

CHAPTER VI

VIBRATION OF STRINGS

48. Vibration of a String—In Physics a string is defined to be a portion of a material medium having a length very large as compared to its breadth or thickness. Further, it must have uniform density and cross-section. A metal wire is a good example of a string. A piece of wire, fixed at one end and free at the other, can be made to vibrate either transversely or longitudinally. If a sudden jerk is given at the free end at right angles to the length of the wire, transverse pulse, which travels towards the fixed end with a definite velocity, is generated.

When, however, a series of waves is set up a continuous vibratory motion at the free end, these waves are reflected back from the fixed end. These two systems of waves are superposed along every point of the string, giving rise to stationary waves. In the case of a string stretched between two points, stationary waves are also set up when it is plucked, struck or bowed.

Longitudinal stationary waves can be generated in the wire or a rod fixed at one end, by rubbing it along its length with a piece of chamois leather smeared with resin powder or with a piece of wet flannel.

***49. Velocity of Transverse Waves along a String**—Set up a wave in a string under a fixed tension and let the wave move with a uniform velocity V in the direction of the arrow (Fig. 57). If the string as a whole be fixed in space, this wave curve would appear to a fixed observer to move from the left to the right over the string with a velocity V. If now the string system be assumed to move from the right to the left with an equal velocity, the *wave curve* would appear to the observer as *fixed* ; but the string would appear to move through the wave curve with a velocity V in the opposite direction. If the top portion of the wave curve be assumed circular, the previous assumption leads to the case of a uniform circular motion of *small part* of a string having a tangential velocity V, which is equal and opposite to the wave velocity. Consider *any* point P on the curved portion and imagine two very small lengths P*a* and P*b* on either side of the point. Since the length *ab* of the curve is very small, its curvature may be assumed to be uniform. Of this curved portion let O be the centre of curvature. Join O*a* and O*b*. At the points *a* and *b* draw two tangents (on a suitable scale) away from P, the length of each being proportional to the tension of the string.

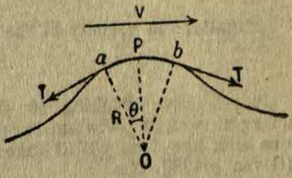


Fig. 57– Transverse pulse

Consider the equilibrium of the small element *ab* of the string in displaced state. Since the element is moving with a definite velocity V along the curvature, it is acted on by a centripetal force in the direction PO towards the centre O of the circular arc. Hence the sum of the components of the tensions at *a* and *b* in the direction PO supplies the necessary centripetal force.

Let the uniform tension of the string $= T$
,, length of each of the elements P*a* and P*b* $= s$
,, mass per unit length of the string $= m$
,, radius of curvature of *ab* $= R$
,, angle subtended by *ab* at O $= 2\theta$
and ,, linear velocity of the wave $= V$

Then the component of the tensions acting at both ends of the element *ab* along PO is $= 2T \sin\theta = 2T\theta$, (since θ is small)

$$= 2T \cdot \frac{s}{R}.$$

Also the centripetal force of the elements $= \dfrac{2msV^2}{R}$.

Hence for equilibrium we have the equation

$$2T \cdot \frac{s}{R} = \frac{2ms\,V^2}{R}, \text{ whence } V = \sqrt{\frac{T}{m}}. \qquad \ldots \quad (49,\ 1)$$

Examples :

1. Find the velocity of transverse wave in a string under a tension of 0·23 kgm., the mass of the string for a length of 65 cm. being 0·52 gm.

Ans. The tension in absolute units $= (0{\cdot}23 \times 1000 \times 981)$ dynes.

Also the mass per unit length of the string $= \frac{0{\cdot}52}{65}$ gm. $= 0{\cdot}003$ gm.

Finally, the velocity of the waves $= \sqrt{\frac{0{\cdot}08 \times 1000 \times 981}{0{\cdot}008}}$ cm./sec.

$= 5310$ cm./sec.

2. A rope weighing 50 grams per anti metre is stretched by a tension of 25 kgm. weight between to points, 30 metres apart. If the rope is plucked at one end, how long will it take the resulting disturbance to reach the other end? (Given $g = 980$ c.g.s.)

Ans. In absolute units the tension $T = (25 \times 1000 \times 980)$ dynes and the mass per length of the rope $= (50 \div 100)$ gm./cm. $= 0{\cdot}5$ gm. per cm.

$\therefore$ the velocity V of the rope wave $= \sqrt{\frac{25 \times 100 \times 980}{0{\cdot}5}}$ cm./sec. $= 7000$ cm./sec.

Hence the time t for the pulse to reach from one end of the rope to the other is $= \frac{\text{distance}}{\text{velocity}} = \frac{3000}{700}$ sec. $= \frac{30}{7}$ sec.

50. Sonometer (or Monocord)—The apparatus consists of a wire SS stretched over a hollow wooden box AA, about a metre long (Fig. 58). One end of the string is attached to a peg fixed on the box, while the other end passes over a pulley and carries a pan P, on which loads are to be placed. There are two fixed bridges B,B, while a movable bridge C slides under the wire and can fix up *any* known length of the wire to be put into vibration. A second wire S′S′ is

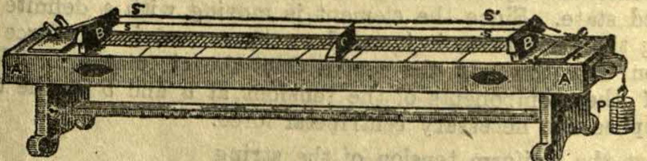


Fig. 58—Sonometer

sometimes used for *comparison* or *reference* and is stretched parallel to the first wire. To read the length of the portion of the string under vibration a metre scale is fixed along the entire length of the sonometer box.

Frequency of Vibration of String—Take a piece of wire of some length stretched over two bridges. When any part of the wire between the bridges is plucked with finger, a wave is set up in it. The wave is reflected again and again from the bridges and gives rise to a system of stationary waves. In the simple form of vibration the wire has an antinode at the middle and nodes at the two bridges. Therefore, the length between the bridges is equal to half that of the wave.

Let the wave-length of the wire $=\lambda$
,, frequency of vibration on the wire $=n$
,, velocity of component wave along the wire $=V$
and ,, length of the wire between the two bridges $=l$

Then from above consideration if follows that $\lambda=2l$.

Also, by equ. (49, 1), $V=\sqrt{\frac{T}{m}}$. Moreover, $V=n\lambda$.

$$\text{Hence } n=\sqrt{\frac{T}{m}}, \quad \text{whence, } n=\frac{1}{2l}\sqrt{\frac{T}{m}}, \quad \dots \quad (50,\ 1)$$

where m denotes the mass per unit length of the wire, and T the tension. If the radius of cross-section of the wire be r, the area of the cross-section is πr^2. Also the volume of the wire per unit length is πr^2. If the density of the material of the wire be ρ, the mass per unit length m of the wire is given by $m=\pi r^2\rho$.

$$\text{Hence from (49,1) it follows that } n=\frac{1}{2l}\sqrt{\frac{T}{\pi r^2\rho}}=\frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}} \quad \dots \quad (50,2)$$

Examples :

1. **Find the frequency of the note emitted by a string 50 cms. long stretched by a load of kilograms, if 1 metre length of the string weighs 2·45. gm. ($g=980$ cms./sec.^{2})** [E. P. U.—1963]

Ans. Here the tension $T=10{,}000\times980$ dynes, the mass per unit length ($m=2{\cdot}45\div100$) or $\cdot0245$ gm./cm. and the length l of the vibrating segment $=50$ cm.

$$\therefore\ n=\frac{1}{2\times50}\sqrt{\frac{10000\times980}{\cdot0245}}.$$

Consulting the log table, we get

$$\begin{array}{r|r|r|r} \log 10{,}000=4{\cdot}0000 & 6{\cdot}9912 & 2\ \underline{|\,8{\cdot}6020} & 4{\cdot}301 \\ \log\ 980=\underline{2{\cdot}9912} & \log \cdot0245=\underline{\bar{2}{\cdot}389} & 4{\cdot}301 & \log\ 100=\underline{2{\cdot}000} \\ 6{\cdot}9912 & \text{diff}=8{\cdot}6020 & & 2{\cdot}301 \end{array}$$

Antilog $2{\cdot}301=200{\cdot}0$ $\therefore$ the frequency $n=200$ cycles/sec.

2. **A wire 23 cms; in length and 0·25 gm. in weight, produces, when plucked, a fundamental note of frequency 200. What is the tension in gm. wt. in the wire ? (Given $g=980$ cm./sec^2.)** [Pat. U.—1952]

Ans. Here $l=25$ cms. ; mass m per unit length $=0{\cdot}25/55$ gm./cm. $=0{\cdot}1$ gm./cm. and $n=200$. For the sonometer we now get

$$\frac{1}{9\times35}\sqrt{\frac{T}{\cdot01}}=200, \text{ whence the tension } T=10^6 \text{ dynes.}$$

$$=\frac{10^6}{980} \text{ gm/wt. (since 980 dynes}=1\text{ gm. wt.)}$$

$$=1020{\cdot}4 \text{ gm./wt.}$$

3. **A copper wire, 100 cms. long and 1·8 mm. in diameter, vibrates 47 times per second when stretched with 20 kilos. Calculate the density of copper.**

Ans. Usual notation being stuck to, $\rho=\frac{T}{4l^2r^2\times n\pi}$.

Here $l=100$ cms., $r=0{\cdot}09$ cm., $n=47$ and $T=(20\times1000\times981)$ dynes.

$$\therefore\ \rho=\frac{4}{4\times100^2\times(0{\cdot}09^2)}\times\frac{20\times1000\times981}{47^2\times3{\cdot}14} \text{ gms./c.c}=8{\cdot}72 \text{ gm. c.c.}$$

4. Two tuning forks A and B produce 4 beats per second when sounded together. A resounds to 32·4 cms., of stretched wire, while B is in unison with 32 cms. of the same wire. Find the frequencies of the forks. [Mysore U.—1952]

Ans. **Let T be the tension, and m the mass per unit length, of the stretched wire. If the fundamental frequencies of lengths 32·4 cms. and 32 cms. of the wire be n_1 and n_2 respectively, we have,**

$$n_1=\frac{1}{2\times32}\sqrt{\frac{T}{m}} \text{ and } n_2=\frac{1}{2\times32}\sqrt{\frac{T}{m}}, \text{ Or } \frac{n_1}{n_2}=\frac{64}{64{\cdot}8}.$$

Evidently n_2, is greater than n_1. Since the two tuning forks have frequencies n_1, and n_2, for 4 beats to be produced per sec. $n_2-n_1=4$, i.e., $n_2=n_1+4$. Substituting the value of n_2, in the expression for frequency ratio, we obtain

$\frac{n_1}{n_1+4}=\frac{64}{64{\cdot}8}$, or $n_1=320$ and so $n_2=324$. Thus the frequencies are found.

5. A steel wire, 0·4 cm. in radius, is fixed to a rigid support at one end and is wrapped round a cylindrical tuning peg, 0·25 cm. in radius, at the other end. The length of the wire between the peg and the support is 65 cm., when the wire is stretched and the tension is negligible. What would be the frequency of the wire, if it is tightened by giving the peg a quarter of a turn? Given that the density of steel is 7·6 gms./cc. and that its Youngs' modulus is $2{\cdot}0\times10^{12}$ dynes/cm.².

Ans. **The circumference of the cylindrical peg of length $2\pi r$ is $=(2\times3{\cdot}14\times0{\cdot}25)$ cm.**

So for a quarter turn it coils the wire by the length $\frac{2\times3{\cdot}14\times0{\cdot}25}{4}$ cm., *i e*, $(1{\cdot}57\times0{\cdot}25)$ cm.

$$\text{Again Young's modulus}=\frac{\text{force/area of cross-section}}{\text{increment in length/original length}},$$

$$\text{or } 2\times10^{12}=\frac{\text{Force}/3{\cdot}14\times({\cdot}65)^2}{1{\cdot}57\times0{\cdot}25/65}, \text{ or } \frac{\text{Force}\times65}{3{\cdot}1\times({\cdot}04)^2\times1{\cdot}57\times{\cdot}25}=2\times10^{12},$$

$$\text{whence the tension on wire}=\text{Force}=\frac{2\times10^{12}\times3{\cdot}14\times({\cdot}04)^2\times1{\cdot}57\times{\cdot}25}{65}\text{ dynes.}$$

Again mass per unit length of the wire $=\pi r^2\rho=3{\cdot}14\times({\cdot}04)^2\times7{\cdot}8$ gm. Hence the frequency (n) of the fundamental tone is given by

$$n=\frac{1}{2\times65}\sqrt{\frac{2\times10^{12}\times3{\cdot}14\times({\cdot}04)^2\times1{\cdot}57\times{\cdot}25}{60\times3{\cdot}14\times({\cdot}04)^2\times7{\cdot}8}}=\frac{1}{130}\sqrt{\frac{2\times10^{12}\times1{\cdot}57\times{\cdot}25}{65\times7{\cdot}8}}.$$

Fron the log table we found :

log 2 = ·3010	log 65 = 1·8129	log 130 = 2·1159
log 10^{12} = 12·0000	log 7·8 = ·8921	
log 1·57 = ·1959	2·7050	4·5949
log ·25 = $\bar{1}{\cdot}3979$		2·1132
11·8948	219·1898	2·4810 ; antilog 2·481 = 302·7
2·7070	4·5949	
9·1898		

∴ the frequency n is 302·7 cycles per second.

51. Laws of Transverse Vibration of Strings : their Verification—

It is evident from (50·1) that the frequency of a note given out by the transverse vibration of strings depends upon the length, tension and mass per unit length of the string.

(i) **Law of length :** $n\propto\frac{1}{l}$, when T and m are constants ; *i.e.*, *the frequency of transverse vibration at a stretched string is* inversely *proportional to its vibrating length* for the same or similar string (m remaining constant) under a constant stretching force.

Take a number of tuning forks of frequencies n_1, n_2, n_3, ... Stretch a sonometer wire with a fixed load. Strike the tuning fork of frequency n_1 and place it on the board. Adjust the length of the wire by a movable bridge till it is in unison with the fork. Measure the length of the vibrating segment of the wire. Take three readings for the length and find the mean. Call it l_1.

With the other forks repeat the same operation. The resultant length of the wire is found to vary with different tuning forks. Call the mean lengths l_1, l_2, l_3,...corresponding to the forks of frequencies n_1, n_2, n_3,...... Then

$$n_1 l_1 = n_2 l_2 = n_3 l_3 = \dots$$

$$i.e., \quad nl = \text{a constant}.$$

The relation between n and l is graphically represented by a rectangular hyperbola. (*Cf.* Boyle's Law : PV = const.)

(ii) **Law of Tension** : $n \propto \sqrt{T}$, when l and m are constants ; *i.e.*, for a given length of the same string or a similar string, *the frequency varies* directly *as the square root of the stretching force.*

Take the sonometer provided, in addition to its main wire, with a reference wire under some tension. Stretch the main wire with a known tension T_1. Tune a certain length of it with a length l_1 of the reference wire. Change the tension of the wire to T_2 without alteration in its length. Tune it again with another length l_2 of the reference wire. In this way find a number of lengths l_3, l_4... of the reference wire corresponding to the tension T_3, T_4... of the main wire.

Now by the law of length, the frequencies n_1 and n_2 of the reference wire are inversely proportional to the length l_1 and l_2. This means that $l_1 : l_2 = n_2 : n_1$.

It is found experimentally that $\frac{T_1}{T_2} = \frac{l_2^2}{l_1^2}$.

$$\text{Thus } \frac{T_1}{T_2} = \frac{n_1^2}{n_2^2}, \ i.e. \ \frac{n_1^2}{T_1} = \frac{n_2^2}{T_2}$$

$$n \propto \sqrt{T}.$$

(iii) **Law of Mass** : $n \propto \frac{1}{\sqrt{m}}$, when T and l are constants ; *i.e.*, when the tension and the length are kept constant, *the frequency varies* inversely *as the square root of the mass per unit length.*

Take a sonometer provided with a reference wire. Stretch a piece of wire over the movable bridges of the sonometer. Let the mass per unit length of the wire be m_1. Take any fixed length of this wire and tune it with a length l_1 of the reference wire. Under the same load replace another wire of mass m_2 per unit length, Take a length of it equal to that of the first wire and tune it with a length l_2 of the reference wire.

Experiments demonstrate that $\frac{m_1}{m_2}=\frac{l_1^2}{l_2^2}$. But $\frac{l_1^2}{l_2^2}=\frac{n_2^2}{n_1^2}$, by the First Law.

$$\text{Hence } \frac{m_1}{m_2}=\frac{n_2^2}{n_1^2},$$

or $m_1n_1^2=m_2n_2^2$, which means $mn^2=\text{constant}$, *i.e.*, $n\propto\frac{1}{\sqrt{m}}$.

Example :

A string 50 cms. long and stretched by a weight of 16 kgms. makes 256 vibrations per second. How could the frequency of the note be raised to 384, (*a*) by altering the length of the string, (*b*) by altering the stretching weight ?

(Gau.—1962)

Ans. **When the length is altered, other things remaining constant, we have from the law of length that $n_1l_1=n_2l_2$.**

Thus $348l_2=256\times50$, whence the altered length $=33{\cdot}5$ cm.

Again when the tension is changed, other things remaining constant, the law of tension gives

$$\frac{n_1^2}{T_1}=\frac{n_2^2}{T_2},\ \text{or,}\ \frac{384^2}{T_2}=\frac{246^2}{10},\ \text{whence the required tension}=22{\cdot}5\ \text{kgm.}$$

52. Determination of Pitch by Sonometer—The pitch of a musical sound (*e.g.*, a tuning fork) may be determined with a sonometer. Take a sonometer board and stretch a metal wire of a suitable diameter over it. Tie one end of the wire to the terminal peg and suspend a weight hanger at the other. Now place the wire over the sonometer pulley, so that the hanger is freely suspended. Place a load of 1 or 2 kilograms on the hanger so as to stretch the wire with some tension and put two movable bridges under the wire.

Strike the tuning fork against a padded hammer and place it upon the board. The tuning fork gives out a uniform sound. Now pluck the sonometer wire in between the bridge and compare the two sounds. If the note of the wire appears *more acute* than of the tuning fork, increase the vibrating segment of the wire by drawing apart the bridges In this way adjust the length of the string between the two bridges till the string on being plucked at this region gives out a note of the same pitch as that of the tuning fork. When the two notes are in unison, a small paper rider, on being placed upon the string, will be thrown away as soon as the stem of the vibrating tuning fork is placed on the board. When such a condition is attained. measure the length of the string between the two bridges. For the same stretching load of the wire take three or four readings of the resonant length. Call the mean length l cm.

Now take a sample of wire of the same specification as the sonometer wire. Measure the length of this sample wire accurately by taking a number of readings. Suppose that the mean length of this wire is L cm. Then measure the mass of the wire. Let the mass is recorded from 2 or 3 observations be M gm. Then the mass per unit length m of the wire is M/L gm/cm. Express the stretching force in absolute units and call it T dynes. Thus knowing l, m and

T, the frequency n can be determined from equ. (50,1). (For further details, vide J. Chatterjee's *Intermediate Practical Physics*).

***53. Harmonics in a String**—When a string vibrates in on *segment* (Fig. 59, I) there are two nodes at the fixed ends and an antinode at the middle. This is the simplest type of vibration of the string and its frequency is given by equ. (50, 1). The sound emitted by the string under this condition is called the fundamental tone or the *first harmonic*.

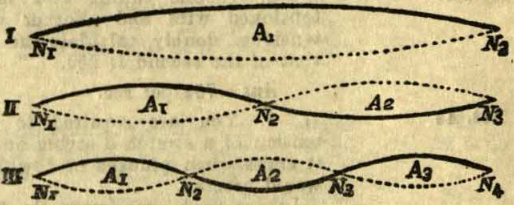


Fig. 59—Harmonics in String

When the string under some stretching load is held softly at its middle point and bowed, it vibrates into equal segments and emits a more acute sound. There are now three nodes present in the string,—two at the ends and one at the middle (Fig. 59, II). If the length of the string be l, it is obvious that $l = \lambda$. Hence from equ. (50, I) it follows that

$$n_2 = \frac{1}{\lambda}\sqrt{\frac{T}{m}} = \frac{1}{l}\sqrt{\frac{T}{m}} = 2n,$$

where n_2 is the frequency of this note. The frequency n_2 is double that of fundamental n and is called its **second harmonic.** In the standard musical scale, this doubling of a frequency is known as a higher octave of a scale.

If the string is similarly held at one-third of its length and if its shorter segment is bowed, it vibrates into three segments of equal length (Fig. 59, III). A still more accute sound is audible at this stage. Here $l = \frac{3}{2}\lambda$, whence n_3 is three times the fundamental tone and is called its **third harmonic**. In this way a particular wire may be made to vibrate into any number of segments and emit its higher harmonics.

Under the circumstances stated above, the frequencies of the harmonics are multiples of the frequency of the fundamental. When the string is bowed at random, it vibrates into segments which are not very regular and the frequencies of the higher tones are not always exact multiples of that of the fundamental. These higher tones are known as **overtones** or **upper partials.**

EXERCISES ON CHAPTRR VI

Reference

1. Two similar wires of equal length are stretched weights of 4 lbs. and 9 lbs. Compare the frequencies of the notes emitted by the wires, when they are vibrating transversely. **Art. 49**

Ans. 2 : 3.

2. A stretched wire under a tension of 1 kgm. weight is in unison with a tuning fork of frequency 320. What alteration in **Art. 49**

Reference

tension would make the wire vibrate in unison with a fork of frequency 256 ? [B. H. U.—1971]

Ans. 0·36 kgm.

Art. 49 3. If one tuning fork is in unison with 128 cm. of a tensioned wire and another with 130 cm. of the same wire tensioned doubly, calculate the frequency of the first fork, if that of the second is 250. [Utk. U.—1969]

Ans. 752 per sec.

Art. 49 4. You are required to find the relation between the tension of a stretched string and the frequency of a note which it emits when plucked or bowed. Explain in detail how you would proceed.

In one such experiment the following results were obtained ;—

frequency···	260	320	80	480	520
tension ...	1·69	2·46	3·61	5·76	6·76 lb.

Plot a graph to show in the clearest possible manner. the relation between the frequency and tension. State in words the conclusion to be drawn form results. [Dac. U.—1961]

Art. 51 5. Describe how you would prove that the frequency of transverse vibration of a string is proportional to the square root of the stretching force. [U. P. B.—1966]

Art. 51 6. Upon what factors does the vibration frequency of a stretched string depend ?
(*Cf.* Poo. U.—1960 ; E. P. U.—1953 ; Raj. U.—1962, '73]

Art. 51 7. Calculate the pitch of fundamental note of a string 25 cms. in length and of mass ·003 gm. per cm., the stretching weight being 3 kilograms. [Utk. U.—1959]

Ans. 626 per sec.

Art. 50 8. A steel wire of length 60 cm. and diameter 8·5 mm. emits a note of 240 vibrations per sec., when stretched with a certain weight. A second steel wire bears the same weight but is of length 40 cm. and diameter 0 6 mm. Find the frequency of its fundamental tone.

Ans. 300 per sec.

Art. 50 9. On increasing the weight of stretching on given string by 2·5 kilograms, the frequency is altered in the ratio 3 : 2. Find the original stretching weight. [U. P. B.—1971]

Ans. 2 kg.

Art. 50 10. A sitar wire is 30 cm. long and it emits a note of 288 vibrations per second. How far from the top may it be pressed, so that it may emit a note of 312 vibrations per second ?

Ans. 6·2 cm.

Art. 50 11. What is the fundamental frequency of transverse vibration of a steel wire, 1mm. in diameter and 1 metre in length, hanging vertically from a rigid support with a mass of 20 kilograms attached to its lower end ? (Density of steel—7 9 gm./cc.) (Utk. U.—1967)

Ans. 85·5 per sec.

Art. 50 12. Two tuning forks when sounded together give 4 beats per second. One is in unison with a length of 26 cm. of a sonometer wire under constant tension, and the other with 97 cm. of the same wire. Find the frequencies of the forks. (Raj. U.—1965 ; Bom. U.—1969)

Ans. 388 per sec. and 384 per sec.

Reference

13. When will be the frequency of the note emitted by a wire, 50 cm. in length, when stretched by a weight 25 kilograms, if 2 metres of the wire be found to weigh 4·79 grams ? Art. 50

Ans. 320 per sec.

14. Two steel wires of the same length emit their fundamental notes, when stretched by 729 gm. and 324 gm The ratio between the frequencies produced is 1 : 2. Find the ratio between the diameters of the two wires. Art. 50

Ans. 3 ; 1.

15. State the laws of fundamental vibration of stretched string. (U. P. B.—1962 ; Gau. U.—1959 ; And. U. 1970 ; V. U.—1952, Bom. U.—1961, *Cf.* B. H. U.—1972) Arts. 50 & 51

16. State the laws of transverse vibration of a stretched string and describe experiments to verify them. (Utk. U.—1959, '73, Bom. U.—1964, C. U.—1950, '64, Del. U.—1951, '63, Pat. U.—1972, V. U.—1954, Guj. U.—1955, '66, B. H. U.—1951, '59, U. P. B.—1960, '63) Art. 51

17. A sonometer wire is stretched with a certain force. If the force is increased 4 times and the length of the wire is reduced to half its original value, how is the frequency of the note altered ? (Bom. U.—1971) Art. 50

Ans. Increased 4 times.

18. Describe some accurate method of determining the frequency of the note emitted from a given source. (Del. U.—1960, '64 ; U. P. B—1952, '63, '75) Art. 52

19. How do you determine the pitch of a musical note, with a sonometer ? (Mys. U.—1951, '72, And. U.—1960, '72, Bom. U.—1961, V. U.—1953, Nag. U.—1972, Dac. U.—1961, '63) Art. 52

20. What are harmonics ? How will you demonstrate their formation in a sonometer wire ? What important part is played by them in musical notes ? (U. P. B.—1960 ; *Cf.* Gau. U.—1965) Art. 53

21. The distance between the adjacent nodes of a string in transverse vibrations is 10 cm. By how much should be the tension be altered so that this distance is halved ? If it is desired to secure the same effect by altering the diameter how should the latter be changed ? (V. U.—1962) Art. 53

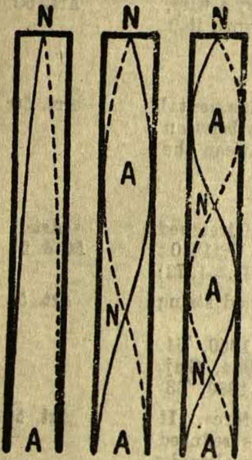
Ans. $T_2 = \frac{T_1}{4}$; $D_2 = 2D_1$.

CHAPTER VII

VIBRATION OF AIR COLUMN

54. Stationary Vibration in Closed Pipes—In one class of musical instruments (*e.g.*, *bugles*, *clarionets*, etc.) the vibration of air within the instruments produces the characteristic notes. For simplicity we shall now study the nature of vibration of air column enclosed within a pipe of uniform cross-section.

Pipe closed at one end—Suppose that there is a uniform pipe closed at one end and open at the other. To set the air column within it into vibration, a source of sound (*e.g.*, a tuning fork) is placed near its open end. When the source of sound vibrates in front of the open end of a pipe, it sends forth within the pipe alternate compressions and rarefactions. These waves travel through the pipe and strike the closed end, whence they are reflected back towards the open end. As they reach the open end, they meet with the freshly generated incoming waves. Thus the two systems give rise to stationary waves and the whole column is thrown into a forced vibration with a frequency equalling that of the tuning fork, when the source of sound is of such a frequency that the air column is thrown into resonance, the nodes and antinodes of the stationary waves occupy fixed positions within the pipe. The air at the open end of the pipe is free to move inwards and outwards with the maximum possible freedom. An antinode is, therefore, produced at this end. On the other hand, the layer of air in contact with the closed end cannot move freely, since the latter acts like a rigid wall. For this reason, a *node* is formed at a closed end.



(a) (b) c)

Fig. 60

Fundamental of a closed pipe—The simplest case of resonance in a pipe occurs when an antinode A is formed at the open end and a node N at the closed end (Fig. 60a). The air has got the maximum amplitude of vibration at the open end, gradually diminishing to no vibration at the closed end. The pulsations of the air column can be well compared to those of a spiral spring of the same length as that of the pipe, and having one terminal attached to the fixed end, the other being free (Fig. 61). On producing slow oscillations at the free end of the spring, this end is found to vibrate with the maximum amplitude. The amplitude gets gradually smaller in the successive coils, till at the fixed end it becomes zero.

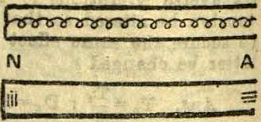


Fig. 61

If l be the length of the pipe, it is clear that the distance between a node and the next antinode of the stationary air waves formed within the pipe is l. Now the distance between any two adjacent node and antinode is a quarter of the wave-length. Therfore $l=\frac{\lambda}{4}$, where λ stands for the wave-length of the constituent wave.

But since $V=n\lambda$ and $\lambda=4l$, it follows

$$n=\frac{V}{4l}. \qquad \dots \qquad \dots \qquad \dots \qquad (54,1)$$

The note, having a frequency equal to the velocity of propagation divided by four times the length of the air column, is called the fundamental or *first harmonic* of the closed pipe. When a sounding source having a frequency equal to the fundamental frequency of the pipe is placed at the open end, the sound is much intensified due to resonance.

Harmonics in a closed pipe—The next possible case of a stationary vibration in a closed pipe is that in which there are two antinodes and two nodes alternately placed along the pipe as shown in Fig. 60(b). In this case the wave-length of the stationary waves is smaller. Call it λ_3, and the corresponding frequency of the note n_3. The spring can also exhibit such a state of vibration, when the oscillations are suitably timed. It is seen from Fig. 60(b) that the length of the pipe l is three-fourths of the wave-length λ_3; so $\lambda_3 = \frac{4}{3}l$. But since $V = n_3 \lambda_3$ and $\lambda_3 = \frac{4}{3}l$, we get

$$n_3 = \frac{3V}{4l} = 3n. \qquad \ldots \qquad \ldots \qquad \ldots \qquad (54,2)$$

The note corresponding to n_3 *has a frequency* three *times that of the fundamental* and is thus the *third harmonic*. When the closed pipe is blown more strongly this note is emitted.

Fig. 60(c) represents the next stage of the vibration, in which the pipe contains three nodes and three antinodes, evenly placed along its length. If λ_5 is the wave-length corresponding to this vibration, then $\lambda_5 = \frac{4}{5}l$, so that

$$n_5 = \frac{5V}{4l} = 5n. \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (54,3)$$

The note corresponding to n_5 *has frequency* five *times that of the fundamental* and is thus the *fifth* harmonic. Such a note is emitted by the pipe, when it is blown still more strongly.

Proceeding in this way, it can be shown that a pipe closed at one end can emit a succession of tones of higher harmonics (along with the fundamental) of the following frequencies :—

n, $3n$, $5n$, $7n$, etc.

It is to be noted that *only* odd *harmonics are present in the vibrations of a* closed *pipe.*

If an organ pipe is taken with a movable plunger or a piston at one end, the note emitted by the pipe is found to rise in pitch, as the plunger is slowly pushed inwards, and to fall in pitch as the piston is pulled outwards.

Examples :

1. What will be the length of a closed pipe, if the lowest note has a frequency of 256 at 20°C ? (Velocity of sound at 0°C = 332 metres per sec.) [E.P.U.—1962]

Ans. **Since the velocity of sound at 0°C is 332 m./sec.**
The velociy at 20°C is 332 (1 + ·00183 × 20) m./sec. = 344·44m/secs.

If the length of the closed pipe be l, and if we assume no end-correction, $n=\frac{V}{4l}$ or $l=\frac{V}{4n}=\frac{344\cdot15}{4\times256}$m. $=0\cdot336$ m. $=33\cdot6$ cm.

2. A closed pipe has a frequency of 250. Find the change of frequency, if the air inside the pipe is replaced by carbon dioxide. (Velocity of sound in air is 350 metres per second, and in carbon dioxide 273 metres per second). [And. U. 1952]

Ans. Let the length of the closed pipe be l cm. Then the velocity of sound V is given by $4nl$, where n is the frequency. Now for air $V=35000$ cm./sec, and $n=250$ per sec.

Hence $250\times l\times4=35000$, whence $l=35$ cm.

For carbon dioxide $V=27300$ cm. per sec.

Hence $n\times4\times35=27300$ whence $n=195$ per sec.

Thus the change in frequency $=(250-195)=55$ per sec.

55. Pipe open at the ends—When a source of sound vibrates in front of an open pipe, it generates a series of alternate compressions and rarefactions within the pipe. These pulses, as they reach the open and, come back as a reflected system; the reflected pulses meeting the open end, come back as a direct pulses give rise to the stationary waves. As the ends have free communication with the outside air, a layer of air at these regions can vibrate with the utmost freedom. When air within the open pipe vibrates in resonance, *antinodes are* always *formed at the* open *ends.*

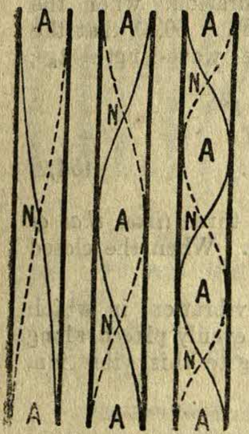


Fig. 62

Fundamental of an Open Pipe—As there are antinodes at the end of the pipe in a stationary vibration, the simplest kind must have a node at its middle. This condition is graphically represented in Fig. 62(a).

The nature of vibration of the air volume in an open pipe can be compared to that of a spring having the same length as that of the pipe but fixed at the middle part (Fig. 63). In this case oscillations must be generated simultaneously at both ends of the spring. It is evident from Fig. 62(a) that if l be the length of pipe, $l=\frac{1}{2}\lambda$ where λ is the wave-length of *fundamental* tone.

$$\text{Thus} \qquad n=\frac{V}{\lambda}=\frac{V}{2l}. \qquad \dots \qquad \dots \qquad (55,1)$$

It may be noted that the frequency of the fundamental of an open pipe is twice that of the fundamental in a closed pipe of the same length. Hence the tone in the former is more acute than the latter. If an open pipe while producing a note is suddenly closed by hand or otherwise, the note changes to a less sharp one; *i.e.*, its pitch decreases. The effective length of the pipe here may be altered by including a metal plate across the open end. The pitch also may be slightly altered sliding a plate across the open end.

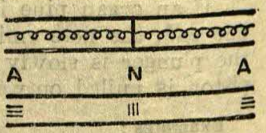


Fig. 63

Harmonics in an Open Pipe—The next possible case of vibration in open pipe is shown in Fig. 62(b) in which the length of the pipe contains three antinodes. The length of the pipe then corresponds to one wave-length. This means that $l = \lambda_1$. Accordingly,

$$n_2 = \frac{V}{\lambda_2} = \frac{V}{l} = 2n. \quad \ldots \quad \ldots (55,2)$$

Hence *the next higher tone is the second harmonic and stands an octave to the fundamental.*

The next higher harmonic is given out when stationary vibrations are represented as in Fig. 62(c), *i.e.*, when $l = \frac{3}{2}\lambda_3$. In that case,

$$n_3 = \frac{V}{\lambda_3} = \frac{3V}{2l} = 3n. \quad \ldots \quad \ldots (55,3)$$

Evidently this is the *third harmonic.* Proceeding in this way, it can be shown that the open pipe can emit notes of frequencies n, $2n$, $3n, \cdots$

Thus *when air vibrates in an open pipe, all the harmonics may be present.* An open pipe is for this rearon more suitable for making organ pipes and other wind intruments.

Examples :

1. The pitch of the fundamental note of an open organ pipe, 100 cm. longs, is the same as that of a sonometer wire 200 cm. long and of mass one gram per cm. Find the tension of the wire. (Velocity of sound in air = 332 metres per sec.) [Pat. U.—1953]

Ans. For an open pipe of length l cm. the pitch of the fundamental note if given by

$$n = \frac{V}{2l} = \frac{33200}{200} = 166 \text{ which is the fundamental of the given pipe.}$$

This frequency is the same as that of a sonometer wire of length 200 cm. and of mass of 1 gm. per unit length. Hence utilising the formula, we find that if the tension be T dynes,

$$\frac{1}{2\times 200}\sqrt{\frac{T}{1}} = 166, \text{ whence } T = (166\times 400)^2 = 440896\times 10^4 \text{ dynes.}$$

2. Open at both ends, an organ pipe 2 ft. 6 in. long, produces 5 beats per second with a similar pipe slightly short in length and the same number of beats with tuning fork at 0°C. Calculate the number of beats produced by the shorter pipe with the tuning fork at 22°C. (Velocity of sound at 0°C is 1100 ft. per sec. and at 23°C is 1144 ft. per sec.).

Ans. For an open organ pipe of length l and for a velocity of sound v, the frequency n of the fundamental note is given by

$$n = \frac{V}{2l} = \frac{1100}{2\cdot 5} \text{ per sec.} = 440 \text{ per sec. at } 0°\text{C.}$$

The general formula $2nl = V$ shows that a slightly shorter open organ pipe would have a slightly higher frequency. If further 5 beats are produced when the two organ pipes are sounded together, the slightly shorter organ pipe must have a frequency 440+5 or 445 per second. Since the tuning fork prodoes 5 beats with the longer pipe, it has a frequency of either 435 or 445 per second.

The length of the shorter pipe at 0°C is 1100/890 ft. Neglecting the slight alteration in length 22°C, the fundamental frequency at this temperature is clearly

$$= \frac{1144\times 445}{1100} \text{ vib./sec.} = 463 \text{ vibrations per second (nearly).}$$

Thus with a shorter pipe at 22°C and with a tuning fork frequency 435 or else 445 the number of beats heard is either 463 − 435, *i.e.*, 28 per sec. or else 463 − 445, *i.e.*, 18 sec.

56. Resonance Air Column—A tall glass jar G (Fig. 64) is nearly filled with water. A glass tube T open at both ends is partly dipped into water and is kept vertical by a suitable clamp C. The surface of water standing within the glass tube forms the closed end. Since the tube can be fixed with any length exposed, it is an arrangement for a closed pipe of adjustable length.

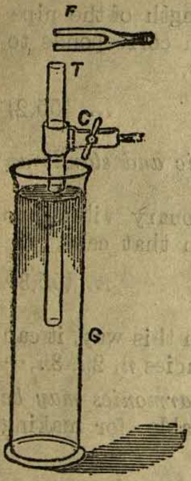


Fig. 64

Experiment—Hold the pipe with a very small part above water. Strike the tuning fork, whose frequency is to be determined, and hold it near the upper end of the glass tube. Gradually, increase the length of the exposed column till there is resonance. Measure the length from the surface of water to the open end of the pipe and call it l. At this stage, $n = V/4l$, by eqn. (54,1). So knowing the value of V, we can determine n. In actual practice, a number of readings for l is taken, both when the pipe is gradually raised up and lowered down.

Rayleigh, however, made a correction for the open end. He worked out mathematically that the antinode is not exactly at the open end, but is at a distance of $0{\cdot}6 \times r$ beyond this, where r is the radius of the tube. This is known as the *end-correction*, So according to Rayleigh, if l be the actual length of the exposed part of the pipe at resonance, the effective length between the node and the antinode of a stationary wave is $l + 0{\cdot}6 \times r$. Taking the first resonance and making the end correction, the frequency of vibration n is, by eqn. (47,1), given by :

$$n = \frac{V}{4(l + 0{\cdot}6 \times r)}. \quad \ldots \quad \ldots \quad (56,1)$$

As the end-correction is rather uncertain it may be eliminated by the following procedure. Raise the tube gradually till a second resonance is obtained. The exposed column is now three times the original length. The fundamental of this air column is one-third of n so that the next higher (odd) harmonic is $3n$. The fundamental of the tuning fork is then resonant with the first odd harmonic of the air column.

If the lengths of the air column in the two cases be l and l' and if the end-correction be x. then $\lambda = 4(l + x)$ and $3\lambda = 4(l' + x)$, whence on elimination of x we get $\lambda = 2(l' - l)$.

As $V = n\lambda$, we find $n = V/\lambda = V/2(l' - l)$ ... (56,2)

The value of n on being known, that of V can be calculated. It is important to note here that the value obtained gives the velocity of sound in moist air above the water column at the temperature of the laboratory. To get the velocity in dry air at 0°C, the value so obtained should be corrected for temperature and humidity. (For details vide J. Chatterjee's *Intermediate Practical Physics.*)

The mechanism, by which the intensity of a sound increases when air resonates in a pipe open at one end, can be explained in the following manner : Imagine a tuning fork of period T to be vibrating in front of the pipe (Fig. 64). At any instant in course of its vibration, when a prong moves from B towards C, it is just sending forth a pulse of compression, which travels down the length of the pipe. As a pulse of compression it reflects at the closed end E and travels upwards. When the pulse comes back to the end D it is again reflected at the end E without changing its nature and travels towards D. As it finally reaches D, it is transformed into a pulse of compression. Hence starting along DE, a wave of compression at D becomes a wave of compression at the same place after travelling *four* times the length DE. If V be the velocity of sound within the tube and l be the distance DE, the time to move through the distance $4l$ is $4l/V$. If during this time the prong moves from B to C, C to A and back to B, *i.e.*, if it executes one complete vibration, it is just producing another pulse of compression, which will be superposed on the previous pulse, so that the amount of compression, will be greater now. Therefore, as the fork vibrates, the magnitude of condensation (and hence the extent of oscillation of the layer at the mouth of the pipe) is gradually increased. The loudness of the sound being dependent on the amplitude of vibration, it will thus be magnified enormously.

Hence $T=\frac{4l}{V}$ or $n=\frac{V}{4l}$ since $n=\frac{1}{T}$. So $V=4nl$.

Example :

Sounded over a resonance tube, a tuning fork shows resonance with an air column 24·0 cm long and again with a column 74·1 cm. long. Explain this and calculate the frequency of the fork and the end-correction of a tube, the velocity of sound at the room temperature being 340 metres per second.

Ans. A resonance tube is a pipe closed at one end by a liquid level and the effective length of the pipe can be altered. For such a closed pipe, the vibration of the enclosed air column can take up any *odd* multiple of its frequency n, (such as n, $3n$, $5n$, etc.) where n is given $V/4l$, V being the velocity of sound and l the length of the air column within the pipe.

Now if we take a resonance tube and adjust its effective length so that its fundamental frequency is n then on exciting a number of tuning forks of frequencies n, $3n$ and $5n$, each such fork would resonate with the tube, although the effect is less as the harmonics are higher. Again, if we take only one tuning fork with a frequency n it would resonate with the tube of length l emitting its fundamental tone. If now we increase the effective length of the tube till it comes to $3l$ or very near it, the fundamental of the tube changes to a frequency n such that

$$n=\frac{V}{4\times 3l}=\frac{1}{3}\cdot\frac{V}{4l}=\frac{n}{3}=\frac{1}{3}\times\text{(the original fundamental frequency).}$$

For this length of the pipe the new resonant frequencies are n', $3n'$, $5n'$, etc. Now $3n'=n$. So the fork of a frequency n would also actuate the resonance tube when the length is made very nearly thrice the original.

If $l=24{\cdot}0$ cm. $l'=74{\cdot}1$ cm. λ= the wave-length and x= the end-correction,

$\lambda=4(l+x)=4(24+x)$ and $3\lambda=4(l+x)=(74{\cdot}1+x)$.

Thus $3\times4(24+x)=4(74{\cdot}1+)$, whence $x=0{\cdot}2$ cm.

Again $\lambda=2(l'-l)=100{\cdot}2$ cm. = wave-length of fundemental frequney.

Thus $n=\frac{V}{\lambda}=\frac{34000}{100{\cdot}2}$ m /sec. $=338{\cdot}3$ m./sec.

57. Organ Pipes—Broadly speaking, there are two classes of musical instruments working upon the vibration of air column. These are (*i*) fixed lip instruments and (*ii*) vibrating reed instruments. These have already been described in Art. 2.

Position of Nodes and Antinodes—It has already been stated that the air is unable to vibrate at the closed end of a pipe, so that a node is always formed at this region. But at the end itself the air will suffer considerable compression and rarefaction due to forward or backward movements of air in its vicinity and so here the variation of pressure would be the greatest.

Again, at the open end the air is free to move and a slight variation of pressure, would cause considerable movement of air which is, therefore, not affected by pressure ; thus the variation of pressure at the open end would be very small. Hence, in general, where the displacement is maximum, the variation in pressure is minimum and *vice versa*. The positions of the nodes and antinodes in a vibrating air column, as in an open organ pipe, were demonstrated by Koenig in the following way.

A wooden organ pipe is taken, one side of which is replaced by a glass plate. A light pan is suspended by a long string, on which a little sand is sprinkled. The organ pipe is held in a vertical position with its mouth downward and is sounded. The pan with the sand is now slowly lowered into the pipe. When the pan is at a position where the air vibrates, the motion of air is communicated to the pan. Due thereto the sand particles are seen to the dance on the pan. It is found that at certain positions, the sand particles remain practically motionless. Such places are nodes. At certain others, they dance with maximum vigour and the rattling sound caused by the collision of the sand particles with the membrance can be distinctly heard. Such places are antinodes.

The **variation of pressure** in a gas due to sound waves can be demonstrated by the **manometric flame** method, also due to Koenig. The apparatus consists of a stretched elastic thin rubber membrane B dividing a chamber into two compartments (Fig. 65). One of these may be connected to the speaking the tube T, while the other forms a reservoir for coal gas and is connected to two narrow pipes. Either of these pipes C acts as the inlet for gas, while the other D terminates in a pin-hole burner. By turning the tap attached to C, the pressure of the gas can be so adjusted that the flame F becomes very sensitive to slight alteration of pressure. At this stage a slight rise of pressure will increase the height of the flame to a marked extent, while if the pressure falls very slightly, the flame is extremely small.

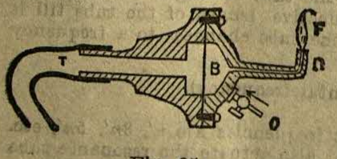


Fig. 65

To show nodes and loops of an organ pipe, holes are bored at regular intervals on the body of the pipe and a manometric flame is fitted at the hole.

If now a sound is produced in the organ pipe, the membrane will be set into a periodic vibration in proportion to the amount of pressure falling upon it. This membrane in its turn communicates the same changes of pressure to the other chamber, which produces a corresponding fluctuation in the flow of the gas to the burner. Consequently, the flame is caused to jump up or down to an extent proportional to the pressure of the sound waves falling upon the diaphragm at any instant. The frequency of these jumps is equal to that of the wave. The manometric flame flickers most at the node but scarcely at an antinode. Since the variation of pressure at the node is very rapid, not all the jumps of the flame can be visualised by the naked eye. Only some sort of flickering may be observed. Provided with plane mirrors on its four sides, a rectangular box is rotated about a vertical axis in front of the flame. Successive stages of the flame are then seen reflected in the mirrors (Fig. 66).

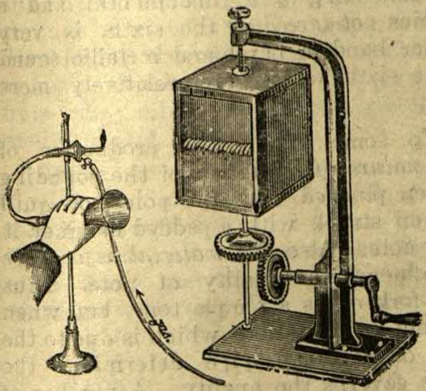


Fig. 66

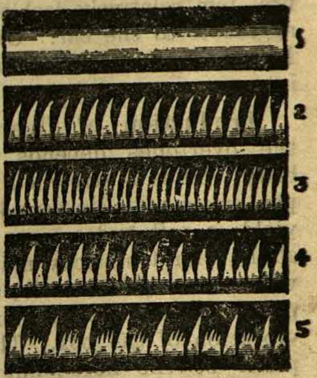


Fig. 67

Figure 67 is an enlarged view of the flame dancing in the revolving mirrors, when different sounds are produced before the collecting funnel. The uppermost one (Fig. (67, 1) resembles a straight and uniform band of light. Obviously the flame is steady and no sound has been produced before the funnel. Figure (67, 2) is that of the flame, when a tuning fork is sounded. Figure (67, 3) represents the type of the flame when another tuning fork having double the frequency is placed before the collector. Not that the teeth of flame are exactly half of those in figure (67, 2). The next figure (67, 4) is due to the two forks being simultaneously sounded. Figure (67, 5) is obtained, when the vowel O is uttered along with the note of the tuning fork used in figure (67, 3). Thus from a study of the forms of the flame reflected in a rotating mirror, the pressure variation in air can be detected. Nowadays an oscilloscope with a microphone is used. Various types of sound curves are formed, which may be examined.

58. **Quality of Musical Note**—As already stated, the quality of a musical note is that property which distinguishes a note produced by one instrument from a note having the same pitch and intensity

but produced by another instrument. A musical sound can be analysed by presenting it successively to a series of resonators of different sizes. It is found that any particular note excites a *number of such resonators* and that the magnitude of excitation is *different in different resonators*; the series having the lowest frequency gives out always the maximum resonance. From this it can be concluded that a musical note is a *mixture* of several simple tones. Of these the one having the lowest frequency and known as the fundamental is *most intense* and defines the pitch of the note. The other tones or overtones, which also are present, *vary in number and relative intensities*. The difference in quality of notes in due to the presence of the overtones in unequal numbers and intensities.

Physiological Effect of Overtones—Helmholtz made a series of observations regarding the effect of overtones on the sweetness of a sound. He found that a note, consisting of a fundamental and a first few relatively feebler harmonics *not exceeding* the sixth, is very aggreable to the ear. On the other hand, a harsh and metallic sound has got harmonics higher than the sixth which are relatively more intense.

Production of Overtones—To some extent the production of the overtones depends on the *nature of excitation* of the sounding body. Thus a stretched wire when plucked with the point of quill pen emits metallic sound, but when struck with a padded hammer it yields a soft and a more pleasant note. Also the *material and shape* of the vibrating body markedly influence the quality of note. Thus when sounded in air, a tuning fork emits a simple tone, but when placed upon a hollow box, gives out a fuller note which is due to the vibration of the box. The length of an organ pipe determines the pitch of the note; but its shape governs the quality. A number of organ pipes of different materials and shapes but of the same length emits quite distinguishable notes, although they are of same pitch.

Detection of Overtones—A series of resonators, having natural frequencies in a gradually ascending order, is arranged on a table. Each resonator is provided at its bottom with a collecting funnel fitted with a tube which leads to the ear of the observer. The source of sound is taken over the resonators, one after the other. The resonators, which 'speak', *i.e.*, respond to the composite note, are marked. The natural frequencis of the resonators can be determined from their dimensions. Thus the overtones present in the note can be detected.

59. **Projection of the Displacement Curve**—For the demonstration of the wave nature of musical notes or speeches, an apparatus, known as the **phonedeik** (Fig. 68) and constructed by Miller, is used. It consists of a conical horn or *mouth-piece* C. Approximately 0·0002 inch in thickness, a glass *diaphragm* D is stretched in a suitable holder across the narrow end of the horn. One end of a *thread* (or of a fine platinum wire) is attached to the centre of the diaphragm. The thread is coiled a few times over a small *pulley* P and is finally attached to a *spring* S. The pulley is mounted on a spindle, about

which it can rotate almost without friction. A very light and small mirror M is rigidly fixed to the spindle. A thin pencil of light from B passes through the convex lens L. On being reflected from the mirror M, it is made to converge on a sensitized film F, which can be drawn continuously in one direction by a clock-work arrangement.

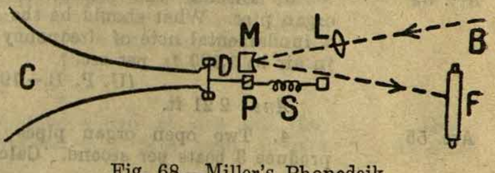


Fig. 68.—Miller's Phonedeik

Whenever a sound is made near the mouth-piece C, the waves move towards the narrower end and finally impinge upon the diaphragm. So the waves produced by the sound get more and more intensified. The motion of the diaphragm is communicated by the silk thread to the pulley which is made to oscillate either way through a small range proportional to the displacement of the diaphragm. Being rigidly connected to the spindle, the mirror makes a movement similar to the pulleys. A spot of light reflected from the mirror on the photographic film, further magnifies the oscillations of the mirror.

When no sound is made near C, the mirror is steady and on moving the film a straight band of light is observed. But when a source of sound is placed before it, the morror moves sideways with the frequency of the waves and the spot of light makes a periodic curve on a moving film. Figure 69 shows the nature of a sound curve on a moving film. For the recording of sounds there are also other mechanical and electrical devices ; but this method is considered to be the most efficient, because by this optical arrangement the movement of the diaphragm is magnified about 2500 times and gives a film record about 2½ inches wide.

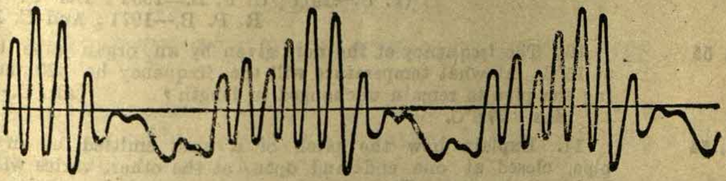


Fig. 69—A Sound curve on a film

EXERCISES ON CHAPTER VII

Reference

1. Describe, with a diagram, an open organ pipe, and explain how stationary waves are formed when it is excited. Arts, 55 & 56
(*Cf.* Gau. U.—1965, Pat. U.—1971)

2. The pitch of the fundamental note of an open organ pipe, 63 cm. long, is the same as that of a stretched wire, 20 cm. long vibrating transversely. If the mass of the wire be 0·1 gm. per cm., find the tension of the wire. Velocity of sound in air = 330 meters/sec.) Art. 55
(P. U.—1971)

Ans, 10^7 dynes,

Reference

Art. 55 3. Discuss the formation of nodes and loops in an open organ pipe. What should be the length of such a pipe to produce a fundamental note of frequency 256, of the velocity of the sound in air be 1,130 ft, per sec. ?

(U. P. B.—1963 ; Dac. U.—1961 ; All U.—1965)

Ans. 2 21 ft.

Art. 55 4. Two open organ pipes of lengths 50 cm. and 50·5 cm. produce 3 beats per second. Calculate the velocity of sound in air,

Ans. 330 m./sec, (P.U.—1971)

Arts, 54 & 55 5. The frequency of the fundamental of an open organ pipe is 128 c. p. s. What are frequencies of their first three overtones ?

Ans. 156, 384, 512. (Gau. U.—1961)

Art 55 6. You are to prepare an open organ pipe, having its fundamental frequency 240, What is the length of the pipe so prepared ? (Velocity of sound in air at N.T.P. = 330 metres/sec.) (P. U —1962)

Ans. 68·75 cm.

Art. 56 7. If the velocity of sound in air at 0°C is 332 metres per second, find the shortest length of the tube, open at both ends, that will be thrown into resonant vibration by a tuning fork of frequency 256, when the temperature of air is 50°C. (Raj U.—1972)

Ans. 7·078 cm. (nearly).

Art. 55 8. Describe the modes of vibration of the air column in a close pipe. (Dac. U.—1963 ; Utk. U.—1962 ; And. U.—1970 *Cf.* Gau. U.—1963 ; Del, H. S. 1972)

Arts. 53 & 54 9. Describe the possible modes of vibration of air column in a closed and open organ pipes. Show that the frequencies of overtones are related to that of the fundamental note.

(P. U.—1971 ; U. P. B.—1964 ; Del U.—1972 ; R. P. B.—1971 ; And U. 1964.)

Art. 55 10. The frequency of the note given by an organ pipe is 312 at 15°C. At what temperature will the frequency be 320 supposing the pipe to remain unchanged in length ? (All. U.—1963)

Ans. 29·6°C.

Art. 54 11. Explain how the pitch of a note emitted by an organ pipe, closed at one end and open at the other, varies with the length of the pipe. Find also the frequencies of harmonics given by such a pipe, when that of fundamental is 256 vibrations per sec.

Ans. Any odd multiple of 256. (U. P. B.—1974)

Arts. 54 & 55 12. Explain the formation of nodes and antinodes in closed pipe. How would you detect them experimentally.

(Del. U.—1969 ; P. U.—1966)

Art. 55 13. Describe how you would proceed to find the velocity of sound in air with the help of a tuning fork of known pitch and a tube of a adjustable length which could be closed at one end.

(Dac. U —1966 '68 ; P. U.—1968 ; Utk. U.—1956, '68, '73 ; Anna. U.—1970 ; Pat. U —1971, '53 ; And. U.—1971)

Art. 56 14. Describe briefly how the frequency of a tuning fork is measured by the resonance of an air column.

What is the state of air in an open pipe when the first harmonic is produced in it ?

Art. 56 15. Describe an accurate laboratory method of determining the velocity of sound in air. Deduce the formula used.

(Bom. U.—1960 ; E. P. U.—1962, '73 ; Del. U.—1973)

Reference

16. A fork of frequency 250 is held over a tube and maximum resonance is obtained when the column of air is 31 cm. or 97 cm. long. Determine the end-correction and the velocity of sound. (U. P. B.—1962) Art. 56

Ans. 2 cms.; 330 m./sec.

17. The second overtone of a pipe closed at one end, has the same frequency as the third overtone of a pipe open at both ends. Arts, 54 & 55

Neglecting end effects, compare the lengths of the pipes. (Gau. U.—1968)

Ans. 5 : 8.

18. Explain the resonance air column and describe how you will use it for finding the velocity of sound in air. Art. 66

(Dac. U.—1961; Del. H. S.—1958, '69, '71;
Del. U.—1960, '71; P. U.—1966)

19. How can the existance of nodes and antinodes in a sounding organ pipe be demonstrated. Art. 55

An open organ pipe emits its fundamental note. Find its length, if the velocity of sound in air is 330 metres per sec. and it vibrates in unison with a violin string of length 3·3 cm. under a stretching force of $7{\cdot}89\times10^{3}$ dynes, the mass of the string per centimetre 0·0012 gm. (C. U.—1957)

Ans. 1·34 metres.

CHAPTER VIII

VIBRATION OF RODS PLATES AND MEMBRANES

***60. Longitudinal Vibration of Rods**—When a rod is firmly clamped at its middle point and is rubbed length-wise with a piece of linen or soft resined leather, it gives out a shrill note. This is due to the longitudinal stationary vibration of the rod. The rod is alternately elongated and compressed in its course of vibration, producing nodes and antinodes at a number of points within its body.

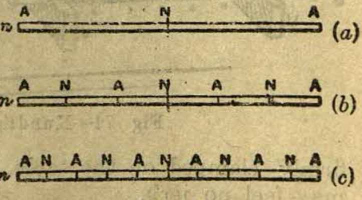


Fig. 70

The free ends of the rod have got the maximum vibration and are always the seats of antinodes. The middle part, being clamped, is fixed and has got a node. Therefore the simplest possible vibration occurs when there are two antinodes at the ends together with a node at the middle. The wave-length of the compressional wave within the material of the rod is consequently equal to twice the length of the rod. So the entire length of the rod is equal to half the wave-length of the disturbance travelling through it (Fig. 70*a*).

To find the velocity of sound through the rod, the pitch of the note emitted by the rod should be determined by comparison with a sonometer wire. Let the frequency be n.

Then
$$V = n\lambda = n \times 2l.$$

From this V can be experimentally determined. But the pitch of the note is very high. So it is extremely difficult to find its frequency by a method of comparison For determining frequency of vibration of the rod there is however, an indirect method, which is as follows. We know that

$$V = \sqrt{\frac{Y}{\rho}} = n \times 2l,$$

$$\text{or} \qquad n = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}. \qquad \cdots \quad (60, 1)$$

where Y stands for the Young's modulus and ρ the density of the material of the rod. On knowing the Young's modulus of the material and the density of the material, and by measuring the length of the rod, the frequency of vibration may be found.

For example, if the rod is brass of length 2 metres, then Young's modulus of brass 10×10^{11} dynes/cm^2 and density is 8·5 gm./c.c. Substituting such values in the above equation, we get,

$$n = \frac{1}{400}\sqrt{\frac{10 \times 10^{11}}{8{\cdot}5}} = 8{\cdot}57 \times 10^{2} \text{ cycles per second.}$$

61. Kundt's Experiment—To study the velocities of sound in different materials Kundt devised an experimental method. The apparatus, called Kundt's tube, consists of a metal rod AB (Fig. 71) of uniform circular cross-section and of a metre or two in length. It is clamped at the middle and carries a small circular cork or cardboard disc D at one end. The disc and small length of the rod are so inserted a few centimetres within a glass tube G as not to touch its wall. The other end of the glass tube is closed by a movable piston R. The whole system is firmly clamped on the table, such that during experiment the tube may feel no jerk.

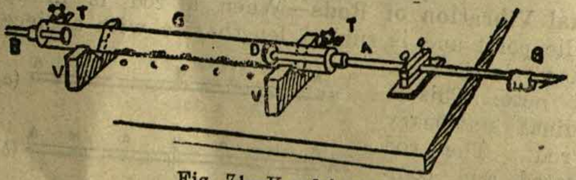


Fig. 71—Kundt's tube

To start the experiment the tube is taken out and is thoroughly dried by passing a current of hot air from a blower. It is then fixed in position and a thin layer of dried cork dust (or lycopodium powder) is evenly distributed into the tube in a column of a few millimetres wide running up to the disc. On now rubbing the rod lengthwise with a piece of resined leather, a stationary vibration is set up in the rod and the disc D vibrates backwards and forwards with the frequency of the note emitted. The air inside the glass tube is thrown into forced vibration. By reflection from the piston head R, stationary waves also are formed. During stationary vibration the movement of the air molecules sets the cork powder into violent agitation. In order to secure the best possible effect, the point R is slowly adjusted, while the rod is being excited by rubbing.

Under the steady stationary vibration the powder in the glass tube is seen to be violently agitated and thrown away from certain regions and collected in heaps at others. At the antinodal regions the powder thins out, since the vibrations at those places are maximum. At the nodes the powder collects in heaps. Varying with the length of the tube between the disc and the piston head, there are in general five or six such heaps. The average distance between two consecutive heaps is equal to half the wave-length in air.

Let the mean length of each loop be l, the length of the rod be L and the velocities of sound in air and in the rod be V_a and V_r respectively.

Then $V_a = 2nl$ and $V_r = 2nL$

$$\therefore \quad \frac{V_r}{V_a} = \frac{L}{l} = \frac{\text{Length of the rod}}{\text{Length of the loop}}. \qquad \ldots \quad (61,\ 1)$$

Thus the velocity of sound in either air or rod medium being known, that in the other can be found. Or, even the frequency of vibration of the rod being known, V_a or V_n can be indirectly determined.

The experiment also offers a possibility of measuring γ which is the ratio of the specific heats of a gas. Put the gas inside the Kundt's tube at a known pressure P. The method already described so as to find the velocity of sound in the gas. Let the velocity be V. Then by any method find the density ρ of the gas at the temperature and pressure of experiment.

$$\text{Then,} \quad V = \sqrt{\frac{\gamma P}{\rho}}, \quad \text{or} \quad \gamma = \frac{V^2 \rho}{P} \qquad \ldots \quad (61,\ 2)$$

Thus V, P and ρ being known, γ may be determined.

To compare the velocities of sound in the gases, fill the gas tube with one gas and repeat the experiment as described. Find the average distance between the heaps. Call it l_1. Repeat the experiment with the other gas and find the length between the heaps in it. Denote it by l_2.

$$\text{Then} \quad \frac{V_1}{V_2} = \frac{n.2l_1}{n.2l_2} = \frac{l_1}{l_2}. \qquad \ldots \quad (61,\ 3)$$

V_1 and V_2 being the velocities of sound in the two gases respectively. This ratio gives the desired comparison.

Examples :

1. A glass rod, 2·5 feet long, is clamped at the middle. It is set into longitudinal vibration. What is the velocity of sound in glass, if the freqency is 2590 per second ?

Ans. Since the rod is clamped at the middle, the free ends of the rod in course of vibration form antinodes. Hence if the desired velocity be V, then from the formula $V = 2nl$ we find that

$$V = (2 \times 2590 \times 2{\cdot}5) \text{ ft./sec.} = 12950 \text{ ft. sec.}$$

2. A rod, 140 cms. long and clamped in the middle, is in resonance with an air column, 50 cms. long. When the rod is set into longitudinal vibrations, the powder in the tube arranges itself in a loops. Find the velocity of sound in the rod, the velocity of sound in air being 330 meters per second. [Guj. U.—1965)

Ans. In a length of 50 cms. of air there are 5 loops. Hence the length of each loop = 10 cms.

Now since the velocity in air is given to be 330 m./sec., we have

$2n \times 10 = 330$, the symbol n carrying the usual significance,

$$\text{or } n = \frac{33}{3}$$

Hence the required velocity in the rod $= (2 \times n \times 140)$ m./sec. $= (33 \times 140)$ m./sec. 4620 m./sec.

*6?. Transverse Vibration of Rods

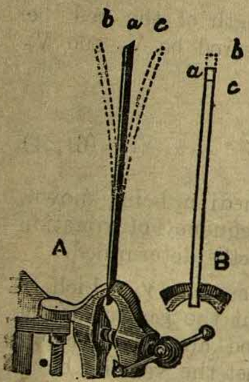


Fig. 72

Transverse Vibration of Rods—Clamp a uniform rod of any material having a rectangular cross-section at its lower end in a vertical position (Fig. 72). When the rod is drawn aside and then released, it begins to vibrate sideways with the fixed end as a node and the free end as antinode. This sort of vibration of the rod at right angles to its length is known as the **transverse vibration** of the rod. The frequency of such a vibration depends upon many factors, *viz.*, the elasticity and density of the material, the length of the bar and the thickness in the plane of vibration. Moreover, the bar can vibrate in various segments giving out very feeble overtones. These tones are non-harmonic and are not in the ratio of exact natural numbers unlike the case with a string.

It can be shown that the ratio of the vibration frequency of the first over-
tone and that of the fundamental is *approximately* 6, so that if a rod vibrating
transversely gives out a fundamental tone of frequency n, the first overtone of
frequency (approximately $6n$) will be feebly present in it. Still higher overtones
are almost undetectable.

36. Vibration of a Tuning Fork

36. Vibration of a Tuning Fork—A tuning fork is a uniform
U-shaped steel bar of rectangular cross-section with a stem attached
at the bend. The vibration of the two prongs of such a fork can be
studied by considering the successive stages of vibrations of a bar
which is gradually bent to its usual shapes. Figure 73,I shows a
straight piece of a bar vibrating with the two nodes equidistant from
the middle. As it is bent slowly in the middle, the nodes come
closer together (Fig 73, II-IV) until when it takes the usual shape
of a tuning fork (Fig. 73, V), the nodes are very close to each other
at the bend.

The vibration of two prongs at this stage is analogous to the vibration of two similar bars clamped at their lower extremities, Although the attachment of the stem, as in a turning fork, considerably modifies the problem, the frequency with which each limb vibrates, obeys the same law as that of a bar clamped at one end, *viz.*, that the frequency of the fundamental tone varies inversely as the square of the length and directly as the thickness in the plane of vibration. It does not *sensibly* depend upon the width of the prongs perpendicular to the direction of vibration. For this reason *tuning forks of low frequencies are thin and long, while those of high frequencies are very stout and short*. The two prongs of the fork should be exactly alike.

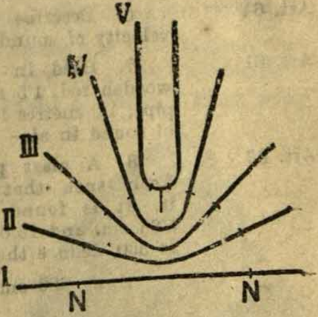


Fig. 73

Since the high frequency overtone of bar, clamped at one end, is extremely feeble, *the tone given out by a fork*, unless violently struck, consists entirely of the fundamental and is, therefore, the *purest possible* one. The vibration of an ordinary tuning fork is simple harmonic in nature and therefore approaches the ideal sine curve. The frequency is not appreciably affected by a moderate change of temperature or the magnetisation of the steel. For this reason a *tuning fork is used as a standard of frequency source.*

***64. Vibration of Membranes and Diaphragms**—The membranes have definite surface area. The frequency of vibration depends upon the shape, the restoring force, thickness and the density of the material of the membrane. The note emitted by the natural vibration of a plate or a membrane is not musical. A stretched circular membrane, when loaded at its centre, produces a partially agreeable note. A familiar example of this is the Indian *Tabla*.

From what has been stated above it may be supposed that a diaphragm gives out a non-musical sound. But the gramophone sound box (Vide Chap. IX) or the modern form of loud-speakers (Vide 'Electricity') used in radio or talkie machines are all provided with some sort of diaphragms, the forced vibration of which produces music.

EXERCISES ON CHAPTER VIII

Reference

1. Explain fully Kundt's Tube method of finding velocity of sound in air. (Raj. U.—1970; R. P. B.—1971; Del. U.—1969; And U.—1962; Guj U.—1965; P. U.—1963, '67) Art. 61

2 Give briefly the construction and mode of action of a Kund's Tube. Show how it may be used to compare the velocity of sound in a gas with that in air, (Bom. U.—1965; P. U.—1966) Art. 61

3. The distance between consecutive heaps in a Kundt's Tube experiment is 12 cms. and the length of the rod is 120 cms. What is the velocity of sound in the material of the rod, the velocity in air being 330 metres per second? Art. 61

Ans. 3300 m./sec.

4. How would you determine experimentally with a Kundt's Tube the velocity of sound in (a) solid; (b) air? (E. P. U.—1970) Art. 61

Reference

Art. 61 5. Describe the working of the Kundt's Tube and indicate its various applications. (Nag. U.—1973)

Art. 61 6. Describe with a neat sketch, some method of measuring velocity of sound in glass rod. (U. P. B.—1970)

Art. 61 7. Held in the middle and set in vibration longitudinally, a wooden rod, 1·5 metres long, gives the same note as an open organ pipe, 1·3 metres long, Calculate the velocity of sound in rod. (Vel. of sound in air—330 m./sec.)

Art. 63 8. A glass plate with its surface smoked is allowed to fall freely such that a vibrating tuning fork traces a wavy curve on it, It is fonnd that two consecutive groups of 30 waves occupy 5·88 cm. and 15·68 cm, respectively. If the frequency of the fork is 300, deduce the value of g, the acceleration due to gravity.

Ans. 980 cms./sec.^{2}

CHAPTER IX

ACOUSTICAL TECHNOLOGY

65. Recording and Reproduction of Sound—The first attempt at a mechanical recording of speech and music may be traced back as early as 1864 when Scott and Koening invented the *Phonautograph.* Thirteen years later, an improved model, known as *Phonograph* was devised by Edison. This could *record as well as reproduce* sound. Later on, remarkable improvements in the design of the instrument were made by Bell, Tainter and Berliner. In recent years the electro-optical method of recording and reproduction has come into existence due to the efforts of Rankine, Case and many other scientists.

Phonograph—The apparatus consists of a metallic ring S (Fig. 74), which is provided at this lower end with a very thin glass or mica diaphragm uniformly stretched in suitable holders. This is called the *sound-box.* A sharp metal style is fixed rigidly to the centre of the diaphragm. The sharp end of the style touches the surface of a cylinder D which can be revolved about its axis by a handle K or by a clock-work arrangement. The cylinder is mounted upon a screw A, which gives it a continuous lateral motion as it revolves. In order to allow a uniform rotation the drum is provided with a heavy fly-wheel F. For purposes of magnification of sound there is a horn H which can be screwed to the sound-box S.

Principle of Recording—The source, whose sound is to be recorded, is placed before the horn. The air pulsations, produced by the source, travel down the horn and being thus intensified, strike against the diaphragm. The diaphragm is, therefore, thrown into a forced vibration which is exactly synchronous with the source. The style is consequently made to move up and down against the cylinder which is wrapped up with a sheet of soft tinfoil or a layer of solid wax.

When the cylinder is rotated at a uniform speed, a spiral trace of varying depths due to the up-and-down movements of the style is engraved on its surface. In this way curves due to sound waves are engraved on the cylinder, which we might call the *original record.*

Principle of Reproduction—In order to reproduce the sound, the sharp style attached to the sound-box is replaced by a rounded sapphire point. On allowing this point to travel a second time over the same groove, the diaphragm is made to follow the same sequence of vibrations and the original sound is reproduced in the same order in which it was produced.

Drawbacks of a Phonograph—The following are the chief *defects* of a phonograph.

(*i*) The materials, with which records are made, are soft and consequently the impressions made by the engraving style rapidly deteriorate.

(*ii*) The groove. through which the reproducing pin run, can move the diaphragm in one direction only, *viz.*, *upwards.* The reverse motion had to be reproduced by the elasticity of the diaphragm. Thus it presses somewhat heavily upon wax to enable it to follow the rapid vibration.

(*iii*) The quality of tone in reproduction is poor.

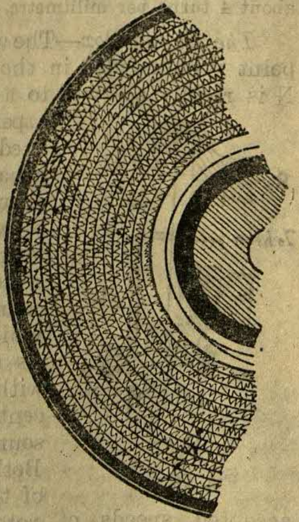


Fig. 75—Disc Record

66. Gramophone—The defects, stated above, were largely rectified by Berliner by making improvements on sound-box and also making the records in the form of flat discs in which spiral grooves are cut from the rim to the centre. Instead of being of varying depths, these grooves contain small transversely etched periodic curves of varying width, representing speech of music with approximately uniform depth (Fig. 75). Thus if a needle point is placed in the groove, it would oscillate sideway, along with the rotation of the record the movement being imparted by the transverse curves in the groove. The original form of

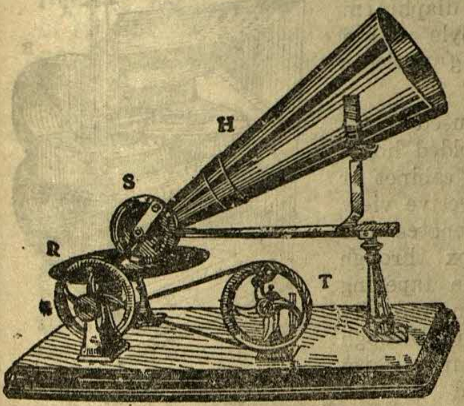


Fig. 76 -Original form of Gramophone

gramophone is shown in figure 76. The disc record is rotated by the handle T.

The record, on which the first impression is made, is composed of a mixture of wax. Then an electroplate of it is made on a metal plate, known as the *parent record*. The records, that are available in the market, are made of a composition of some inert powder with shellac or some similar material as the binder. On heating this composition, it assumes a soft state. It is then rolled, cut into usual shape and then pressed upon the electroplate. The spiral trace is about 4 turns per millimetre.

The Sound-Box—The sound box is provided with a rounded needle point which slides in the groove of the record (Fig. 77). The needle N is rigidly screwed to a lever system L with its fulcrum hinged on a pair of V's. The other end of the lever is fixed to the centre of the diaphragm. Made usually of mica, the diaphragm M, which is of about 4·5 cm. to 6 cm. in diameter, is mounted between rubber rings R, R called *gaskets* and forms the front of a small cylindrical box of some metal called the **sound-box.** This is connected to a conical pipe called the *tone arm* which can move freely, on a vertical axis. As the record revolves, the tone arm together with the sound-box gradually moves to the centre of the disc along the groove. The sound is magnified finally through a horn. Both in recording and reproduction the speed of the disc is maintained uniform. The various accepted speeds of rotation of the records are 78, 45, 33 and 25 revolutions per minute. The lower speed records are popularly called the *long playing* records.

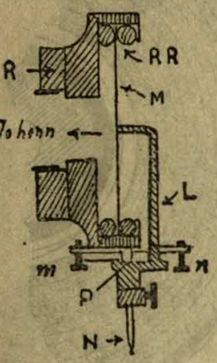


Fig. 77

The Recording Style—A sharp metal style is pivoted place of the needle. When under the impulse of the incoming sound waves the diaphragm moves sideways moving the style within blank grooves producing zigzag periodic curves.

The gramophone machines used now-a-days are provided with folded horns which are fitted within the cabinet or box. Figure 78 gives a perspective view of the nature of horn. 'A' denotes the pipe connected to the sound-box through the tone arm and forms the tapering end of the horn. The tube then gradually widens and enters into the horn through the two openings. It is a compact form of a horn, which is folded in the peculiar way.

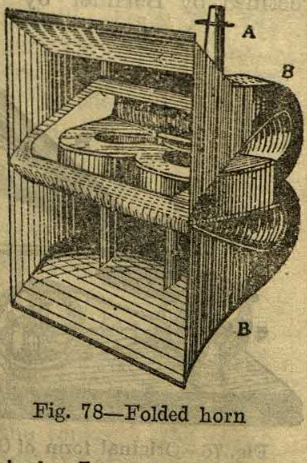


Fig. 78—Folded horn

The modern method of recording is electrical. Instead of having a mechanical sound-box the reproducing machine is sometimes provided with an electrical *pick-up* which produces a modulated induced current as the 'pick up' runs through the groove. This feeble modulating current finally passes through a loud-speaker, before it gets amplified through a combination of radio valves or transistors.

67. **Hot-wire Microphone**—This is an apparatus for detecting electrically a periodic disturbance in air. It is very sensitive and is able to receive both continuous and discontinuous sound waves. For this reason such microphones are largely employed for sound ranging purposes. It essentially consists of a fine wire of platinum stretched across the mouth of a hollow air vessel. The ends of the wire terminate in binding screws S, S and are carefully insulated from the body of the vessel.

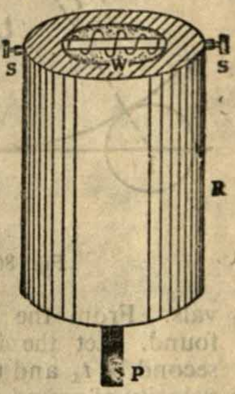


Fig. 79

Whenever a series of air waves due to the sound from a source falls upon the vessel, an antinode is formed at its neck. The vibration at this region is the greatest, when the air vessel is tuned to the wave frequency. Due to the vibration of air there is a cooling of the platinum wire resulting in its change of resistance. If an electric current is previously established in the wire, then with sound waves falling on the microphone, there would be periodic variation in current passing through the circuit and a galvanometer placed in the circuit would be similarly affected. The oscillation of the spot of light reflected from the mirror of the galvanometer is received on a moving photographic film. In order to have a very sensitive reception, the vessel should be approximately tuned to the frequency of the incoming waves. The principle of such a microphone has been developed mainly by Tucker and Paris.

*68. **Sound-Ranging**—The location of the hostile guns behind trenches became matter of absolute necessity during the Great War of 1914-18 and a special device known as **Sound-Ranging** was adopted on war fields. The principle upon which this method depends, is given below :

A number of sound-detecting stations, each provided with a hot-wire microphone, is selected on a base line at a distance of four to five miles from the enemy front line. Usually five or six in number, these are separated from each other by a few hundred years. These microphones are electrically connected to a central station, where the reception of a sound wave in any of these microphones due to a gun-fire is automatically recorded on the same photographic film. Nearly a mile ahead of the base line there is an observation post, from which by turning a switch the electrical connections between the central and the detecting stations can be established.

Whenever the sentry, placed at the observation post, receives any sound of explosion, he puts on the switch with a view to operate all the microphones. As the explosion wave sweeps over the stations in successions, momentary impulses of air fall upon the microphones, one after the other. Thus a short wavy line corresponding to each microphone is recorded on the photographic film which is moved at a known constant speed. Since some time elapses between the receptions of the sound at different stations, waves are traced one behind the other at some fixed distances on the photograpic film, depending on the corresponding intervals. From the rate of motion of the film these intervals can be found. Let the interval between receptions at the first station and second be t_1 and that between the first and the third be t_2. Let the velocity of sound of the explosion be V.

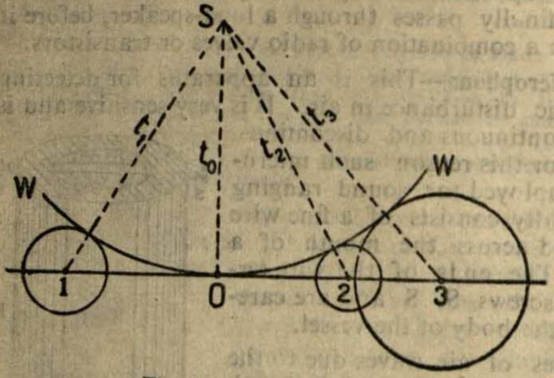


Fig. 80—Sound-Ranging

The position of the source sound is geometrically located by the following construction. Adopting a suitable scale, arrange the system of stations 1, 2, 3, etc., on a piece of paper (Fig 80). With 1 as centre and Vt_1 as radius on the same scale draw a circle. Again with 2 as centre and Vt_2 as radius draw another circle. Locate the centre S of the circle that will pass through O and touch these two circles. The centre S is the *position of the source.* The circle, which has been found by trial, evidently represents the wave front when the first station O has received the report. At this instant the wave front lies at distances of Vt_1 and Vt_2 from the two stations marked 1 and 2 respectively.

69. Velocity of Sound from a Projectile—So long as the velocity of the sounding body is less than that of the sound produced, the number of waves per second from it reaching an observer would be governed by the Doppler's principle. But when the projectile is a high speed bullet shot out from a rifle, its vilocity exceeds that of sound. At

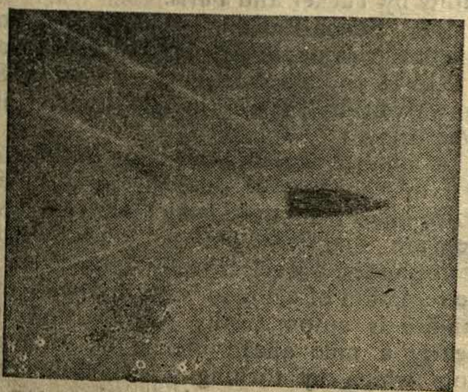


Fig. 81.—Photograph of moving Bullet

every point in its path of flight, the bullet generates spheres of compression which expand with the velocity of sound, while it moves faster than the rate at which these spheres are growing. As a result of this, it produces a conical wave front which at any assigned instant lies along the tangent planes of these spheres with the bullet at the apex (Fig. 81). An apparatus for taking photograph of the condensation waves in air due to a flying bullet has been designed by Prof. R. W. Wood. A similar type of wave is generated on the surface of water, when a steamship makes its way through a calm sea or even when a pointed end of a stick is drawn quickly along the surface of still water.

Just after the War, a thorough investigation regarding the explosive sounds of shells has been made by Esclangon, Villard and others. They hold that there are two distinct types of waves associated with an explosion. One has very low frequency below the audible limit and is described by Esclangon *infra-sounds* or **infra-sonics** ; the other is the ordinary audible sound. Both these waves can be detected by hot-wire microphone.

***70 Hydrophone**—It is an under-water microphone receiver and is used for finding the direction of a source of sound in each depth-sounding and also for sound-ranging purposes. It consists of a heavy metallic circular ring A to which a metal diaphragm D is rigidly attached. Figure 82 shows a hydrophone in cross-section, and figure 83 its perspective view. The centre of a metal diaphragm is connected to a movable metal plate in a box containing carbon granules C, the combination forming a microphone receiver. Wire from a cable are connected separately to the diaphragm and the other end of the box. The cable is finally connected to a telephone receiver and the battery of cells in series. In order to cut off any reception of sound from the back side of the microphone box which is provided with a screen called the *baffle* P (Fig. 83).

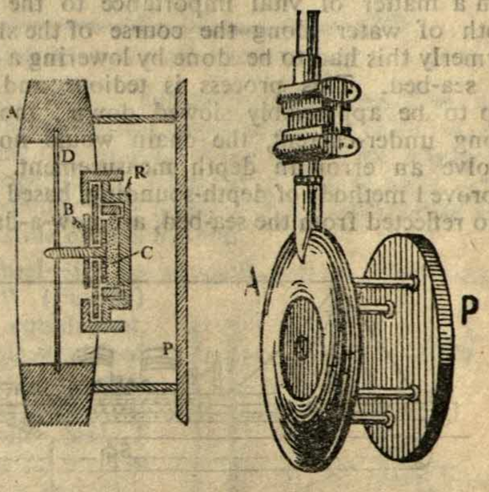


Fig. 82 Fig. 83

Hydrophone

When a compressional wave-motion passing through water impinges normally on the diaphragm, the latter is thrown into a forced vibration. Any movement of the diaphragm produces a fluctuation of resistance of carbon granules in the microphone box producing a corresponding modulation of electrical current througn the telephone receiver. (Vide VOLTAIC ELECTRICITY).

The telephone is applied to the ear of the operator. The hydrophone is then gradually lowered into water by means of the cable attached to it. It is then gradually rotated in all possible directions until the maximum sound is heard. In this position of the microphone, the source is in the direction normal to the diaphragm. The directional property of reception acquired by the hydrophone is due to the introduction of the heavy metal annulus. When the direction of propagation of the waves is such that they normally impinge upon the baffle, no sound is heard in the phone. The accounts for its nomenclature *deaf side*.

By means of two or more microphones, commonly called the *sentry microphones* and placed at a definite distance along the bed of the sea, the method of sound-ranging is often employed to detect the position of a *submarine*, the source of sound in this case being the rotating propellers of the submarine. Hydrophones are also used to locate submerged objects and icebergs by the process of an echo depth-sounding.

71. Echo Depth-Sounding—The measurement of the depth of the sea is called the *depth-sounding*. From very early ages it has been a matter of vital importance to the sailors to measure the depth of water along the course of the ship for a safe navigation. Formerly this had to be done by lowering a *lead line* till it touched the sea-bed. This process is tedious and requires the speed of the ship to be appreciably slowed down ; moreover, when there is a strong under-current, the chain would not go vertically down and involve an error in depth measurement. For this reason, various improved methods of depth-sounding, based upon the reception of an echo reflected from the sea-bed, are now-a-days employed.

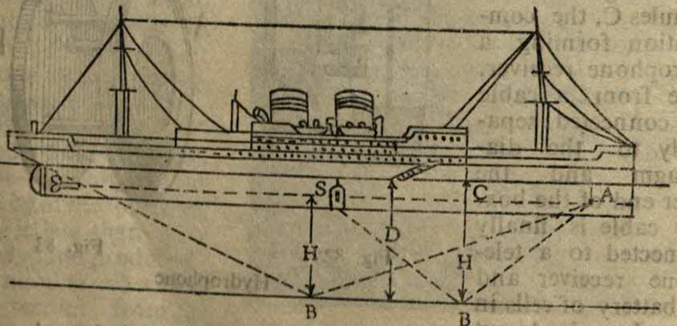


Fig. 84 Echo Depth-Sounding

In one system of measurement the source of sound, producing a very short train of a high frequency waves, is placed at the bottom of the moving ship at the region S in question (Fig. 84). The train of waves travels out in all directions in water. The hydrophone is placed in water at the bottom of the ship at S. When any sound is produced, a sound beam travels from S to all directions in water. A sound beam is reflected from the bottom B of the sea in a direction BA and would actuate the hydrophone. When it has moved to the position A let the angle, at which it receives the sound be ϕ. If the distance between the stern and the keel be x and the depth of the ship's bottom from the water surface be D, then the depth of water D is obtainable from the relation $D=C+\frac{1}{2}x \tan \phi$. Specially, when the depth of the sea is large, the propeller of the ship may as well be chosen as the source of sound.

Utilising the echo depth-sounding method, the distance of a moving submarine under water can be determined from another ship not very far from it. The first procedure is to determine the depth of the sea by the depth-sounding method as given above. Suppose the ship at the middle is to determine the distance R_1 of the submarine in its front (Fig. 85). On being reflected from the sea-bed the sound of the propeller of the submarine is received by the hydrophone of the ship ot an angle θ_1, say. Then from the properties of triangles it can be shown that $R_1=2D \cot \theta_1$. In a similar way location of a submarine in any other direction is also possible by this method.

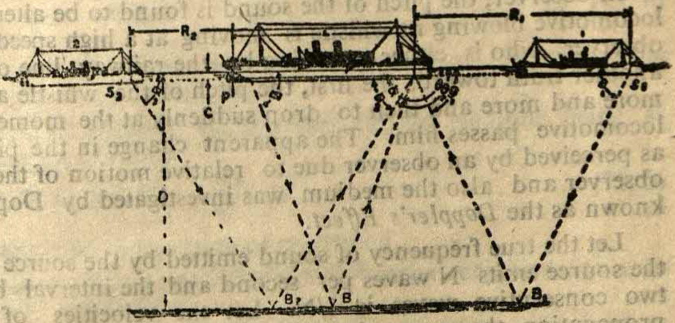


Fig. 85—Location by Echo Method

*72. **Stroboscopic Wheel**—In its simplest form a stroboscopic wheel consists of a wheel W (Fig. 86) having a number of equidistant, radial and rectangular slots, arranged symmetrically on the rim of the wheel which can be mechanically driven at a known speed. The tuning fork T under examination is placed on one side of the wheel in such a way that the plane of vibration of the prong is parallel to the plane of the wheel and the longer side is parallel to the longer axis of the slots. The tuning fork is run electrically and a strong light is focussed on the top of the prong facing the slots. The wheel is then gradually set to motion and observation is taken on the prong horizontally from the other side of the wheel.

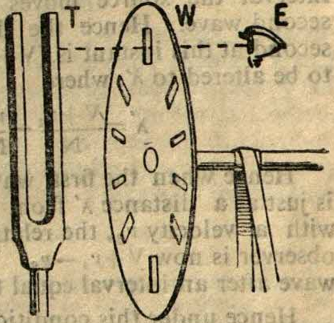


Fig. 86—Stroboscopic Wheel

When the speed is so increased that the interval, in which one slot is replaced by the next, becomes comparable with the period of the fork, the prong seems to be oscillating slowly. When the motion is such that interval is made exactly equal to the period of the fork the prong would appear to be stationary.

From the number of slots on the wheel and its rate of revolution, the interval between the appearance of the two consecutive slots at the same position can be calculated, and hence the frequency of the fork can be determind. Rayleigh devised an electrical arrangement for the automatic synchronous movement of such a wheel and it is known as the **phonic wheel.**

73. Doppler's Principle—If a sounding body is moving relatively to an observer, the pitch of the sound is found to be altered. Thus if a locomotive blowing its whistle is moving at a high speed towards an observer, who is either standing by the railway line or moving in another train towards the first, the pitch of the whistle appear to rise more and more and then to drop suddenly at the moment when the locomotive passes him. The apparent change in the pitch of sound as perceived by an observer due to relative motion of the source, the observer and also the medium was investigated by Doppler and is known as the *Doppler's Effect.*

Let the true frequency of sound emitted by the source be N. Then the source emits N waves per second and the interval between any two consecutive waves is 1/N. Let the velocities of the sound propagation, the source, the wind and the observer be denoted by V, ν, ν_1 and ν_2 respectively. Suppose that all are moving in the same directian ; (say, from the right to the left, so that the source moves towards the observer, the observer moves away from the source and the wind is blowing in the direction from the source to the observer.

Suppose that at any instant a wave starts from the source. The next wave is generated after an interval of 1/N sec. The apparent velocity of sound is now $V+\nu_1$. So during this interval the first wave moves over a distance equal to $(V+\nu_1)N$. But during the same interval the source moves through a distance ν/N and emits the second wave. Hence the distance between the first wave and the second at this instant is $(V+\nu_1)/N-\nu_1/N$. So the wave-length appears to be altered to λ', where

$$\lambda'=\frac{V+\nu_1}{N}-\frac{\nu}{N}=\frac{V+\nu_1-\nu}{N}. \qquad \ldots \quad (73,\ 1)$$

Hence when the first wave reaches the observer, the second wave is just at a distance λ' from him. As the observer is moving away with a velocity ν_2, the relative velocity between the sound and the observer is now $V+\nu_1-\nu_2$. So the observer will receive the second wave after an interval equal to $\lambda'/(V+\nu_1-\nu_2)$ second.

Hence under this condition, the number of waves N′ reaching the observer per second is given by

$$N'=\frac{V+\nu_1-\nu_2}{\lambda'}=\frac{V+\nu_1-\nu_2}{V+\nu_1-\nu}.\ N, \text{ by } (73,\ 1) \quad \ldots \quad (73,\ 2)$$

As the pitch of a note depends directly on the frequency of the source, the above expression shows that the pitch is altered from N to N′.

Now consider the following three cases :

(i) *Motion of the observer*, the medium and the source being at rest—In this case, $\nu=0$ and $\nu_1=0$.

From (72, 2) it follows $N'=\frac{V-\nu_2}{V}.\ N,$ ... (73, 3)

This shows that the pitch of the *note falls* as the observer *moves away* from the source.

If the observer moves *towards* the source, ν_2 should be taken negative, so that

$$N'=\frac{V+\nu_2}{V}.\ N, \qquad \text{... (73, 4)}$$

which shows that the pitch of the sound appears to *rise* as the observer moves *towards* the source.

(ii) *Motion of the source*, the observer and the medium being at rest—Here $\nu_1=0$ and $\nu_2=0$.

From (73, 2) we get $N'=\frac{V-\nu}{V}.\ N,$ ... (73, 5)

which indicate a *rise* in the pitch of the sound.

If the source moves *away* from the observer, negative sign is to be prefixed to ν. In that case,

$$N'=\frac{V}{V+\nu}.\ N, \qquad \text{... (73, 6)}$$

which indicates a *fall* in the pitch of the sound.

(iii) *Motion of the medium*—When the medium moves *in the direction* of the observer (the source and the observer being fixed), in equ. (73, 3) $\nu=0$ and $\nu_2=0$ and so

$$N'=\frac{V+\nu_1}{V+\nu_1}.\ N, \qquad \text{... (73, 7)}$$

or N′=N, *i.e*, there would be no change in pitch.

The Doppler's Effect was also found in the *astronomical observations* on steller bodies. When the relative velocity between a star and the earth is such that the two are approaching, the frequency of the particular radiation as given out by the star increases ; when, on the other hand, the two are receding, the frequency diminishes, This affords a means of studying spectroscopically whether during any particular period the distance between a star and the earth is diminishing or increasing.

Examples :

1. The engine of a railway train approaching an observer with a speed of 60 miles an hour emits a whistle which appears to have frequency of 501. What would be the frequoncy of the note heard when the train is moving away from the observer with the same speed ? (Velocity of sound in air =1120 ft./sec.)

Ans, Velocity of 60 miles/hour=88 ft./sec. Let the frequency of the whistle be n per sec. When, therefore, the engine is approaching the observer, then $\frac{1120}{1120+88}n=501$, whence $n=461·6$.

When the engine is receding from the observer, let the apparent frequency be n_2 per sec. Then

$$\frac{1120}{1120+88}n_2=461·6, \text{ whence } n_2=428.$$

2. Two express trains travelling at 60 miles per hour meet each other, when one sounds a whistle. If the frequency of the note emitted be 800 per second, find its apparent frequency to an observer in the other train (*a*) before the trains meet and (*b*) after they have passed each other. What will be the apparent frequency to a stationary observer near the line? (Given that the velocity of sound in air—1100 ft./second).

Ans. The velocity of the source $v=60$ miles/hour $=88$, ft./sec. Let the velocity of the observer be v ft./sec. The frequency (N′) of the source is 800 per sec. As there is no motion of the medium, let the apparent frequency as heard by the observer be N per sec. Thus,

(*a*) when the trains are approaching each other.

$$N'=\frac{1100+v}{1100-v}N \quad \therefore \quad N'=\frac{1100+88}{1100-81}\times 800=933·1\ ;$$

(*b*) when tke trains are receding from each other.

$$N'=\frac{1100-v}{1100+v}N \quad \therefore \quad N'=\frac{1100-88}{1100+88}\times 800=681·5\ ;$$

(*c*) when the observer is stationary and the trains are approaching,

$$N'=\frac{1100+88}{1100}\times 800=864\ ;$$

(*d*) when the observer is stationary and the train is receding,

$$N'=\frac{1100-8}{1100}\times 800=736.$$

EXERCISES ON CHAPTER IX

1. Describe a phonograph and explain its action.
(And. U.—1961 ; Dac. U.—1962 ; C. U.—1962)

2. Describe a gramophone. How is sound recorded and reproduced?
(B. H. U.—1973, P. U.—1978, '82 ;
(U. P. B.—1979 ; Nag. U.—1978)

3. Summarise your knowledge about a gramophone sound box.
(U. P. B.—1984)

4. Describe with a sectioral diagram an apparatus for receiving sound in water.

5. How can the depth of the sea be measured by acoustical method?
(Del. U —1982)

6. Describe a Stroboscopic Wheel. How can the frequency of a tuning fork be determined with it?
(Raj. U.—1983)

7. Explain how the pitch of a note is affected by the motion of the source and the observer.
(Nag. U.—1980 ; R.P.B.—1979)

8. What is Doppler's Principle? Explain it.
(W.B.S.B.—1981, '83 ; Nag. U.—1962)